INFLUENCE OF LOCAL EXTREMA ON THE EFFICIENCY OF GRADIENT ALGORITHMS FOR LASER BEAM CONTROL

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The control over high-power laser beam based on the aperture sounding algorithm and its modification is considered in this paper using methods of numerical experiment. A domain of the problem parameters is shown to exist for which local extrema appear in the space of control coordinates. A decrease in the efficiency of correction for the beam thermal blooming due to these local extrema is estimated. The method for seeking the global (basic) maximum is proposed.

1. INTRODUCTION

A comparison of algorithms for phase control over laser beams in a nonlinear medium carried out by many authors[1–3] shows that none of the algorithms known at present is free of serious drawbacks. Thus, for example, it is characteristic of the phase conjugation method that the correction becomes unstable as the radiation power increases.[3,4] The aperture sounding[5] has a higher stability and, perhaps, a faster performance. At the same time a decrease of the efficiency of this algorithm (and others based on the gradient method of seeking the efficiency function extremum) is observed if local extrema are present in the space of the beam control coordinates.[1] Up to now this problem has been investigated less thoroughly than it is needed. In particular, the domain of the problem parameters where local maxima appear is not determined, even approximate estimations of a decrease in the efficiency done. No possibilities to overcome the troubles due to such local extrema have been considered so far. These problems are discussed in the present paper.

The study is carried out based on the methods of numerical experiment. Analysis of the high-power beams became feasible owing to the increase in the power of modern computers and corresponding increase of the dimensionality of the calculation grids used.

2. NUMERICAL MODEL AND THE SOFTWARE VERSION OF THE MODEL

Beam propagation was considered in the approximation of stationary wind refraction for the case of a homogeneous medium. Under such conditions the complex amplitude of a field, E, can be described by the following system of equations[2]:

$$2ik\,\frac{\partial E}{\partial z} = \Delta_\perp E + 2\,\frac{k^2}{n_0}\,\frac{\partial n}{\partial T}\,TE; \qquad (1)$$

$$(\mathbf{V}_\nabla)\,T = \frac{\alpha}{\rho_0\,C_p}\,I, \qquad (2)$$

where k is the wave number, n_0 is the undisturbed magnitude of the refractive index n, z is the coordinate axis along which the beam propagates, T is the temperature of a medium, $\mathbf{V}$ is the wind velocity vector, α is the absorption coefficient; notations of other physical values are of common use.

The interaction between the beam and the medium is characterized by the dimensionless parameter

$$R_v = \frac{2\,k\,a_0^2\,\alpha I}{n_0\,\rho_0\,C_p\,V}\,\frac{\partial n}{\partial T},$$

that is proportional to the intensity I and to the initial radius of the beam a_0, and depends on other parameters of the medium and radiation.

Nonlinear distortions in the observation plane $z = z_0$ can be described using the following criterion:

$$J(t) = \frac{1}{P}\iint \exp\left(-(x^2+y^2)/r_a^2\right) I(x, y, z_0, t)\,\mathrm{d}x\,\mathrm{d}y; \qquad (3)$$

the function $J(t)$ is, in fact, the radiation power within a given aperture. In formula (3) r_a is the radius of the receiving aperture and P is the total power of the beam.

We have considered the correction for nonlinear distortions based on two algorithms. The aperture sounding algorithm[1] according to which the change of adaptive corrector control coordinates $\mathbf{F} = \{F_1, F_2, \ldots, F_N\}$ is performed according to the following formula:

$$\mathbf{F}(t) = \mathbf{F}(t - \tau_d) + \beta(t - \tau_d)\,\mathbf{grad}\,J(t - \tau_d), \qquad (4)$$

and a modified aperture sounding algorithm[6]

$$\mathbf{F}(t) = \mathbf{F}(t - \tau_d) + \beta(t - \tau_d)\,\mathrm{sign}\left(\frac{\Delta J(t - \tau_d)}{\Delta F_i(t - \tau_d)}\right), \qquad (5)$$

where F_i is the ith component of the vector of control coordinates (coefficients of Zernike polynomials or adaptive mirror drive displacements can be used as the components), $\beta(t - \tau_d)$ is the coefficient whose value decreases at the iterations that had led to a decrease in

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the control efficiency function (in our problems it is the criterion $J(t)$, formula (3)), sign is the function of taking a sign. The algorithms (4) and (5) are differ by that, according to expression (4), in the process of test variations the value and direction of the iteration step are determined, while according to the algorithm (5) only the motion direction (direction to an extremum) is sought, and the step value is completely specified by the coefficient $\beta(t - \tau_d)$.

The program that realizes the calculation scheme is developed using the methods of object-oriented programming (programming language C++). The main panel of the program interface is shown in Fig. 1 where the adaptive system is represented schematically, the basic input physical parameters of the problem are also presented in the block-diagram. Those are the path length, nonlinear layer length (its extension, in the general case, may be different from the path length), and the non-linearity parameter. Characteristics of the calculated model are also pointed out in the form of the dimensionality of the calculation grid and the number of phase screens along the beam propagation path. Thus, all data characterizing the variant of the calculation chosen are displayed on the main panel.

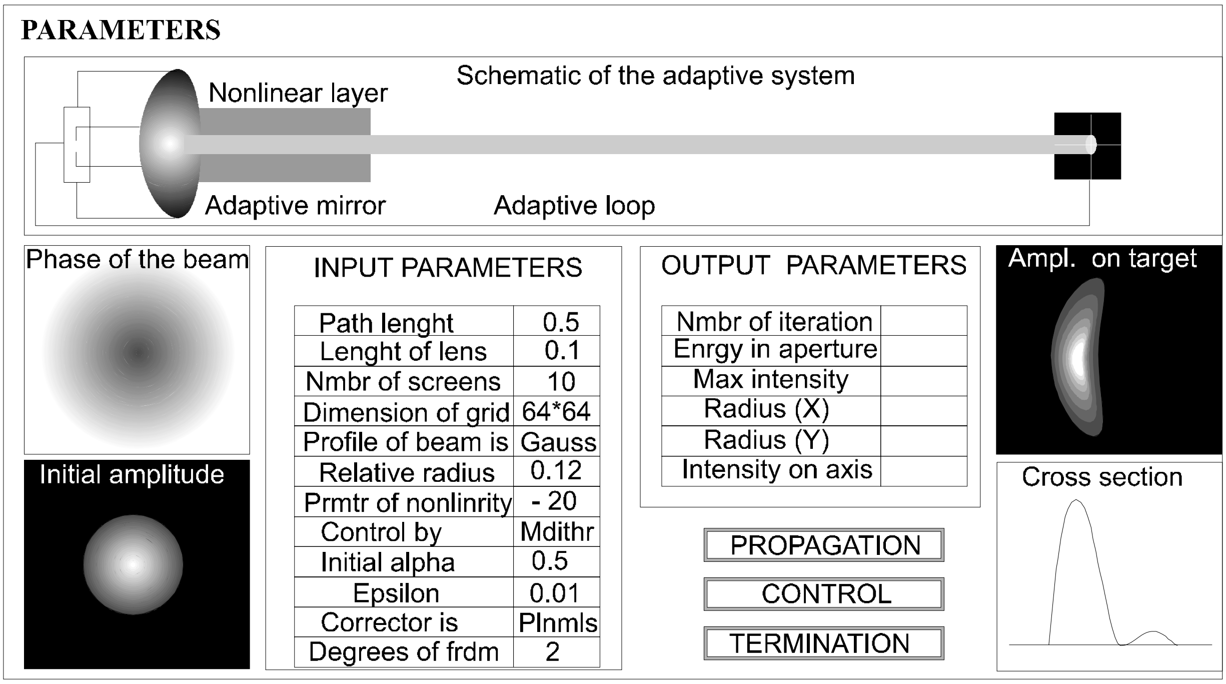

FIG. 1. Interface of the program developed for the numerical model used.

During the calculations the basic parameters of a beam in the observation plane are being displayed on the panel either. Those are the field distribution, criterion J (formula (3)), maximum intensity, energy radii of the beam along the axes that are perpendicular to the propagation direction.

The input parameters and the control algorithm are specified by the interface panels that are accessible through the program menu. Calculation results are stored in a file.

3. ACCURACY OF DETECTING THE PRINCIPAL MAXIMUM AGAINST THE BACKGROUND OF LOCAL EXTREMA

The aperture sounding algorithm (and its modification) can be considered as a gradient method of seeking of the efficiency function extremum. The extremum is being sought in the space of control coordinates F_i. Therefore, to reveal the algorithm peculiarities, in order to simplify the problem, it is possible to consider the procedure of seeking the global extremum against a relatively simple analytical function that has local maxima. The basic relationships obtained from simplified analysis can also be correct for more complicated problems of the adaptive optics.

We have considered a function of one variable which has a global maximum and two local ones disposed symmetrically relative to its global maximum. Regardless of local extrema present the algorithm could, in certain cases, identify the main peak. The relative width and height of the local extrema (normalized to the corresponding parameters of the global extremum) have been varied. That allowed us to determine the boundary of the domain where the algorithm always isolates the global maximum. For the algorithm (4) the results are presented in Fig. 2*a*. The region of the parameters, for which a global maximum is determined, is above the curve. That means that the algorithm easily “steps overB the narrow and high local extrema, but “stopsB on the relatively wide ones regardless that their heights can be low (in comparison with the height of a principal maximum). This is connected with that the iteration step is determined by the derivatives ($\Delta J/\Delta F_i$), that is, by the rate with which the function changes. So, if the derivative is sufficiently large (local extremum is “narrowB and “highB), then the algorithm “steps widelyB and “steps overB a hill.

As to the algorithm (5) the corresponding data are presented in Fig. 2*b*. In the algorithm considered the step value is determined as $\beta \cdot \mathrm{sign}(\Delta J/\Delta F_i)$, i.e., it does not depend on the value of the derivative, therefore the ability of the algorithm to “step overB an extremum or not is determined by the width of the maximum only while being independent of its height.

On the whole a conclusion can be drawn, in a one-dimensional case, that a choice of the algorithm is determined by the characteristics of the function under study (by the shape of local extrema).

For the sake of clarity in describing the “hillB and peculiarities of seeking the maximum, let us consider the control, in the problem of compensation for nonlinear distortions, to be performed in the space of two coordinates (slope and focusing). In this case the “hillB of the efficiency function is a distribution of the criterion J (formula (3)), each value of which corresponds to fixed values of the slope and focusing. As an example, that type of a “hillB is shown by the lines of equal level in Fig. 3. The results have been obtained for the case of stationary wind refraction, when no transient processes due to the interaction of the radiation with the medium are not taken into account. Distribution of the criterion J in the “slope–focusingB space has one extremum, the search trajectory of an adaptive system for which is shown by the dashed line (the aperture sounding algorithm).

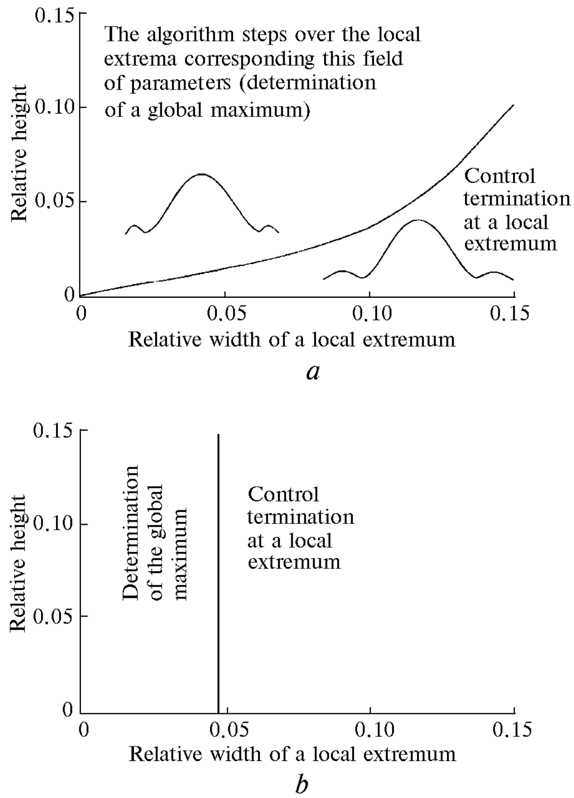

FIG. 2. The boundaries of the region where the determination of the global maximum in the presence of local ones is possible (as found for the function specified analytically): the algorithm (4), above the curve the control stops at the global extremum, below the curve the control stops at a local extremum (a); the algorithm (5), to the left of the straight line is the global extremum, to the right of the straight line is a local extremum (b).

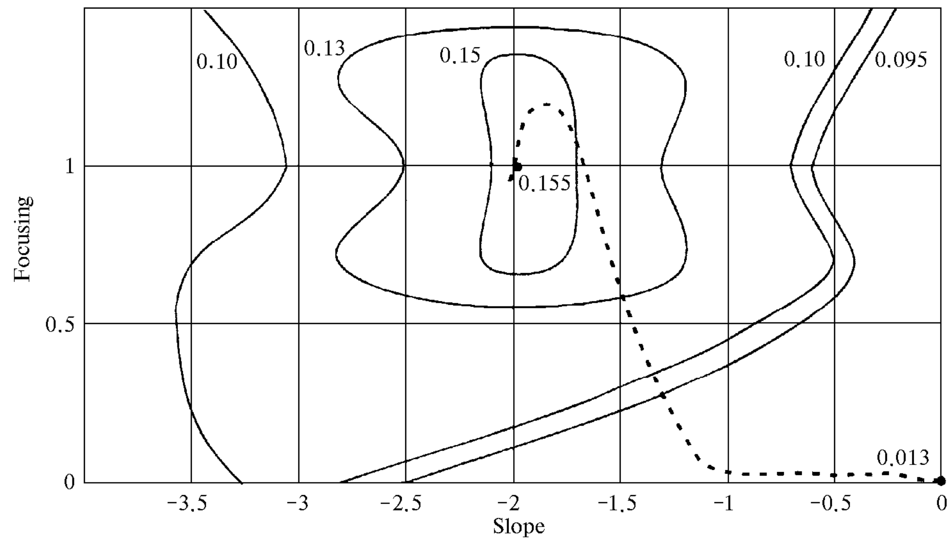

FIG. 3. The control efficiency function (the criterion J, expression (3)) in the space of “slope–focusing” coordinates which is represented by the lines of equal level (numbers are the values of J on a line). In the same figure a trajectory of motion to an extremum for the algorithm (4) is shown (dashed line). Parameters: $Z = 0.5\ Z_{\mathrm{d}}$, $Z_{\mathrm{nl}} = 0.1\ Z_{\mathrm{d}}$, $R_v = -70$, $r_{\mathrm{a}} = a_0/4$.

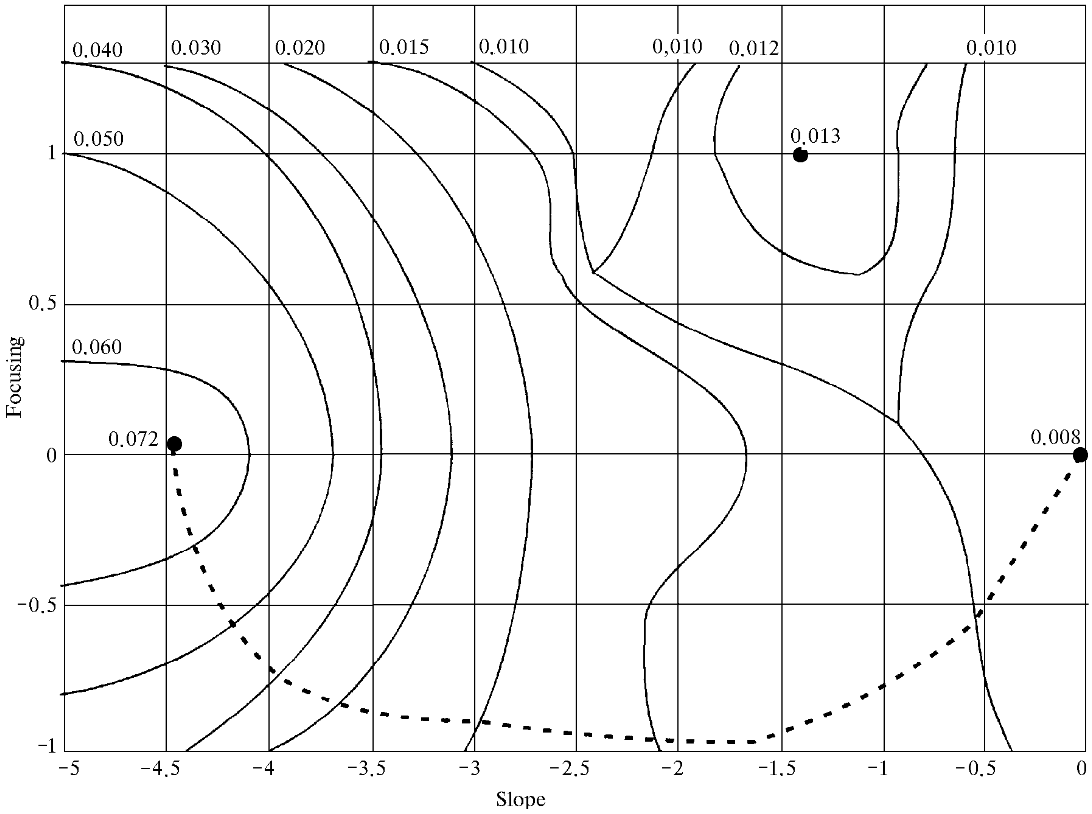

FIG. 4. The efficiency function distribution in the space of "slope–focusing" coordinates when the extension of nonlinear layer is increased relative to that in Fig. 3. Parameters: $Z = 0.5\,Z_d$, $Z_{nl} = 0.25\,Z_d$, $R_v = -70$, $r_a = a_0/4$.

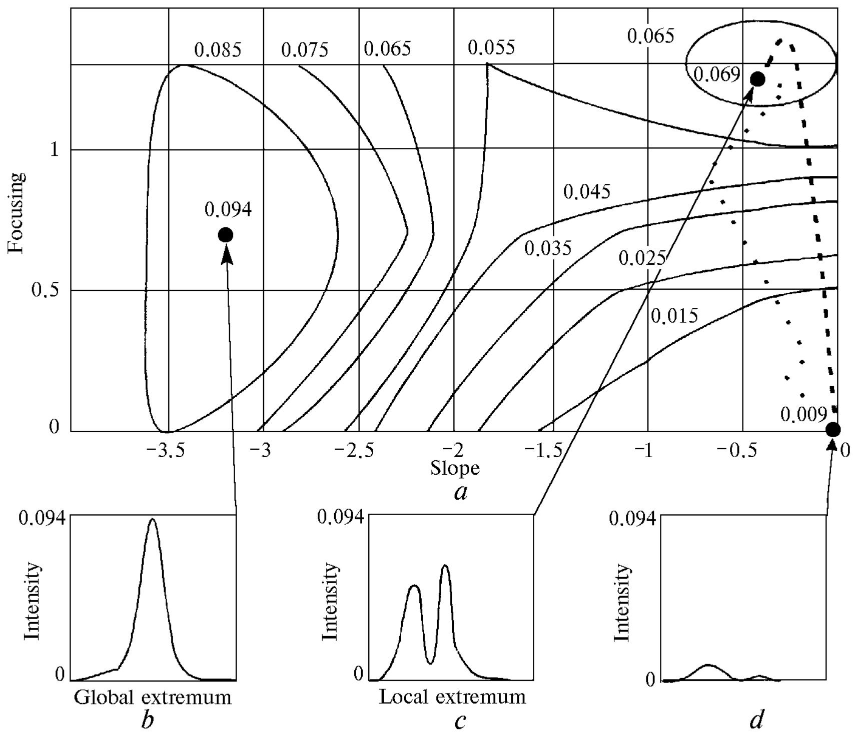

FIG. 5. The control efficiency function (the criterion J, expression (3)) in the space of "slope–focusing" coordinates (a). In the same figure a trajectory of motion to an extremum for the algorithm (4) (dashed line) and (5) (dotted line) is shown; the beam cross section at the global extremum (b); the beam cross section at a local extremum (c); the beam cross section at the control beginning (d). Parameters: $Z = 0.5\,Z_d$, $Z_{nl} = 0.1\,Z_d$, $R_v = -100$, $r_a = a_0/4$.

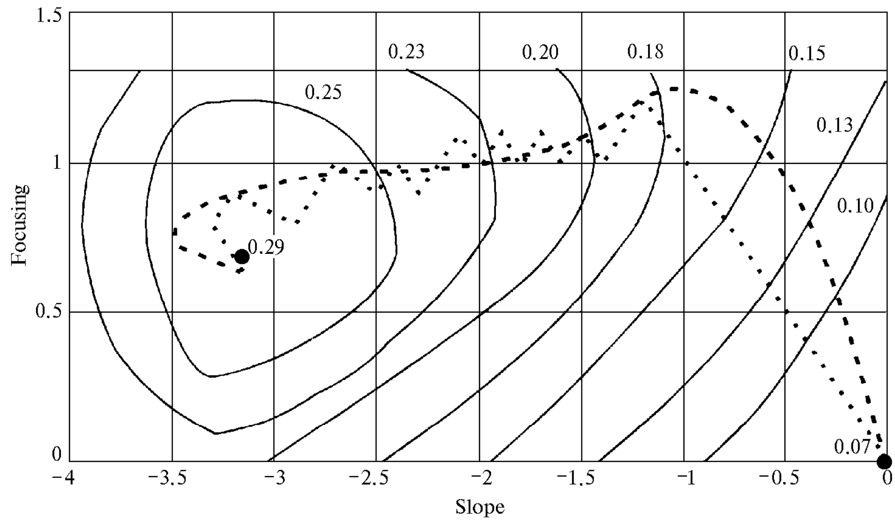

FIG. 6. The control efficiency function (the criterion J, expression (3)) in the space of "slope–focusing" coordinates. Parameters correspond to those used in Fig. 5, except for the aperture radius that is increased.

When the extension of a nonlinear layer (Z_{nl} = 0.25 Z_d) increases without changing the rest parameters of the problem a secondary maximum appears in the space of control coordinates. The height of this maximum is approximately one sixth of the principal maximum height (Fig. 4). That means that if the system stops at a local extremum, then the control efficiency will strongly decrease. This does not happen in the example considered, and the algorithm moves in the direction of a larger slope to a global extremum.

When the nonlinearity of a medium (modulus of the parameter R_v) increases and the rest characteristics are kept unchanged there are two extrema in the criterion distribution (5), with the height difference between these extrema being about 30%. The intensity distributions over the beam cross sections which correspond to the points of maxima and the intensity distribution in the control beginning (i.e., for zero values of the slope and focusing) are presented in the same figure. The local maximum corresponds to the situation when the intensity distribution has two extrema of practically equal heights.

At the global extremum the distribution of light field is Gaussian and the value of maximum intensity is close to the diffraction-limited one.

Use of the algorithms (4) and (5) with the above chosen parameters yields identical results: the compensation for distortions stops at a local extremum (motion trajectories are shown in Fig. 5*a*). Further increase of focusing does not lead to the growth of the criterion *J* (overfocusing), increase of the slope (and thus approaching to the global extremum) is also impossible because of the peculiarities of the algorithms (4) and (5). An increase of the gradient step β does not allow one to approach to global maximum either.

A sufficiently simple method of "smoothing the hillB when local maxima disappear is to increase the receiving aperture radius r_a. The "hillB calculated for the same parameters used in the example considered above, but for $r_a = a_0$ is presented in Fig. 6. One can see from this figure that only one maximum with coordinates corresponding approximately to the coordinates of local extremum in Fig. 5*a* is observed in this case.

Thus, we can draw a conclusion that one of the possible methods to solve the problem of local maxima influence is to increase the receiver aperture. In this case the coordinates of an extremum are found a little bit less accurately than in the case of small receiving areas. But, after an approximate determination of the coordinates of a maximum a decrease of the aperture and more accurate localizing of the extremum are possible.

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