METHOD OF THE SEA WATER ANOMALIES RECOGNITION BASED ON DATA OF AIRBORNE LASER SOUNDING

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In this paper we propose certain criteria to characterize the efficiency of sea disturbances recognition using airborne laser sounding. To calculate the magnitudes of disturbances in a gradient and spectral representations, we use logical procedures. The image recognition algorithm based on potential functions (kernel estimations) method with the control of class representation adequacy is treated. The results of the disturbances recognition in a real sea obtained by the algorithm are presented.

A variety of hydrophysical processes in sea medium, wide range of their spatial and temporal scales increase complexity to the problem on classifying these processes based on remote observations.

The necessity of detecting and classifying hydrophysical processes is associated with the problem of monitoring the ocean when solving different scientific and applied problems.

Evolution of a hydrophysical process in the sea medium is accompanied by the medium disturbances that manifest themselves in an increase variability of the hydrophysical parameters. The signs of the disturbances have no any general forms and depend on a variety of factors.

Information on the disturbance parameters may be extracted from the backscatter signals obtained at sounding the sea medium or sea surface by laser (see Ref. 1) or radar (see Ref. 2) and optical (using surface illumination) means (see Ref. 3). Since the variability of the sea waves is secondary as compared to the processes within the sea depth, one can expect to improve the recognition using signals of laser sounding in the range of "a transmission window" which bear information on the state of the region where the assumed hydrophysical process evolution free of immediate atmospheric action.

As the research practice has demonstrated the layer with an increased concentration of scattering particles in the region of seasonal thermal wedge exhibiting fine structure is a good indicator of the disturbances. Therewith both the layer position in depth and its structure vary what has an immediate impact on the shape of the Mie backscatter of laser pulses.

In the present paper detection of a desired perturbation class is associated with the image recognition in the form of multidimensional vector in the space of formalized signs. The algorithm proposed is based on the principles of image recognition theory (see, for instance, Ref. 4). The values of the criteria are calculated using logical procedures which incorporate the parameters of backscattered pulses (PBP). Therewith the gradient and spectral representations of the PBP have been used (see Refs. 5, 6).

As far as we know, during the research present period (1988–1989) no consideration has been given to the problem on recognition of a hydrodynamic process images with the use of gradient and spectral parameters of airborne lidar return signals.

1. RECOGNITION METHOD

1.1. Basic scheme

Let $A = \{\mathbf{x}_i\}$, $i = 1, \dots N$ be standard (training) set of criteria vectors from a known class, for instance, from the "background" one ("background" means unperturbed state of the medium). The image of a given class which is they used to construct the decision rule is reconstructed from this information. Therewith the control over the reconstruction adequacy is provided.

The principle of reconstruction of the class $K(A)$ from a given sample set $A = \{\mathbf{x}_i\}$ in the space of criteria by the method of potential functions (kernel estimates) is that the concentration density function $f_k = (\mathbf{x})$ of points in R^n (n–dimensional Euclidean space) is approximated by the following expression:

$$\hat{f}_k(\mathbf{x}) = C \sum_{i=1}^{N} \frac{1}{h_i^n} \varphi\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_i}\right), \quad (1)$$

where C is the normalized constant, for instance, $C = 1/N$; $\varphi(\mathbf{x})$ is the kernel function of the smoothing window, for instance,

$$\varphi(\mathbf{x}) = \exp\left(-\frac{1}{2}\|\mathbf{x}\|^2\right), \quad (2)$$

where $\|\cdot\|$ is the Euclidean norm in space R^n; $\{h_i\}$ is the appropriate system of smoothing window parameters.

Function $\hat{f}_k(\mathbf{x})$ (the image of $OK(A)$ class) is taken as the measure of proximity of a point $\mathbf{x}$ to the class $K(A)$ and is used to classify any given vector of criteria $\mathbf{x}$.

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1.2. Decorrelation and spheroidizing

The use of round smoothing window given by Eq. (2) is justifiable only after application of the decorrelation and normalization procedures to the training set A. For this purpose all vectors from both sets, K and A, (the latter is to be recognized) are subject to the orthogonal transformation of decorrelation and spheroidizing. To avoid introduction of a new designation for coordinates **x** let us write this procedure in the form of the assignment operator

$$\mathbf{x} := D^{-1/2} o^{T} \mathbf{x}, \tag{3}$$

Here decorrelation matrix o is such that

$$o^{T} C_A o = D = \begin{pmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{pmatrix}. \tag{4}$$

The matrix columns are the orthogonal eigenvectors of the covariance matrix C_A of the set A, considered as a random vector sample with the matrix C_A eigenvalues λ_1, ..., λ_n.

The spheroidizing transformation is performed by the matrix

$$D^{-1/2} = \begin{pmatrix} \lambda_1^{-1/2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n^{-1/2} \end{pmatrix}. \tag{5}$$

The covariance matrix of the set $A = \{\mathbf{x}_i\}$ after the transformation by Eq. (3) becomes a unit one (while the set inertia hyperellipsoid becomes a hypersphere).

1.3. Optimal window system

When constructing a measure of proximity in the form of Eq. (1) the following algorithm for choosing optimal window parameters h_i was applied. For each point $\mathbf{x}_i \in A$ one can define some value h which maximizes the proximity measure of the type expressed by Eq. (1) of the point $\mathbf{x}_i$ to the set $A - n\,\mathbf{x}_i$ with the parameter h being independent of the summation index. Here $n\,\mathbf{x}_i$ is a small vicinity around the point $\mathbf{x}_i$ including, probably, only one point from the set A, namely, $\mathbf{x}_i$. This value h is calculated as a solution to the following equation:

$$\frac{\mathrm{d}}{\mathrm{d}h}\, p(\mathbf{x}_i, A{-}O\mathbf{x}_i, h) = 0, \tag{6}$$

where $p(\cdot)$ is the proximity measure with the constant parameter h:

$$p(\mathbf{x}_i, A{-}O\mathbf{x}_i, h) = \frac{C}{h^n} \sum_{\mathbf{x}_j \notin O\mathbf{x}_i} \exp\left(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2h^2}\right). \tag{7}$$

Here $p(\mathbf{x}_i, A{-}O\mathbf{x}_i, h) \to 0$ both at $h \to 0$ and $h \to \infty$. This means that this function has its peak on the half-axis $(0, \infty)$. Upon a simplification Eq. (6) reduces to the form

$$h^2 = f(h^2) \equiv \frac{\sum\limits_{\mathbf{x}_j \notin O\mathbf{x}_i} \|\mathbf{x}_i - \mathbf{x}_j\|^2 \exp\left(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2h^2}\right)}{n \sum\limits_{\mathbf{x}_j \notin O\mathbf{x}_i} \exp\left(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2h^2}\right)}. \tag{8}$$

As a rule the iterative process

$$h_{k+1}^2 = f(h_k^2), \tag{9}$$

rapidly converges to some value h_i^* which provides for $\max\limits_h p(\mathbf{x}_i, A{-}O\mathbf{x}_i, h)$. The distance from $\mathbf{x}_i$ to the nearest point from the set A can be taken as the initial approximation h_0.

It is obvious that maximizing of the sum expressed by Eq. (7) is primarily achieved owing to those terms which correspond to the point $\mathbf{x}_j$ being the nearest to $\mathbf{x}_i$ one (except for "excessively close" points). This means that h_i^* is the local-optimal window parameter for the vicinity around the point $\mathbf{x}_i$. Besides, this provides a basis for accepting and experimental investigation of the following proximity measure with the system of windows optimal in the above sense:

$$p_{\text{opt}}(\mathbf{x}_0, A) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{\left(h_i^* \sqrt{2\pi}\right)^n} \exp\left(-\frac{\|\mathbf{x}_0 - \mathbf{x}_i\|^2}{2h_i^{*2}}\right). \tag{10}$$

The estimation of the radius ρ_δ in the vicinity $O\mathbf{x}_i$ based on the approximate value of the density of points in the normal array can be written in the form:

$$\rho_\delta = \frac{1.2\,\sigma \sqrt{n}}{\sqrt[n]{N}} \cong \frac{1.2\, r_{\text{mean}}}{\sqrt[n]{N}}, \tag{11}$$

where $r_{\text{mean}} \cong \sqrt{M(\|\mathbf{x}\|)^2} = \sigma \sqrt{n}$ is the mean radius of the normal sampling considered including N points from the set $A \in R^n$.

1.4. Determination of the class boundary

For the 3-digit answer in the decisive rules of recognition of arbitrary sign point to be obtained it is necessary to determine the boundary or boundary strip ("Δ"–boundary) of the class $K(A)$. Let the set $\Gamma(A)$

$$\Gamma(A) = \{\mathbf{x} \in R^n, p_{\text{opt}}(\mathbf{x}, A) = \alpha \max_i p_{\text{opt}}(\mathbf{x}_i, A))\}, \quad (12)$$

where $0 < \alpha < 1$ be called the boundary of the class $K(A)$.

The value α is a free parameter of the algorithm. From experience it follows that in most cases it is to be chosen from *a priori* information on possible values of the frequency of occurrence of anomalous formations, on the risk value of missing anomalies and so on rather than from mathematical reasons.

1.5. Adequacy of the class representation

The system of windows should provide the adequacy of approximation of the class $K(A)$ by the function $p(\mathbf{x}, A, \{h_i\})$. The purpose of achieving the adequacy is to avoid the following two inadequacies in the class representation:

a) identification of the class $K(A)$ with the set itself if windows are excessively small (when each of the two sets becomes "absolutely separable");

b) blooming of the class in the case of excessively large windows when all images of the classes are indistinguishable.

There are several methods for estimating the adequacy. Almost all of them involve elements of a subjective choice and, therefore, it is essential to have an ensemble of independent methods for making the choice objective.

1) The adequacy estimate by the method of random perturbation of a set. For a given system of windows $H = \{h_1, ..., h_N\}$ the following value (the measure of inadequacy of the first type) is determined:

$$\varepsilon(H) = \frac{\sum_{i=1}^{N} |p(\mathbf{x}_i, A, H) - p(\mathbf{x}_i, \tilde{A}, H)|}{\sum_{i=1}^{N} p(\mathbf{x}_i, A, H)} \quad (13)$$

or

$$\nu(H) = \frac{\sum_{i=1}^{N} |L(\mathbf{x}_i, A, H) - L(\mathbf{x}_i, \tilde{A}, H)|}{\sum_{i=1}^{N} |L(\mathbf{x}_i, A, H)|}, \quad (14)$$

where

$$L(\mathbf{x}_i, A, H) = \ln p(\mathbf{x}_i, A, H); \quad (15)$$

the set A is a "perturbed" set, where coordinate x_i^k of the point $\mathbf{x}_i$ is replaced by $\tilde{x}_i^k = x_i^k + \varepsilon_i^k$ where ε_i^k is a random number with the Gaussian distribution having the parameters $M(\varepsilon_i^k) = 0$ and

$$\sigma(\varepsilon_i^k) = \rho_\delta \frac{l_k}{\left(\sum_{j=1}^{N} l_j^2\right)^{1/2}}, \quad (16)$$

where $l_k = \max_i x_i^k - \min_i x_i^k$.

Let us consider the measure $\nu(H)$ as a function of some coefficient α

$$\nu(\alpha) = \nu(\alpha\, h_1^*, \dots \alpha\, h_N^*), \quad (17)$$

where $\alpha > 0$, while $\{h_i^*\}$ is a set of optimal windows.

The approximation $p(\mathbf{x}_i, A, H)$ of the class $K(A)$ can be considered as optimal if it is constructed on the set of windows $H = \{h_1^0, \dots, h_N^0\}$ satisfying condition $h_i^0 = \alpha_{\min} h_i^*$, $i = 1, \dots, N$, where $\alpha_{\min}$ is the minimum value of α such that the condition $\nu(\alpha) < \varepsilon_{\text{th}}$ is valid (ε_{th} is some inadequacy threshold, for instance, $\varepsilon_{\text{th}} = 0.1$).

2) The criterion of the proximity measure convergence with increasing number of points in the set A.

When the process of a set volume increase is considered, for the sequence of sets $A_1 \subset A_2 \subset \dots \subset A_l \subset$ the convergence of any sequence of the proximity measure values should take place

$$p(\mathbf{x}_i, A_1, H),\ p(\mathbf{x}_i, A_2, H),\ \dots,\ p(\mathbf{x}_i, A_l, H),\ \dots,$$

for each point $\mathbf{x}_i \subset A_k$, $k = 1, 2, \dots$, if the system of windows H is free of inadequacy of the 1st type. The convergence is to be achieved for any random choice of the initial set A and any random addition of new points to the set.

This method calls for further investigation in order to elaborate an algorithm for making decisions on the presence or absence of the convergence and for the determination of the indices of inadequacy of the 2nd type.

3) Visual adequacy (quality of the class reconstruction on a selected set) estimate by an operator by constructing the plots of one- or two-dimensional sections of the function $p(\mathbf{x}, A, H)$.

Elements of all the above methods of estimating the background class representation adequacy are used when solving this problem.

2. THE ARRAY OF CRITERIA USED

2.1. Algorithms for calculating the criteria values

The n-dimensional ($n = 12$) criteria vector constructed on the base of gradient and spectral representations of the backscattered signal has been used. The system of criteria allows for the dynamics of the process adapting to its phase known *a priori*. The algorithms for calculating the criteria values based on logical procedures are presented below (see Tables I, II). The following designations are used: n is the ordinal number of the criteria vector component; ∇U is the depth gradient of the signal amplitude, normalized to the initial amplitude of the backscattered pulse (BP)

$$\nabla U(k) = [U(k+1) - U(k)]/U(1), \qquad (18)$$

k = I, II, III, IV, V is the number of a fixed depth level (or time channel $k = I, \ldots, K$) when quantizing BP on the time scale; t_i is the time measured from the beginning of the ith tack, performed with an airborne lidar during its flight along coincident tacks; $i = 1, \ldots, (I-2)$, where I is the total number of coincident tacks; $\overline{\eta}_s$ is the spectral energy value in the sth frequency interval normalized to the overall energy of BP

$$\overline{\eta}(s) = W(s)/W_0, \qquad (19)$$

where $W(s)$ is the pulse energy in the sth frequency channel ($s = 1, \ldots, S$); $\Delta F_s = \Delta F/S = N\Delta f$ (operation frequency band $\Delta F = S\Delta F_s = SN\Delta f = L\Delta f$); Δf is the interval of spectrum quantization;

$$W(s) = \sum_{n=1}^{N} P_s(n)\,\Delta f, \qquad (20)$$

where $P_s(n)$ are the values of spectral power density in the frequency interval $F_s - F_{s-1}$.

The total pulse energy

$$W_m = \sum_{l=1}^{L} P(l)\,\Delta f. \qquad (21)$$

$f(B)$ is the truth value of Boolean expression B (see below); $f(B) = 1$ if B = true; $f(B) = 0$ if B = false.

For the gradient criteria 1–4 (see Table I) the ordinal number $n = k - 1$ ($k > 0$). Values of $\overline{\nabla U}$ and $\overline{\eta}$ are obtained as a result of averaging over a sample of sounding pulses. The criteria values are calculated for each triplet of coincident tacks. The criteria 1–4 are calculated for each pair of neighbour depth levels. The ordinal number of spectral criterion $n = s + 4$ ($s = 2, \ldots, 7$). Its maximum value is equal to 4.

TABLE I.

The number of criterion (n)	Calculation algorithm for the o_n criterion
	Gradient criteria
1–4	$o_n = \sum_{i=1}^{I-2} [f(\overline{\nabla U}_k(t_i) > \overline{\nabla U}_k(t_{i+1}) \wedge \overline{\nabla U}_k(t_{i+1}) < \overline{\nabla U}_k(t_{i+2})) + f(\overline{\nabla U}_k(t_i) \le \overline{\nabla U}_k(t_{i+1}) \wedge \overline{\nabla U}_k(t_{i+1}) \ge \overline{\nabla U}_k(t_{i+2}))]$.
5	$o_5 = \sum_{i=1}^{I-2} [f(\overline{\nabla U}_{III}(t_i) > \overline{\nabla U}_{IV}(t_i) \wedge \overline{\nabla U}_{III}(t_{i+1}) < \overline{\nabla U}_{IV}(t_{i+1})) + f(\overline{\nabla U}_{III}(t_{i+1}) \le \overline{\nabla U}_{IV}(t_{i+1}) \wedge \overline{\nabla U}_{III}(t_{i+2}) \ge \overline{\nabla U}_{IV}(t_{i+2}))]$.

TABLE II.

The number of criterion $n = s + 4$	Calculation algorithm for the o_n criterion
	Spectral criteria
6–11	At $\overline{\eta_s}(t_1) < \overline{\eta_s}(t_2)$: $o_n = f[\overline{\eta_s}(t_1) < \overline{\eta_s}(t_2)] + f[\overline{\eta_s}(t_2) > \overline{\eta_s}(t_3)] + f[\overline{\eta_s}(t_3) < \overline{\eta_s}(t_4)] + f[\overline{\eta_s}(t_4) > \overline{\eta_s}(t_5)]$. At $\overline{\eta_s}(t_1) \ge \overline{\eta_s}(t_2)$: $o_n = f[\overline{\eta_s}(t_1) \ge \overline{\eta_s}(t_2)] + f[\overline{\eta_s}(t_2) \le \overline{\eta_s}(t_3)] + f[\overline{\eta_s}(t_3) \ge \overline{\eta_s}(t_4)] + f[\overline{\eta_s}(t_4) \le \overline{\eta_s}(t_5)]$.
12	At $\overline{\eta_1}(t_1) > \overline{\eta_1}(t_2)$: $o_{12} = f[\overline{\eta_1}(t_1) > \overline{\eta_1}(t_2)] + f[\overline{\eta_1}(t_2) < \overline{\eta_1}(t_3)] + f[\overline{\eta_1}(t_3) > \overline{\eta_1}(t_4)] + f[\overline{\eta_1}(t_4) < \overline{\eta_1}(t_5)]$. At $\overline{\eta_1}(t_1) \le \overline{\eta_1}(t_2)$: $o_{12} = f[\overline{\eta_1}(t_1) \le \overline{\eta_1}(t_2)] + f[\overline{\eta_1}(t_2) \ge \overline{\eta_1}(t_3)] + f[\overline{\eta_1}(t_3) \le \overline{\eta_1}(t_4)] + f[\overline{\eta_1}(t_4) \ge \overline{\eta_1}(t_5)]$.

3. FORMATION OF THE "BACKGROUND" AND "SIGNAL" SAMPLES

"Signal" and "background" samples have been obtained in different water areas which differ essentially from each other in hydrological parameters. The "signal" sample was formed when the airborne lidar flew in the regime of spatially coincident tacks, while the "background" one was obtained from the tacks forming a grid.

Since random processes describing the behavior of hydrophysical parameters (HPP) are nonergodic and the correlation of HPP in vertical column significantly exceeds that in the plane (see Ref. 7), such a method of forming the "background" sample has obviously added complexity to the recognition conditions.

Complete data file prepared included 60 "points" from the "signal" (N_s) and 2800 "points" from the "background" (N_b) samples. By "point" is meant a definite spatial interval of the signal averaging.

4. RECOGNITION RESULTS

Several variants of the recognition problem has been calculated. According to the variant chosen "signal" or "background" sample was used as the training one. The indicator files wherein each criteria vector (spatial point R^n) was classified as unrelated ("anomaly") to or involved in the training set were printed out. In these cases the indicator value is equal to 1 or 3, respectively. The indicator value of 2 was assigned to the marginal points located on the training set boundary. When recognition is performed at the level of false alarms $F = 0.1$, points "1" and "2" are assigned to the "anomaly" set. If the false alarm level $F = 0.05$ the marginal points are not included into the "anomaly" region.

When the "background" sample is used as the training one the results of "signal" recognition show high frequency of correct classification (in the limit, the probability of true detection). In some cases the results drastically change with the renewal of the "background" sample. The first situation demonstrates high efficiency of the selected criteria system, while the second one means that, as expected, the "background" situation similar to "signal" one under conditions of high level of small-scale sea medium variability in the horizontal plane is quite probable.

Besides it should be noticed that the criteria set selected provides stable results if the condition of stationarity of the random processes describing the behavior of sea medium parameters is fulfilled. Nevertheless, the same "point" can be simultaneously subjected to several hydrophysical processes of different scales. Therefore the best volume of a sample corresponds to the interval on which the quasi-stationary approximation is valid. It is apparent that the volume can vary with water area and specific hydro- and meteorological conditions. In terms of the spatial scale the selection of 20 values of the criteria vectors is quite acceptable but inadequate for forming clear boundary of a set in a multidimensional criteria space. Nevertheless, an increase of the volume of training (in that case "background") sample and removing from the boundaries of the quasi-stationary approximation region result in "blooming" of the boundaries of the training set including increased number of points from the "signal" sample.

The above reasoning is confirmed by Fig. 1. demonstrating the frequency of true classification of "signal" sample (3×20 values of the criteria vector) versus training "background" one whose volume varies with a step of 20 values in the range from 20 to 300. Solid and dotted lines correspond to the false alarm level $F = 0.1$ and $F = 0.05$, respectively.

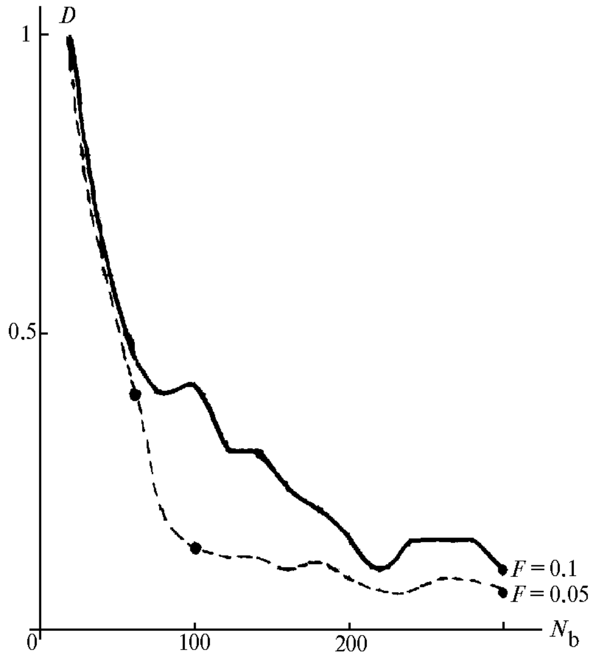

FIG. 1. Results of recognition versus voluHe of the training "background" saHple $D = f(N_b)$.

Another one variant of the problem is recognition of the overall "background" data array (2800 values) divided into segments calculated because of limitations in the computer operative memory. As it follows from Table III (two right columns) reliable separation ($D = 0.75$ at $F = 0.1$) of the representative "background" sample relative to the "signal" set is evident.

TABLE III. Recognition results.

Segment number	Number of points			$D = N_1/N$	$D=(N_1+N_2)/N$
	N (total)	N_1 ("1" anomaly)	N_2 ("2" boundary)	$F = 0.05$	$F = 0.1$
4	320	189	29	0.59	0.68
5	«	224	28	0.7	0.79
6	«	226	18	0.71	0.76
7	«	196	28	0.61	0.7
8	«	256	23	0.8	0.87
9	«	205	32	0.64	0.74
10	«	185	53	0.58	0.74
11	«	192	31	0.6	0.7
12	240	169	18	0.7	0.78
				$\bar{D}$	
Σ	2800	1842	260	0.66	0.75

5. CONCLUSIONS

1. The presented results of application of the image recognition theory to the problem on classification of hydrophysical processes using signals of laser sounding show that despite of the effects of multiple scattering along with the signal aberrations appearing in a double-path propagation of the laser pulse through the random water surface one may expect relatively correct recognition parameters.

2. In the experiment variant considered the 12-dimensional system of criteria used provided the frequency of correct recognition $\bar{D} = 0.75$ at the false alarm level $F = 0.1$.

3. The choice of spatiotemporal intervals for the criteria determination has a decisive role in the recognition problem by virtue of nonstationary behavior of the perturbations developing in a nonuniform sea medium.

This work has been done in 1989–1991 based on the data obtained in the course of a collaborative experiment with the airborne lidar "Sea-gull" performed by the Institute of Physics, USSR Academy of Sciences and CSPIP "Kometa" in 1985 (see Refs. 1 and 5).

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