# QUALITATIVE ANALYSIS OF THE EVOLUTION AND STABILITY OF OXYGEN ATMOSPHERE AND CLIMATE WITHIN THE FRAMEWORK OF LOW-ORDER MODELS

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Some results are presented of qualitative analysis of the ozone cycle behavior within the framework of the model of an oxygen atmosphere and those of the vertical distribution of global temperature within the framework of low-order models oriented to radiation balance considerations. These results indicate that qualitative analysis of climate models themselves or their constituent blocks allows one to reveal the general features in the behavior of the stationary states of a system. A detailed numerical analysis for the parameter values at which the qualitative analysis has already revealed the possible nontrivial behavior of the system should become more efficient tool for analysis of the complex atmosphericoptics systems than direct numerical simulation.

Nowadays, global climate changes attract close attention of researches. Such phenomena as climate warming and ozone depletion, considered to be global, call urgently for tacking questions about their nature and possible evolution. These and other climate problems are considered more and more frequently as problem of physics. This means that global phenomena will be described by low-order models if only the main physical processes are considered on scales of phenomenon being studied. Owing to their simplicity, the low-order models can be used to identify clearly the processes that govern variations of the system parameters and provide a wide opportunity to analyze them qualitatively or in numerical experiments. The models also can provide a basis for developing parametrization techniques for more sophisticated models. We believe that the low-order models already studied for some decades are still far from exhausting their potentialities for an analysis of global changes and the more so model characteristics enumerated above allowed much room for radically new understanding of the nature and evolution of global changes inherent in a complex system.

This paper discusses some aspects that seem to be common for climate models themselves and their constituent photochemical blocks.

The most significant element of photochemical blocks of climate models is ozone that determines the vertical temperature distribution in the stratosphere. Being the main absorber of the UV solar radiation, ozone determines in many respects the incoming solar radiation intensity. The basic cycle of reactions producing the ozone layer in the atmosphere was formulated by Chapman as early as the 30's. In a description of photochemical processes, there is the tendency for consideration of more and more reactions and reagents involved. Numerical solution, however, is always sought for finite time intervals and definite number of compounds and hence gives only a limited amount of information on the process. To elucidate consequences of any change in the conditions, calculation should be repeated. But even in this case some questions still have not answered. There are: How many steady states has the system got? What are they? How do their number and character change with the system parameters? What are results of long-term system evolution? Are there any changes in it perceived as catastrophic? Until these questions are answered, the ozone layer problem will not be spoken about as the resolved one.

The problem of global climate changes caused, for example, by anthropogenic impacts on atmospheric gaseous composition, in many respects closely resembles the logic pattern of the problem just outlined. Climate description evolves to creating complicated models considering in greater detail some or other processes influencing the climate. Here, we face the problems on the presence, character, and evolution of the steady states once again. Questions of this sort will not be answered exhaustively if we lean only upon the numerical analysis of photochemical and climatic processes.

By no means minimizing the significance of a detailed numerical analysis of the behavior of complex atmospheric-optical systems, we suggest that it should follow a qualitative analysis of the examined systems within the framework of low-order models by the methods of nonlinear dynamics. Qualitative analysis is a special division of mathematics (see, e.g., Refs. 1 and 2) intended for investigation of nonlinear systems such as the climatic and photochemical ones, and is just oriented on an investigation into the problems of existence,

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character, and evolution of the steady states mentioned above (see, e.g., Refs. 1 and 2). Being the step that came before complex-model calculations, the qualitative analysis converts in many respects random search of factors that cause qualitative changes in the systems under study, into a goal-oriented process, when the numerical analysis of complex systems is made for those parameter values, for which the qualitative analysis has already revealed the possible nontrivial system behavior.

We note that, the nonlinear dynamics techniques, being not new in studying the general circulation (see, e.g., Refs. 3 and 4), were rarely used in photochemical blocks. At the same time, in current qualitative analysis of the low-order climate models primary emphasis was placed either on paleoclimates or on dynamics of the ice cover (see, e.g., Refs. 3–6).

Below we present some results, of qualitative analysis of the behavior of the ozone cycle in the oxygen atmosphere and vertical distribution of the global temperature within the scope of the low-order models considering the radiation balance.

## 2. OXYGEN ATMOSPHERE

The fundamental question we would like to tackle first is: whether the observed variations of the ozone concentration are natural or caused by anthropogenic activity. As applied to the atmospheric chemical subsystem, this question naturally reduces to the following one: whether or not foreign species and associated extra reactions lead to qualitative changes in the behavior of atmospheric oxygen constituents. As a result of this reduction, the problem is reduced to an analysis of equations of chemical kinetics, that is, of nonlinear equations for concentration, whose parameters are the rate constants of chemical reactions dependent on the temperature and initial values of concentration. For our analysis, we chose an approximation admitting an exact mathematical solution of the problem, namely, the model of purely oxygen atmosphere with the minimum cycle of Chapman's reactions.<sup>7</sup>

The natural question arises: to what extent will the results obtained for this simple model be applicable to the real atmosphere? Since the formulation of Chapman's cycle, the answer has been quite optimistic, because this simple cycle allows one to reproduce a typical vertical profile of the ozone concentration. Quantitative discrepancies play no role in our analysis as yet, since its purpose is elucidation of the tendency of evolution.

We note that in the recent literature there appear publications<sup>8–11</sup> discussing problems of multiplicity of steady states in the atmospheric photochemistry, but they consider other cycles of reactions. Only three papers by Karol,<sup>12</sup> Sahabanova,<sup>13</sup> and Kozak et al.<sup>14</sup> were devoted to qualitative analysis of the pure oxygen ozone cycle, as it is, in the atmospheric photochemistry, but they considered only some particular cases, and the authors did not reveal any qualitative singularity. However, the full phase pattern is still uncertain even for the simplest system. At the same time, knowledge of the quantitative behavior of concentration of oxygen constituents in the pure oxygen ozone cycle, being an important part of all more complicated reaction systems, seems mandatory.

### A. Chapman's ozone cycle

We now write down the reactions of the ozone cycle

$$O_2 + hv \xrightarrow{J_2} O + O , \qquad (1)$$

$$O_3 + hv \xrightarrow{J_3} O_2 + O , \qquad (2)$$

$$O + O_2 + M \xrightarrow{k_2^M} O_3 + M , \qquad (3)$$

$$O + O_3 \xrightarrow{k_3} 2O_2$$
, (4)

$$O_3 + M \xrightarrow{k_2^M} O + O_2 + M, \tag{5}$$

$$O + O + M \xrightarrow{k_4} O_2 + M, \tag{6}$$

under the assumption that the third particle is nonoxygen, that is,  $M \neq O$ ,  $O_2$ , and  $O_3$ . We made a detailed qualitative analysis of this reaction system,<sup>15–19</sup> starting from the simplest cases, to elucidate which reactions are responsible for one or another of features of the phase portrait and whether or not a situation arises in which variations in the parameters of the system result in the change of the character of stability of the steady state or even to its vanishing.

Our analysis has shown that the dynamics of concentration of oxygen constituents is rather manifold. We now dwell on the simplest system in which ozone is still produced, that is, on the ozone cycle without radiation, to illustrate the character of results obtained.

Figure 1a shows the reaction of ozone formation at three-particle collision and of ozone destruction at collision with the atom O. Shown is also the phase space of concentration of oxygen constituents. The system state at a given moment is specified by a point in the phase space. System evolution with time is described by a phase trajectory. Due to the conservation law of the number of atoms of the reagents, the domain of permissible variations of the concentration is the so-called triangle of reactions. The set of the stationary points and phase trajectories is called the phase portrait of the system. In the given case (two reactions shown in Fig. 1a), the system possesses a singular point - a saddle - where the O<sub>3</sub> concentration is zero, and a singular straight line – the axis  $O_3$  with all point being singular. The saddle point is unstable. Any trajectory, even approaching very close to it, goes away from it with time. Points of the singular straight line are stable, that is, a phase point getting to that straight line stays there. We see that the phase trajectories, wherever they started, approach the  $O_3$  axis. We also can say whether the  $O_3$  concentration will decrease or increase as a function of initial conditions when it approaches the steady state. It is also seen that the reaction triangle is divided into two parts with the saddle separatrix and that the trajectories cannot cross it going over from one part to the other. This phase portrait provides all possible qualitative information on the system. Any numerical solution always gives just a certain trajectory segment defined by the initial conditions.

The remaining cases (Fig. 1) illustrate changes in the phase portrait due to incorporation of reactions (5) and (6) in addition to the two reactions mentioned above. As a result, only one singular point remains in the reaction triangle, namely, the stable node with zero values of the O and O<sub>3</sub> concentration, which is unattainable on the model applicability scales owing to long relaxation time.



FIG. 1. Three-dimensional phase portraits and their projections onto the plane (O,  $O_3$ ) for a set of reactions of the ozone cycle without radiation.

In all cases of the ozone cycle without radiation emission considered so far, the stationary point is located on the boundary of the reaction triangle, that is, in the steady state with zero concentration of at least one oxygen constituent. The simplest case in which the stationary point is within the reaction triangle, i.e., when all three oxygen constituents have nonzero values of concentration, is that of three reactions: two reactions (3) and (4) considered above and reaction (1) of molecular oxygen photodissociation. In this system there are two parameters, A and M, being combinations of the reaction rate constants and initial values of concentration. Phase portraits of the system differ drastically for different relationships between the system parameters (see Fig. 2). When AM > 1, there

is a stable node with nonzero values of concentration of all oxygen constituents in the reaction triangle, whereas for AM < 1, the steady state is on the boundary of the reaction triangle, and the only nonzero concentration is that of atomic oxygen. Thus, in the parametric space the curve AM = 1divides the domain with the steady state having values of concentration of nonzero oxygen constituents and the domain without steady state. Incorporation of reaction of ozone photodissociation makes a little change; it simply adds the parameter,  $C_2$ , and alters the position of the dividing line that now is specified by the equation  $A^2M - A - C_2 = 0$ . The domains of existence of the states with nonzero values of concentration of oxygen constituents are shown in Fig. 3 for indicated set of reactions.



FIG. 2. Phase portraits of reaction systems (1), (3), and (4) for the following relationships between the parameters: AM > 1 (case a), AM = 1 (case b), and AM < 1 (case c).



FIG. 3.  $O_3$  concentration in the steady states as a function of the parameters A and M for the reaction set: (1), (3), and (4) (case a); (1)-(4) (case b); (1)-(5) (case c).

Reaction (5) of recombination of the O atoms turned out to play a crucial role for stabilization of the ozone cycle. Its consideration "returnsB the steady state into the reaction triangle, i.e., restores the steady state with nonzero concentration of oxygen constituents (see Fig. 3c).

In essence, the recombination reaction (5) suggests the presence of the O sink and the  $O_2$  source, so we may expect that incorporation of sources and sinks of oxygen constituents in equations for their concentration will lead to "instability restoration.B In fact, it was found that in the presence of linear sink of atomic oxygen, irrespective of the intensities of permanent sources of O, O<sub>2</sub>, and O<sub>3</sub>, the stable steady states with nonzero concentration of oxygen constituents exist only for a limited range of variations of the intensities of sources and sinks (see Fig. 4). In the presence of linear sinks of oxygen constituents other than O, the system of reactions (1)–(6) always has the steady state with nonzero concentration of oxygen constituents.



FIG. 4. Domain of existence of the stable steady states with nonzero concentration of oxygen constituents (hatched) in the space of parameters  $L_{O_2}$ ( $O_2$  source) and  $K_O$  (O sink). [L] a mole/cm<sup>3</sup>·s, [K] = 1/s.

It is well recognized now that the stationary concentration of atmospheric ozone calculated for Chapman's cycle is overestimated. There are two ways of improving the estimate: (1) extension beyond the framework of the oxygen atmosphere and complement of the set of kinetic equations (1) through (6) by reactions with hydrogen, nitrogen, and chlorofluorocarbon compounds, or (2) incorporation of sources and sinks of oxygen constituents into the model of the oxygen atmosphere to consider extra reactions. So, analysis of the situation with sources and sinks of oxygen constituents was continued.

## B. Role of sources and sinks in the vicinity of the steady state

Thus far we have considered the system of reactions (1)–(6) with  $M \neq O$ , O<sub>2</sub>, and O<sub>3</sub>. A further analysis was oriented on determination of conditions existing in the atmospheric ozone layer.  $^{\rm 20,21}$  In the stratosphere, there is a hierarchy of concentration of oxygen constitutents: concentration of molecular oxygen is higher by four orders of magnitude than that of ozone, which, in its turn, is higher by five orders of magnitude than that of atomic oxygen. This system has two characteristic relaxation times: of the order of several seconds for the relaxation of atomic oxygen to the steady state and of the order of several months for the ozone relaxation. Considering the concentration hierarchy of oxygen constituents and the hierarchy of relaxation times in the vicinity of the steady state, we can identify the set of reactions essential for the description of chemical processes in this vicinity. This set includes reactions (1) through (4) with  $M \neq O$ ,  $O_2$ , and  $O_3$  and two reactions of type (3) with  $M = O_2$  and  $O_3$ .

The concentration of oxygen constituents obtained by solutions of the corresponding kinetics equations is called the reference one. We next incorporate sources and sinks of oxygen constituents into the system under study to determine the stationary concentration of oxygen constituents, source and sink intensities in the steady states, and relaxation times in the oxygen atmosphere with one source and one sink.

Here, symbols K and L stand for sink and source, respectively. All calculations presented below were made with the reaction rate constants at 30 km altitude.

At present the examined problem is a matter of convention due to the lack of quantitative data on atmospheric oxygen sources and sinks and their classification, so we assume that the rates of sinks do not exceed  $k_2^{N_2}NA$ , where  $k_2^{N_2}$  is the rate of reaction (3) with  $M = N_2$ , N is the nitrogen concentration, and the source intensities do not exceed  $k_2^{N_2}NA^2$ , where A is the concentration of molecular oxygen in the stratosphere at 30 km altitude. Below we consider the intensities of sources and sinks normalized by these limiting values and varying from  $10^{-13}$  to 1.

It was shown that in systems with the molecular oxygen sink, steady states exist for wide ranges of variations of sink and source intensities, and that the ozone concentration decreases for  $K_{O2} > L_i$  ( $i = O_2, y, z$ ) and increases for  $K_{O2} < L_i$ . When the normalized sink and source intensities are equal, the steady states are stable, with ozone concentration being close to the reference one and the molecular oxygen concentration being equal to  $L_i/(2K_{O2})$  in all three cases. In systems with the ozone sink, stable states exist only for narrow ranges of variations of sink and source intensities, the ozone concentration is less than the reference one in almost all cases.

Now we dwell on the results of qualitative analysis for the model with one source and sink of molecular oxygen.

Figure 5 shows the plot of logarithms of dimensionless concentration of oxygen constituents, with coordinates specifying the steady values of concentration of these constituents. For  $K_{O2} > L_{O2}$ , the concentration of  $O_2$  and  $O_3$  decreases, and vice versa for inverse inequality, the concentration of  $O_2$  and  $O_3$  increases. When the normalized sink and source intensities are equal, the concentration of these constituents remains unchanged and close to the reference one marked by point A in the figure.

Figure 6 shows horizontal sections through surfaces  $O_2$  ( $K_{O_2}$ ,  $L_{O_2}$ ) and  $O_3$  ( $K_{O_2}$ ,  $L_{O_2}$ ), corresponding to the  $O_2$  and  $O_3$  concentration spaced at two orders of magnitude on real concentration scale. Arrows in the figure are for the curves along which the concentration coincides with the reference one and indicate the direction of the concentration decrease.



FIG. 5. Variations of the O, O<sub>2</sub>, and O<sub>3</sub> concentration attendant to variations of sink  $(K_{O_2})$  and source  $(L_{O_2})$  intensities of molecular oxygen.



FIG. 6. Contour plots of the  $O_2$  (dashed lines) and  $O_3$  (dot-dash lines) concentration in the steady states, brought into coincidence, as functions of the molecular oxygen sink and source parameters.

The incorporation of the source and sink into the oxygen atmosphere model makes all three oxygen constituents independent and introduces the third longest relaxation time of molecular oxygen. Figure 7 shows the curves of the dependence of relaxation times on the molecular oxygen source and sink intensities. The arrows in the figures indicate the direction of the relaxation time decrease, and the numbers designate the order of its magnitude. When going from one isoline to the other, the relaxation time changes by an order of magnitude. From Fig. 7, it follows that the hierarchy of relaxation times remains unchanged for all changes in the sink and source intensities considered here. As sink and source intensities increase, the relaxation time of atomic oxygen increases up to about several days, whereas the relaxation times of  $O_2$  and  $O_3$  decrease down to a week for ozone and to several hundreds of years for molecular oxygen. Along the solid straight line, calculated for  $K_{O_2} = L_{O_2}$ , the stationary values of concentration of oxygen constituents are close to reference ones,

however relaxation times for O,  $O_2$ , and  $O_3$  may differ substantially.



FIG. 7. Dependence of relaxation times on the molecular oxygen sink and source intensities.

Thus, the qualitative analysis shows that, for certain variants of the systems of equations describing the evolution of concentration of oxygen constituents in the oxygen atmosphere, the domains of the parameter values (rate constants, initial values of concentrations, source and sink intensities) exist without physically admissible steady state with nonzero values of concentration of oxygen constituents. In other words, in some cases the equilibrium in the ozone cycle exists only when any oxygen constituent vanishes, not necessarily due to anthropogenic activity.

## **3. LOW-ORDER CLIMATE MODELS**

Calculations elucidating the effect of atmospheric trace gases variability on climate change are mostly run for one-dimensional radiative-convective models (see, e.g., Refs. 22-25). As already mentioned above, our attitude is: detailed calculations for complex models should follow qualitative analysis of simple climate systems by the nonlinear dynamics methods. We addressed ourselves to the one-dimensional balance and radiation climate models that were repeatedly studied by other authors from other viewpoints (see, e.g., Refs. 6 and 26-28), primarily because global climate changes, by their nature, should manifest themselves in simple models. A question arises: how sensitive can be the simple models to variations in the radiation characteristics, for example, to variations in the solar radiation transmission caused by variations of the stratospheric ozone concentration, or to variations in the thermal radiation transmission as a result of the increase of the tropospheric greenhouse gas concentration. We notice that the sensitivity here is meant new qualitative features, such as the appearance or disappearance of steady states, the change in the character of their stability, or at least, the change in the sign of any dependence rather than quantitative changes in the characteristics studied here. This point is illustrated by Fig. 8 showing the simplest two-layer model "surface-atmosphere,B in which the surface temperature  $T_s$  and the atmospheric top temperature  $T_a$ are the independent variables, satisfying the equations

$$\dot{T}_s = \sigma T_a^4 (1 - D_t) - \sigma T_s^4 + F^{\downarrow} D_s , \dot{T}_a = \sigma T_s^4 (1 - D_t) - 2 \sigma T_a^4 (1 - D_t) + F^{\downarrow} (1 - D_s) .$$
(7)

The parameters of system (7) are the incoming solar energy on the atmospheric top,  $F^{\downarrow}$ ; the atmospheric transmission function for the thermal radiation,  $D_t$ , and, the atmospheric transmission function for the solar radiation,  $D_s$ . Such a model is incorporated as a block in models with more detailed vertical resolution. Figure 8 shows the phase portrait of this system possessing only one steady state within the physically allowed part of the plane, namely, the stable node. As is well known, the outgoing radiation is frequently fitted by a linear function of the temperature. Figures 8c and d show the phase portraits of system (7) for indicated parametrizations of the outgoing radiation.<sup>30</sup> While in case shown in Fig. 8c the steady state remains unchanged, in Fig. 8d the steady state is lacking in the physically allowed part of the plane. Due to this fact, one should be careful when using different climate model parametrizations, especially changing the character of nonlinearity. This fact also confirms that the simple models can be sensitive to details in radiation transmission.



FIG. 8. The simplest climate model described by equations (7) (case a) and its phase portrait (case b). Phase portraits of the model for the following parametrizations of outgoing radiation:  $\sigma T_a^4(1-D_t) = \varepsilon T_a$  (case c);

 $\sigma T_s^a D_t + \sigma T_a^4 (1 - D_t) = A + BT_s \quad (case \ a).$ 

Figure 9 shows the behavior of the independent variables  $T_s$  and  $T_a$  of system (7) for the varying system parameters without (Fig. 9a) and with the parametrization (Fig. 9b) of outgoing radiation. The most typical feature of this model is the temperature inversion observed for certain relationships between the system parameters. Once again the presence or absence of the inversion, that is, the presence or absence of the atmospheric greenhouse effect depends strongly on the choice of the parametrization formula for outgoing radiation (see Fig. 9b).



FIG. 9. Behavior of the temperatures  $T_s$  and  $T_a$  in the two-layer climate model described by equations (7) (case a) and equations (7) with the parametrization  $\sigma T_a^4(1 - D_t) = \varepsilon T_a$  (case b) as a function of system parameters  $D_s$  and  $D_t$ .



FIG. 10. Variations of the stratospheric temperature attendant to changes in the transmission functions of the layer in the radiation model consisting of four layers up to 40 km altitude. Here,  $D_{si}$  is the solar radiation transmission function for the ith layer,  $D_{ti}$  is the thermal radiation transmission function for the ith layer: vertical distribution of the temperature (a), variations in the temperature of the third and fourth layers attendant to variations in solar radiation transmission functions for the third and in solar radiation transmission for the third layer and thermal radiation transmission for the first layer (c).

Further we consider a more complex radiation model in which again the equilibrium temperature profile is completely determined by radiation balance, and the convective adjustment is not used. This model has four 10-km thick layers that are vertically stratified. Transmission functions for solar and thermal radiation are specified for each layer separately. The temperatures of the layers are the sought-after coordinates of the steady state. The vertical temperature profile in this simple model has the features common to the radiation models (see for example, Ref. 22), that is, has the minimum at approximately 10-km altitude and then slowly increases with altitude. Not expecting that this model agrees numerically with the observed temperature profiles, we believe that between 30 and 40 km altitudes their character should be reproduced satisfactorily. The question we want to answer now is: Can, in principle, the variations in the transmission functions cause the appearance of any specific feature in the temperature profile at those altitudes? Figure 10 shows the behavior of the temperatures of the third and fourth layers as functions of variations in the transmission function, indicated in the figure. As seen, the position of the temperature inversion boundary depends significantly on the character of variations of the transmission functions. At the same time, the steady state remains the same - the stable node. However, qualitative changes may be even greater. Figure 11 illustrates the situation when the steady state is the stable focus rather than the stable node. Figure 11a shows the behavior of one of the characteristic numbers,  $\lambda$ , governing the character of stability of the state attendant to variations in the thermal radiation transmission of the first and second layers. Real negative values of  $\lambda$  correspond to the stable node. For the stable focus,  $\lambda$  are complex, so they are not shown in the figure. Also shown in Fig. 11b are the behavior of the temperature and the transmission function vs. the altitude for a point with coordinates  $D_{t1}$  and  $D_{t2}$ , falling within the region of the stable focus. The presence of focus means that, as the steady state is approached, the temperature may undergo periodic oscillations with the amplitude that depends on the initial conditions.



FIG. 11. Change in the character of the stable steady state "stable node –- stable focusB for the four-layer radiation climate model described in the text. The characteristic number  $\lambda$  as a function of variations in the thermal radiation intensity of the first and second layers (a) and the vertical behavior of the temperature and of the thermal radiation transmission function for  $D_{t_1}$  and  $D_{t_2}$  values corresponding to the stable focus (b).

Thus, it has been shown that quite diverse qualitative vertical behavior of the temperature is possible for simple models in principle. The variations of the transmission functions that produce qualitative changes will be analyzed further in more detail.

### 4. CONCLUSION

Examples of qualitative analysis of the ozone cycle reactions and vertical temperature distribution for the simple radiation climate models presented in this work indicate that the qualitative analysis of climate models themselves and their constituent blocks allows the general behavior of the steady states of the system to be determined. Qualitative analysis of low-order models preceding comprehensive numerical analysis of the behavior of complex atmospheric-optics systems, in particular, in modeling of atmospheric photochemistry and in the study of global climate changes, is necessary for the determination of approximate ranges of variations of the parameters that may produce radical changes in the system. A detailed numerical analysis should be performed for those parameter values for which the qualitative analysis has already revealed possible nontrivial system behavior. Namely, these domains should be thoroughly studied theoretically as well as experimentally.

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### REFERENCES

1. A.A. Andronov, E.A. Leontovich, I.I. Gordon, and A.G. Mayer, *Qualitative Theory of Dynamical Systems of the Second Order* (Nauka, Moscow, 1966), 568 pp.

2. N.N. Bautin and E.A. Leontovich, *Methods and Techniques for Qualitative Analysis of Dynamical Systems on a Plane* (Nauka, Moscow, 1976), 496 pp.

3. A.S. Monin, *Introduction to the Theory of Climate* (Gidrometeoizdat, Leningrad, 1982), 246 pp.

4. D.G. Seidov, *Sinergism of Oceanic Processes* (Gidrometeoizdat, Leningrad, 1989), 287 pp.

5. M.I. Budyko, Tellus 21, No. 5, 611–619 (1969).

6. E.N. Lorenz, in: *Irreversible Phenomena and Dynamical Systems Analysis in Geophysics*, C. Nicols and G. Nicols, eds. (D. Reidel Publ. Co., 1987), pp. 557–567.

7. S. Chapman, Phil. Mag., Ser. 7 **10**, No. 64, 369–383 (1930).

8. B. Fichtelmamm and G. Sonnenmann, Ann. Geophys. **10**, No. 10, 719–728 (1992).

9. W.H. White and D. Dietz, Nature **309**, No. 5965, 242–244 (1984).

10. J.L. Fox, S.C. Wofsy, M.B. McElroy, and M.J. Prather, J. Geoph. Res. 87, No. C13, 11126–11132 (1982).

11. P. Yang and G. Brasseur, J. Geophys. Res. **99**, No. D10, 20955–20965 (1994).

12. I.L. Karol, Pure and Appl. Geophys. **118**, No. 3, 695–705 (1980).

13. M.V. Shabalova, in: Trudy Gl. Geofiz. Observ., No. 494 (1985), pp. 77–82.

14. J.J. Kozak, G. Nicolis, J.W. Kress, and M. Sanglier,

J. Non-Equilibrium Thermodyn. 4, No. 2, 67-74 (1979).

15. E.P. Gordov, O.B. Rodimova, and Yu.E. Smirnov, Opt. Atm. 1, No. 7, 56–63 (1988).

16. V.G. Monoshkina and O.B. Rodimova, Atm. Oceanic Opt. **5**, No. 2, 93–98 (1992).

17. E.P. Gordov, O.B. Rodimova, and V.A. Sennikov, Atmos. Oceanic Opt. **6**, No. 4, 274–276 (1993).

18. E.P. Gordov, O.B. Rodimova, and V.A. Sennikov, Atmos. Oceanic Opt. 7, No. 7, 484–487 (1994).

19. E.P. Gordov and E.N. Ryzhov, Izv. Vyssh. Uchebn. Zaved., Fizika, No. 10, 117-118 (1994).

20. E.P. Gordov, O.B. Rodimova, V.A. Sennikov, and A.Z. Fazliev, Atm. Oceanic Opt. 7, No. 9, 697–702 (1994).

21. E.P. Gordov and A.Z. Fazliev, Atm. Oceanic Opt. 8, No. 9, 743–750 (1995).

22. S. Manabe and R.F. Strickler, J. Atmos. Sci. 21, No. 4, 361–387 (1964).

23. V. Ramanathan and J.A. Coakley, Rev. Geophys. Space Physics **16**, No. 4, 465–489 (1987).

24. I.L. Karol' and E.V. Rozanov, Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana **18**, No. 11, 1179–1191 (1982).

25. I.L. Karol' and V.A. Frol'kis, Meteorol. Gidrol., No. 8, 59-68 (1984).

26. A.S. Ginzburg, Dokl. Akad. Nauk SSSR **208**, No. 2, 295–298 (1973).

27. G.S. Golitsyn and I.I. Mokhov, Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana 14, No. 4, 378–387 (1978).

28. A.V. Kislov, *Theory of Climate* (Moscow State University Publishing House, Moscow, 1989), 148 pp.

29. O.B. Rodimova, Atm. Opt. **4**, No. 11, 853–854 (1991).