DISTRIBUTION OF THE AEROSOL PARTICLES FLUX EMITTED FROM AN UNDERLYING SURFACE

A.I. Borodulin, B.M. Desyatkov, and S.R. Sarmanaev

Scientific Research Institute of Aerobiology, Novosibirsk
Received January 15, 1997

In this paper we deal with the flux of aerosol particles on an underlying surface. A relation connecting the instantaneous flux with the instantaneous near-surface concentration is obtained for this (generally speaking, random) value. This made it possible to obtain the expression for the distribution function of the particles' flux. Using these results, the mathematical expectation, variance, and alternation of the flux are estimated. Some data on the intensity of radio nuclides ascending in the zone of Chernobyl nuclear power plant are taken as input parameters.

An underlying surface can be a sufficiently powerful source of aerosol particles in some cases. For instance, this applies to the ocean surface emitting aerosols of sea salts. Usually, particle emission is estimated from direct measurements, by indirect gradient methods, and from data on admixture concentration above the underlying surface.[1]

This paper deals with the particle flux at the underlying surface. A relation connecting the instantaneous flux with the instantaneous near-surface concentration is obtained for the parameter which is, generally speaking, a random value. This made it possible to obtain an expression for the probability density function of the particles' flux.

Using these results, we have estimated mathematical expectation, variance, and alternation of the flux by using a one-dimensional model of aerosol admixture dispersal in the boundary layer of the atmosphere. The model is valid for horizontally homogeneous fragments of the underlying surface. The results of the study of nuclide ascent rate in the zone of Chernobyl accident were taken as the initial data.

Let us consider the definition of the flux of aerosol particles from an underlying surface:

$$q = (U_z\, C)\Big|_{z=z_0}\,, \qquad (1)$$

where q is the flux of particles; U_z is the vertical component of wind velocity; C is the aerosol concentration; z_0 is the vertical coordinate of the underlying surface. According to Eq. (1), the flux is a product of two random (due to atmospheric turbulence) values, thus being a random value too. The flux q is an instantaneous (defined at the moment t), single-point (defined at a point with coordinates x, y, and z_0) parameter. Its meaning is the number of particles emitted from a unit area of the underlying surface per unit time.

According to Eq. (1), the value q depends on the instantaneous near-surface value of the admixture concentration, i.e., $q = q(C)$. Let us assume that the concentration is low near the underlying surface. Then, the flux can be approximately represented in the form

$$q(C) \approx q(0) + \frac{\partial q(C)}{\partial C}\Big|_{C=0} C = VC\Big|_{z=z_0}\,. \qquad (2)$$

The right-hand side of Eq. (2) takes into account that the flux always equals zero for zero concentration. Since the derivative of q with respect to C in Eq. (2) is taken for a non-random, zero concentration value, and wind velocity also vanishes on the underlying surface, we suppose that the value V is not random. According to Eq. (2), V has the meaning of the characteristic linear rate of emission of the aerosol particles from the underlying surface.

The relation (2) justifies the fact that the instantaneous value of particles' flux is proportional to the instantaneous value of their concentration near the underlying surface. Similar, but averaged relation is usually taken for an admixture falling onto the underlying surface.[2] Thus, the standard boundary condition for the admixture concentration also implicitly supposes that the near-surface concentration value is low and, therefore, that the Eq. (2) is valid.

According to Eq. (2) the probability density function of the flux q can be assigned if and only if the probability density function of the aerosol particles' concentration is known for $z = z_0$. The solution to the problem of determining the single-point probability density function $f(C)$ for the concentration of passive admixtures dispersed in the turbulent atmosphere, and experimental justification of the obtained theoretical results are described in Ref. 3:

0235-6880/97/09 681-04 $02.00

$$f(C,\,t) = (1-\gamma_C)\,\delta(C) + f^{(1)}(C,\,t);$$

$$f^{(1)}(C,\,t) = = \frac{1}{\pi^{1/2}\,\beta}\left\{\exp\left[-\left(\frac{C-\bar{C}}{\beta}\right)^2\right] - \exp\left[-\left(\frac{C+\bar{C}}{\beta}\right)^2\right]\right\};$$

$$\gamma_C = \operatorname{erf}\left(\frac{\bar{C}}{\beta}\right), \tag{3}$$

where γ_C is the probability of observing non-zero concentration (this parameter is called alternation); $\delta(...)$ is the delta-function; C is the mathematical expectation of the admixture concentration; β is the second parameter of the probability density function; erf(...) is the probability integral.

The second parameter β can be most conveniently determined by the expression for the concentration variance σ^2 (see Ref. 3)

$$\frac{\sigma^2}{\bar{C}^2} = \gamma\left[\frac{1}{2}\left(\frac{\beta}{\bar{C}}\right)^2 + 1\right] - 1 + \frac{\beta}{\pi^{1/2}\,\bar{C}}\exp\left[-\left(\frac{\bar{C}}{\beta}\right)^2\right]. \tag{4}$$

Taking into account the linear connection between q and C, let us write the probability density function of the particles' flux $f_q(q,\,t)$

$$f_q(q,\,t) = \frac{1}{V}\,f\left(\frac{q}{V},\,t\right)\Bigg|_{z=z_0}. \tag{5}$$

Besides, according to Eqs. (2) and (3), we obtain the expressions for the values which will be considered below:

$$\bar{q} = V\,\bar{C}\,\Big|_{z=z_0};\ \sigma_q^2 = V^2\,\sigma^2\,\Big|_{z=z_0};\ \gamma = \operatorname{erf}\left(\frac{\bar{C}}{\beta}\right)\Bigg|_{z=z_0}, \tag{6}$$

where σ_q^2 is the variance of the flux; γ is its alternation.

The results presented below are based on the solution of a semiempirical equation of the turbulent diffusion and the equation for the concentration variance.[2,5] So, let us discuss the boundary conditions on the underlying surface.

The boundary condition for $\bar{C}$ has the form[2]

$$\left[(K_{zz}+\nu)\,\frac{\partial\bar{C}}{\partial z} + V\,\bar{C}\right]\Bigg|_{z=z_0} = 0, \tag{7}$$

where K_{zz} is the turbulent diffusion coefficient corresponding to the coordinate z; ν is the coefficient of particles' molecular diffusion.

The boundary condition for σ^2 follows from Ref. 4:

$$\left[(K_{zz}+\nu)\,\frac{\partial\sigma^2}{\partial z} + 2V\,\sigma^2\right]\Bigg|_{z=z_0} = 0. \tag{8}$$

If the process of particles' dispersal is assumed to be horizontally homogeneous and quasistationary, one can determine $\bar{C}$ and σ^2 by semiempirical equations[2,5]

$$-\frac{\partial}{\partial z}\,(K_{zz}+\nu)\,\frac{\partial\bar{C}}{\partial z} = 0; \tag{9a}$$

$$-\frac{\partial}{\partial z}\,(K_{zz}+\nu)\,\frac{\partial\sigma^2}{\partial z} = 2\,K_{zz}\left(\frac{\partial\bar{C}}{\partial z}\right)^2 - E_\sigma. \tag{9b}$$

For the dissipation rate of the concentration variance E_σ, assume, according to Ref. 5 that $E_\sigma = \varepsilon(q_\sigma b^2)^{-1}\sigma^2$, where ε is the dissipation rate of the turbulent energy b^2, C_σ is an empirical constant.

It follows from Eq. (9a) that the value q does not depend on z:

$$-(K_{zz}+\nu)\,\frac{\partial\bar{C}}{\partial z} = \bar{q};\quad z \ge z_0. \tag{10}$$

Let us first solve the equation (9a) with the boundary conditions $\bar{C}(z_1) = C_1$; $\bar{C}(h) = 0$, where C_1 is the mathematical expectation of concentration at height z_1, and h is the height of the boundary layer of the atmosphere. From Eqs. (9a) and (10) it follows that

$$\bar{q} = \left[-(K_{zz}+\nu)\,\frac{\partial\bar{C}}{\partial z}\right]\Bigg|_{z=z_1}. \tag{11}$$

Then we solve equation (9a) with the boundary condition (11) at $z = z_0$ and $\bar{C}(z_1) = q_1$. Taking into account Eqs. (6) and (11), we obtain

$$V = \bar{q}/[\bar{C}(z_0)]. \tag{12}$$

To obtain the parameter β in Eq. (3), one should solve the equation (9b) with the boundary conditions (8) and $\sigma^2(h) = 0$.

To obtain the mathematical expectation of the flux $\bar{q}$, σ_q^2, and the emission rate V, it is necessary to know K_{zz}. In this paper, this value is assigned based on the hypothesis[5] that

$$K_{ij} = C_\varphi\,\frac{b^2}{\varepsilon}\,\tau_{ij}, \tag{13}$$

where K_{ij} is the ijth component of the tensor of turbulent diffusion coefficients; $C_\varphi = 0.13$; τ_{ij} is the Reynolds viscous stress tensor. The hypothesis (13) is confirmed by laboratory and field experiments.[3]

To obtain the values ε, b^2, and τ_{ij}, we used the algebraic model[5] similar to that described in Ref. 6.

The assumption that the problem is horizontally homogeneous and quasistationary makes it possible to apply simplified equations of the boundary layer dynamics to determining mean values of the wind velocity and temperature. For this purpose, we used a numerical-analytical model.[7]

In the algebraic and numerical-analytical models isolate the boundary atmospheric layer in which the relations of the similarity theory[2] are used. Different influence of the underlying surface on the turbulent regime within the near-surface layer of the atmosphere and out of it makes it necessary to consider two-layer problems when using the algebraic and numerical-analytical models.[2,7]

In Ref. 1, the rate of wind radionuclide ascent from the territories near the Chernobyl nuclear power plant was studied experimentally. The results were used for calculations using the above model. The initial data used were obtained under horizontal homogeneity and averaged over a three-day interval. On the average, thermal stratification of the atmosphere was neutral, so the concentration profiles obtained for radionuclides ^{144}Ce, ^{103}Ru, ^{137}Cs and those of wind velocity up to the height 15 m were close to the logarithmic[1] ones.

In calculations, the wind velocity profile was reconstructed first from the value of average wind velocity at the height $z_1 = 2$ m with the numerical-analytical model.[7] The reconstructed profile was used for computing ε, b^2, τ_{zz}, and K_{zz} using the algebraic model.[6] Then the concentration C_1 from Ref. 1 and normalized by the value $\bar{C}(z = 1\text{ m})$ was used for computing concentration profiles and concentration variance using the algorithm discussed above.

Since the initial data in Ref. 1 were normalized by $\bar{C}(z = 1\text{ m})$, the mean value of the flux and its variance were obtained as normalized by the same value. They are presented below in conventional units. At the same time, this normalization obviously does not influence the flux alternation and the rate of particles' emission from the underlying surface.

Let us consider the obtained results. The Table I presents the values of $\bar{C}$, σ, and alternation of radionuclide concentration γ_C calculated by the above-mentioned algorithm for the following initial data: $\bar{U}(z = 2\text{ m}) = 3$ m/s, $z_0 = 0.1$ m, neutral stratification of the atmosphere, and $\bar{C}$ $(z = 1\text{ m}) = 1$ conv.unit. The calculated values of the normalized radionuclide flux for the isotope ^{144}Ce and the characteristic rate of particles' emission from the underlying surface are $2.5 \cdot 10^{-2}$ conv.units and $5.2 \cdot 10^{-3}$ m/s, respectively. The standard deviation of the flux is $1.9 \cdot 10^{-3}$ conv.units. By virtue of the aforesaid statements, the alternation of the radionuclide flux γ, in fact, turned out to be equal to unity. It is characteristic that the alternation of concentration γ_C (see the Table) is considerably less than unity. This is connected with a strong influence of turbulence on the distribution of concentration over the underlying surface.

TABLE I. Profiles of the radionuclides concentration (^{144}Ce)*, variance, and alternation calculated using the data from Ref. 1*

z, m	0.5	1.0	2.0	3.0	4.0
$\dfrac{\bar{C}}{\bar{C}\ (z = 1\text{ m})}$	1.61	1.00	0.41	0.12	0.02
$\dfrac{\sigma}{\bar{C}\ (z = 1\text{ m})}$	0.32	0.29	0.24	0.16	0.07
γ_C	1.00	1.00	0.91	0.50	0.14

The intensity of wind radionuclide ascent $\alpha = q/p$ and the empirical coefficient of wind ascent $R = \bar{C}(z = 1\text{ m})/p$, where p is the density of aerosol fallout on the underlying surface, were analyzed in Ref. 1. The absence of data on the value p does not allow one to obtain the values α and R. So we computed their ratio $\alpha/R = q/\bar{C}(z = 1\text{ m})$. According to Ref. 1, the value α/R averaged over five experiments is $3.0 \cdot 10^{-2}$ m/s. According to our calculations, it is $2.5 \cdot 10^{-2}$ m/s. The coincidence is, obviously, quite satisfactory if one takes into account that the values of the parameters α and R presented in Ref. 1 have a considerable spread.

Thus, this paper complements the methods for determining particle flux from the data on admixture concentration, which were mentioned in the formulation of the problem, by the possibility of determining other practically important statistical properties. Since the sign of the particle flux was not specified above, the theoretical results obtained are quite applicable to particles' deposition onto an underlying surface, and under conditions of competition between particles' ascent and deposition.

REFERENCES

1. E.K. Garger, G.P. Zhukov, and Yu.S. Sedunov, Meteorol. Gidrol., No. 1, 5–10 (1990).
2. A.S. Monin and A.M. Yaglom, *Statistical Hydromechanics* (Nauka, Moscow, 1965), Vol. 1, 640 pp.
3. A.I. Borodulin, G.M. Maistrenko, and B.M. Chaldin, *Statistical Description of the Aerosol Dispersal in the Atmosphere* (State University, Novosibirsk, 1992), 124 pp.
4. A.I. Borodulin, Izv. RAN, Fiz. Atmos. Okeana, No. 30, 125–126 (1994).
5. W. Rodi, in: *Prediction Methods for Turbulent Fluxes*, ed. by W. Kollman (A. von Karman Institute Book Hemisphere Publishing Corporation, 1980).

6. E.N. Teverovskii and E.S. Dmitriev, *Transport of Aerosol Particles by Turbulent Fluxes* (Energoatomizdat, Moscow, 1988), 160 pp.

7. B.M. Desyatkov, in: *Proceedings of the West-Siberian Regional Scientific Research Institute,* issue 77, 68–75 (1986).