QUANTUM UNCERTAINTY AND ITS CONTRIBUTION TO THE PROCESS OF NONLINEAR PROPAGATION OF SCHRODINGER'S SOLITON THROUGH A LIGHTGUIDE

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Based on the rigourous solution of Schrodinger's quantum equation the evolution of the fundamental optical soliton with the Kerr type of nonlinearity is investigated. It is established that the quantum fluctuations steadily grow large over the entire path of nonlinear propagation that finally results in the annihilation of the soliton. The physical nature of this effect is clarified with the help of tetraphoton interaction model. It is shown that these effects restrict the possibility of production of the squeezed quantum states of light pulses.

INTRODUCTION

One of the most attractive features of Schrodinger's optical solutions, being formed and propagated in the nonresonance media with cubic nonlinearity, is their stability. Constant shape and regular phase of fundamental soliton follow from Schrodinger's classic equation. Moreover, the soliton is stable with respect to original noise modulation and in the process of nonlinear propagation it is subjected to "self–cleaning" from the fluctuation components.[1,2]

Unfortunately, its quantum pattern appears to be not so optimistic. Thus, one of the results of development of the successive quantum theory of pulse evolution in the nonlinear lightguides[3-10] is the conclusion about increasing uncertainty in phase and amplitude as well as about the dispersion spreading of the solitons.[11-13] However, approximations used in the indicated papers restrict the applicability of this statements only by the initial stage of nonlinear propagation.

What happens subsequently? Does classic property of "self–cleaning" compensate for the growth of quantum fluctuations when going over to the far diffraction zone? In fact the soliton becomes free of noise just for long mean free paths. Or does the destabilizing effect of quantum uncertainty intensify with time and finally does it lead to annihilation but not to formation of the ideal soliton? This paper is devoted to finding out the answers on these and other questions.

BASIC RELATIONS

Evolution of the electric field of a one–dimensional radiation field entering the transparent medium with cubic nonlinearity to the second order of the dispersion theory can be described by the following equation:[3-13]

$$(\partial/\partial z + u^{-1}\partial/\partial t)\, E^{(+)}(z,\,t) = \Big[(i/2)\, g\, \partial^2/\partial t^2 + (ik\varepsilon_{nl}/2\varepsilon_0)\, E^{(-)}(z,\,t)\, E^{(+)}(z,\,t)\Big]\, E^{(+)}(z,\,t)\,. \quad (2.1)$$

Here, $E^{(+)}(z, t)$ and $E^{(-)}(z, t)$ are the operators of positive and negative frequency parts of the field in Heisenberg representation slowly varying with time, the z axis is directed along the propagation path, t is time, $u = (\partial k/\partial\omega)^{-1}$ is the group velocity at the carrier frequency ω, k is the carrier wavenumber, parameter $g = \partial^2 k/\partial\omega^2$ characterizes the dispersion of the group velocity, ε_{nl} and ε_0 are nonlinear and linear parts of dielectric constant of the medium. Interaction is assumed collinear, spatial mode is assumed plane, and nonlinearity is assumed instantaneous. The derivation of Eq. (2.1) has been presented in detail for example in Refs. 4 and 9, therefore we will not dwell it specially. We note only that in order to go over to classic equation[1] it is sufficient to replace $E^{(+)}$ and $E^{(-)}$ by the pair of complex conjugate amplitudes A and A^*.

Relation (2.1) is reduced to Schrodinger's nonlinear quantum equation by introducing the nondimensional variables:

$$x = ut - z\,,\ S = gu^2 z/2\,,\ \varphi(S,\,x) = E^{(+)}(z,\,t)/|A_0|\,,$$

$$c = -\,k\,\varepsilon_{nl}|A_0|^2/\,2u^2 g\varepsilon_0\,, \quad (2.2)$$

where x is the displacement from the top of the pulse propagating with the velocity u, S is normalized distance passed by it, $\varphi(S, x)$ is the normalized operator of photon annihilation at the points S and x, and A_0 is the amplitude of the pulse at its top.

According to the notation used in literature, we will replace the variable S by t which actually has the sense of normalized propagation time. Then we obtain

$$i\partial\varphi(t,\,x)/\partial\tau = \partial^2\varphi(t,\,x)/\partial x^2 + 2c\varphi^+(t,\,x)\,\varphi(t,\,x)\,\varphi(t,\,x). \quad (2.3)$$

This equation as well as the operators entering into it has been written in the Heisenberg representation. In addition, the following commutative relations

$$\big[\varphi(t,\,x),\,\varphi^+(t,\,x')\big] = \delta\,(x - x')\,,$$

$$\big[\varphi(t,\,x),\,\varphi(t,\,x')\big] = \big[\varphi^+(t,\,x),\,\varphi^+(t,\,x')\big] = 0\,. \quad (2.4)$$

must be satisfied.

However, Schrodinger's representation turns out to be more convenient for obtaining the rigorous solution. In this case the transformation of the vector describing the system state $|\psi\rangle$ is

$$i\hbar\mathrm{d}|\psi\rangle/\mathrm{d}t = H|\psi\rangle\,. \quad (2.5)$$

where the Hamiltonian is

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$$H = \hbar \left[\left(\int \varphi_x^+(x)\varphi_x(x)\mathrm{d}x \right) + c \int \varphi^+(x)\varphi^+(x)\varphi(x)\varphi(x)\mathrm{d}x \right]. \quad (2.6)$$

Hereinafter, if it does not pointed out specially, integration is performed between the infinite limits.

The soliton–like solution of Eq. (2.5) exists only for negative c. It can be represented as superposition of Fock's states $|n, p\rangle$ with the fixed number of photons n and the momentum p[11]

$$|\psi\rangle| = \sum_n a_n \int g_n(p)\, \mathrm{e}^{-iE(n,\,p)\,t}\, |n, p\rangle \mathrm{d}p \,. \quad (2.7)$$

Here the states $|n, p\rangle$ are eigenstates for the Hamiltonian of Eq. (2.6)

$$|n,p\rangle = (n!)^{-1/2} \int F_{np}(x_1, \ldots, x_n)\varphi^+(x_1) \ldots \varphi^+(x_n)\mathrm{d}x_1 \ldots \mathrm{d}x_n |0\rangle, \quad (2.8)$$

$$F_{np}(x_1, \ldots, x_n) = N_n \exp\left[ip\sum_{j=1}^{n} x_j + (c/2) \sum_{1\le i} \sum_{<j\le n} |x_j - x_i| \right] \quad (2.9)$$

and the normalization factor N_n is determined from the conditions $\langle n', p'|n, p\rangle = \delta_{nn'}\, \delta(p - p')$

$$N_n^2 = |c|^{n-1} (n - 1)\,!/2\pi \,, \quad (2.10)$$

in addition

$$\int |F_{np}(x_1, \ldots, x_n, t)|^2\, \mathrm{d}x_1 \ldots \mathrm{d}x_n = 1 \,. \quad (2.11)$$

Energies $E(n, p)$ are eigenvalues of the Hamiltonian of Eq. (2.6) and of the state given by Eq. (2.8)

$$E(n, p) = np^2 - c^2 n\, (n^2 - 1)/12 \,. \quad (2.12)$$

If the pulse entering the lightguide represents the set of the coherent modes, then the weighting coefficients a_n and the functions g_n obey the Poissonian and Gaussian distributions

$$a_n = \alpha_0^n \exp\left(-n_0/2 \right)/(n!)^{1/2} \,,$$

$$g_n(p) = \pi^{-1/4}\Delta p^{-1/2} \exp\left[-(p - p_0)^2/2\Delta p^2 - inpx_0 \right], \quad (2.13)$$

where $n_0 = |\alpha_0|^2$ is the average number of photons in the pulse, in addition

$$\sum_n |a_n|^2 = 1, \; \int |g_n(p)|^2\, \mathrm{d}p = 1 \,. \quad (2.14)$$

Let us first determine the average amplitude of the pulse in the process of its nonlinear propagation (the derivation is given in Appendix A)

$$\langle\psi|\varphi(x)|\psi\rangle \simeq \sum_n \left[n(n + 1)/|c|q_1 \right]^{1/2} \times$$

$$\times\, a_n^*\, a_{n+1} \exp\left\{ (itc^2 n(n + 1)/4 + \right.$$

$$+ \left[(ip_0(x - x_0 - p_0 t) - (x - x_0)^2\Delta p^2/4 \right]/q_1 \Big\} \times$$

$$\times \int \exp\Big\{ \Big\{ -\left[(\Delta p^{-2} + i2t + 4t^2 n(n + 1)\Delta p^2 \right] p^2 +$$

$$+\, i2(n + 1/2)\,(x - x_0 - 2p_0 t)\, p \Big\}/q_1 \Big\} \times$$

$$\times\, \mathrm{sech}\,(2\pi p/|c|)\, \mathrm{d}p \,, \quad q_1 = 1 + it\Delta p^2 \,. \quad (2.15)$$

In contrast to Ref. 11, no restrictions on the mean free path (on the parameter t) have been used in the derivation of this relation. The approximation sign refers only average number of photon n_0 which must be much greater than unity. It is applicable in practice.

At the initial stage ($t\Delta p^2 \ll 1$) when the condition

$$|c| \ll \Delta p \ll n_0|c| \quad (2.16)$$

relation (2.15) is reduced to the superposition of the pulses with envelopes in the form of hyperbolic secant, i.e., of classical solitons. If we neglect the spreading in the number of photons n and in the momentum p taking $n = n_0$ and $p = p_0$, then we obtain the fundamental classical soliton in a pure form

$$\langle\psi|\varphi(x)|\psi\rangle \simeq 2^{-1}(n_0 - 1)|c|^{1/2} \times$$

$$\times \exp\left[i(n_0 - 1)^2 c^2 t/4 + ip_0(x - x_0 - p_0 t) \right] \times$$

$$\times\, \mathrm{sech}\left[2^{-1}(n_0 - 1)|c|(x - x_0 - 2p_0 t) \right] . \quad (2.17)$$

However, the characteristic of quantum treatment consists in the fact that the exact values of the number of photons n and the momentum p, by virtue of the uncertainty principle, give rise to absolute uncertainty in phase and coordinate of this pulse. And this means that its average amplitude will be zero, and its envelope will be independent of x.

Thus, the necessary condition for existing the soliton is the spread of energy and momentum, that, as we will further convince, leads to quite unfortunate consequences, appearing in the process of nonlinear propagation.

EVOLUTION OF THE SOLITON SHAPE AND OF THE PHOTON NUMBER FLUCTUATIONS

For clarification of the pulse dynamics let us first calculate the average intensity

$$\langle N(x)\rangle \equiv \langle\psi|\varphi^+(x)\, \varphi(x)|\psi\rangle = 2|c|^{-1} \exp(-n_0) \times$$

$$\times \sum_n \left(n^2 n_0^n/n!) \right) \int G_n(x, p)\, \mathrm{d}p \,, \quad (3.1)$$

$$G_n(x, p) \simeq p\, \mathrm{sh}^{-1}(2\pi p/|c|) \exp\left[-(\Delta p^{-2} + 4t^2 n^2\Delta p^2) \times p^2) + \right.$$

$$\left. +\, (i2n(x - x_0 - 2p_0 t)p \right] . \quad (3.2)$$

Approximation sign in Eq. (3.2) refers only to the requirement on the average number of photons in the pulse: to satisfy the condition $n_0 \gg 1$. In analogy with Eq. (2.15), no recommendations for the calculation are given in Appendix B.

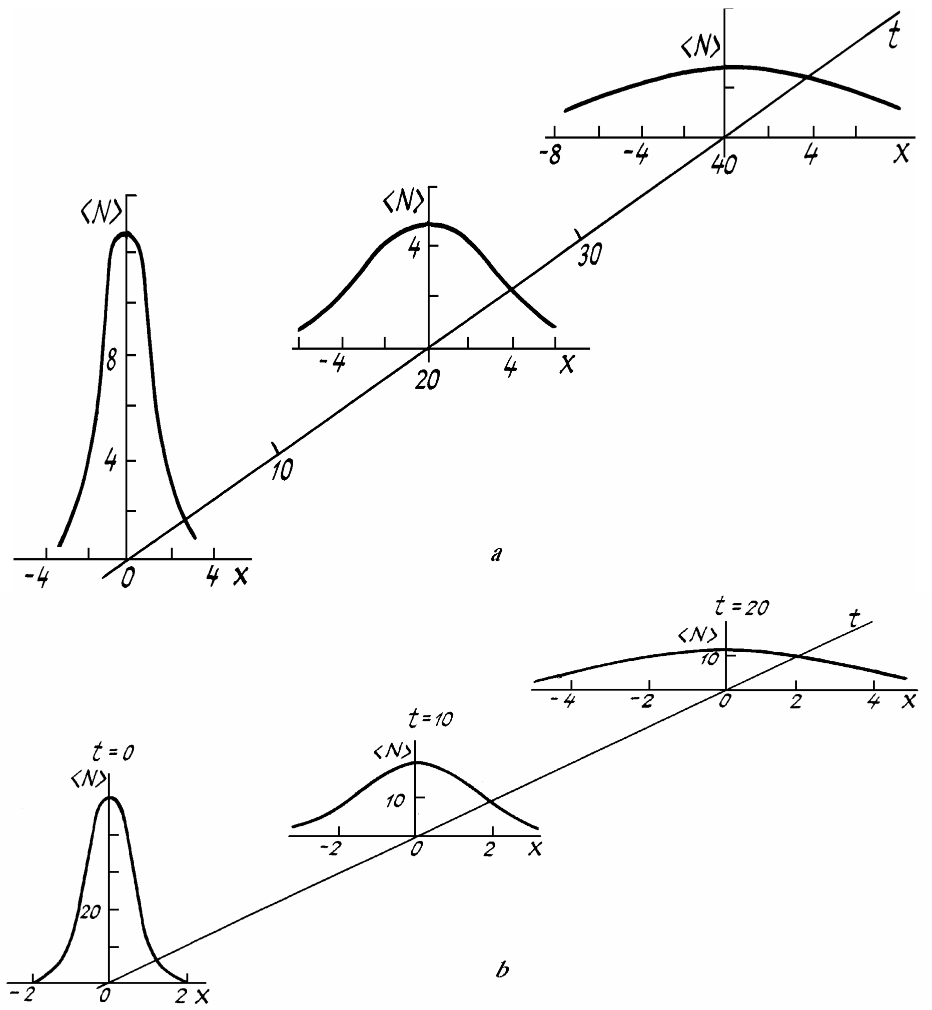

FIG. 1 Evolution of the soliton envelope in the process of its nonlinear propagation: a) $n_0 = 40$ and (b) $n_0 = 80$. The rest of the parameters are the same in both cases: $c = \pi/100$, $\Delta p = 0.1$, and $p_0 = x_0 = 0$.

For analytical evaluation of the evolution of the soliton let us first assume that the spread of its components in n is of secondary importance for this process by taking $n = n_0$. Then the integrand in Eq. (3.1) with the exception of phase exponent is the pulse spectrum. It is seen that this spectrum gets narrower with time due to the term $\exp[-(2tnp\Delta p)^2]$ that, in its turn, gives rise to continuous spreading of the soliton in the process of its nonlinear propagation.

Further analysis of Eqs. (3.1) and (3.2) shows that characteristic time of a double pulse broadening depends on the ratio between the parameter c and the range of variation of the momentum distribution function Δp. In addition, three following modes can be identified:

$$t_{ch} \simeq 2/n_0|c|\Delta p \text{ for } \Delta p \gg |c| , \quad (3.3)$$

$$t_{ch} \simeq 2^{1/2}/n_0|c|\Delta p \text{ for } \Delta p \simeq |c| , \quad (3.4)$$

$$t_{ch} \simeq 3^{1/2}/\ 2n_0\Delta p^2 \text{ for } \Delta p \ll |c| , \quad (3.5)$$

Relation (3.3) has been obtained previously in Ref. 11, however, only for small t. Here it is generalized to the case of arbitrary time.

Correctness of the assumption and of these estimates is confirmed by numerical calculations made according to Eqs. (3.1) and (3.2). Results of calculations are shown in Figs. 1 and 2. It can be seen that the soliton completely annihilates with time. The correctness of this conclusion becomes more evident after determining the variance of the photon number fluctuations

$$\langle \Delta N^2(x)\rangle \equiv \langle\psi|\varphi^+(x)\,\varphi^+(x)\,\varphi(x)\,\varphi(x)|\psi\rangle + \langle N(x)\rangle - \langle N(x)\rangle^2 \simeq \Big[\exp(-n_0)/3\Big] \times \sum_n n^2(n^2-1)\,(n_0^n/n!) \int (1 + 4p^2/c^2) \times G_n(x, p)\,\mathrm{d}p + \langle N(x)\rangle - \langle N(x)\rangle^2 . \quad (3.6)$$

The derivation of this relation is given in Appendix 3. Results of numerical computations are shown in Fig. 2.

Unfortunately, relation (3.6) becomes more critical to the validity of the condition $n_0 \gg 1$ than Eqs. (3.1) and (3.2). Therefore, the regions of quantum uncertainty in the number of photons are not shown in Fig. 1. However, for $n_0 = 500$ approximate relation (3.6) is quite applicable.

It follows from the given data that the process of nonlinear propagation of the soliton is accompanied not only by the spread in its envelope but also by continuous growth of the amplitude fluctuations that intensifies the process of its gradual annihilation. So, the photon statistics which initially obey the Poisson distribution with $\langle\Delta N^2(x)\rangle = \langle N(x)\rangle$, transforms into the super–Poisson one with $\langle\Delta N^2(x)\rangle > \langle N(x)\rangle$. This important conclusion could not be made in previous works[3–13] because of inadequate model for large mean free paths. For example, both the Hartree approximation[11] and the quasistatic approximation of a given channel[8–10] lead to the conclusion about the invariability of the photon statistics in the process of nonlinear propagation and keeping it the Poisson one.

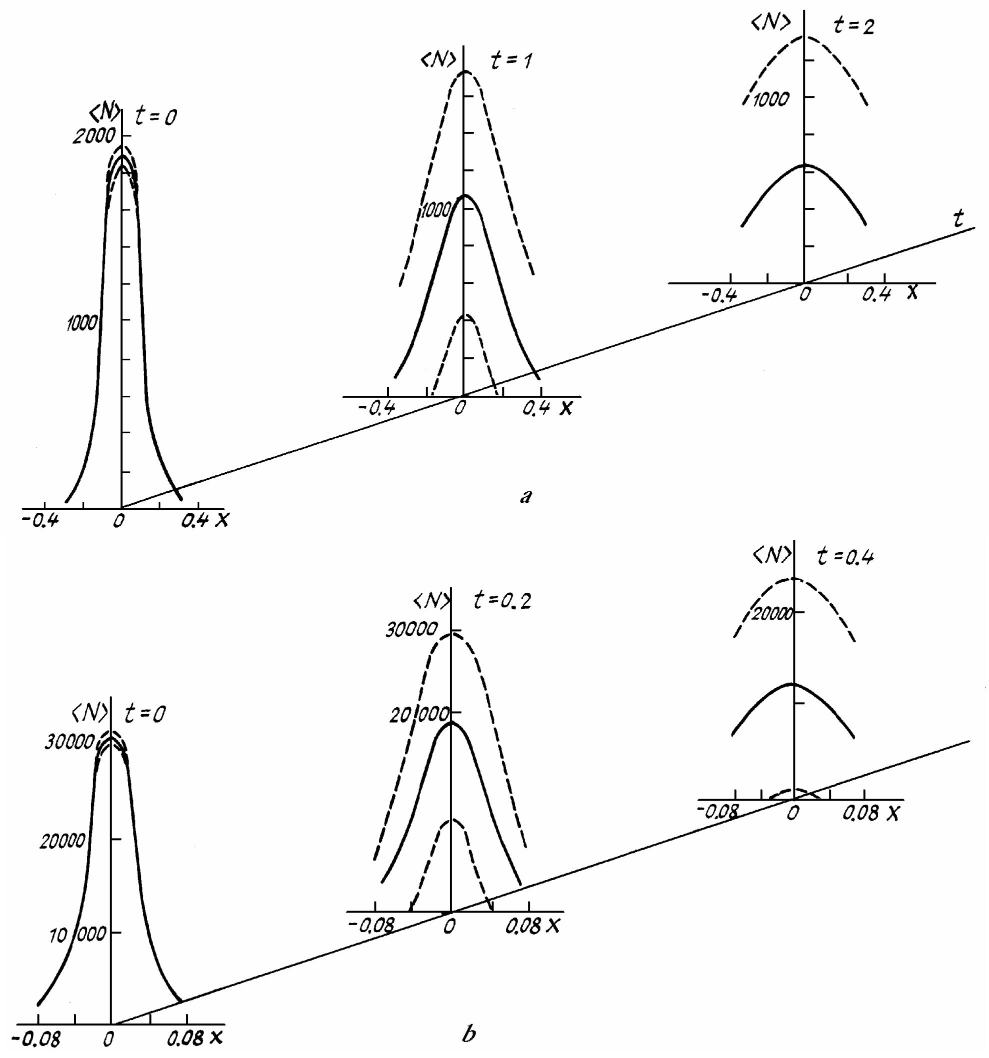

FIG. 2 Plots illustrating the annihilation of the soliton in the process of its nonlinear propagation. Dashed curves indicate the boundaries of the region of quantum uncertainty in the number of the photons calculated in accordance with Eqs. (3.2) and (3.6). a) $n_0 = 500$; b) $n_0 = 2000$. The rest of the parameters are the same as in Fig. 1.

Results shown in Fig. 2 suggest also that practically complete "noising" of the pulse happens exactly at the stage of its double broadening.

What is the reason of such a behavior? Why the classical property of "self–cleaning" of the soliton from the fluctuations is not manifested?[1,2] To explain the situation let us apply the following model. The soliton entering the lightguide and representing the set of modes in coherent states with different amplitudes may be represented as the superposition of classic envelope in the form of hyperbolic secant (being the regular component of the signal) and quantum fluctuations of the vacuum (being the noise).

In the process of soliton propagation the initial noise modulation existing only within the limits of the soliton pulsewidth is "thrown" on the wings and gradually is self–cleaned". However the soliton cannot become free of the stationary vacuum noise, because this "through–off" is accompanied by the "drift" of the fluctuations initially being outside the soliton.

But why these phenomena opposite in their action do not finally compensate for each other and the pattern does not stabilized at the certain level of the increased fluctuations? To answer this question let us analyze the nonlinear evolution of the vacuum noise in the presence of the intense regular component of the soliton. Let us use for this purpose the Heisenberg representation of Schrodinger equation (2.1) or (2.3). Let us linearize it in the fluctuational components and consider, for clarity of representation, the single–mode interaction. As a result we obtain that the average number of noise photons is[13]

$$\langle N_{\mathrm{n}}\rangle = \Psi^2 , \qquad (3.7)$$

where $\Psi = t(cn_0)^2/2$ is the nonlinear run–on of the phase acquired in the process of the propagation and n_0 is the average number of photons in the mode.

Linearization in the fluctuational components actually means that we use the tetraphoton parametric interaction model in the field of a given classical pumping (of a regular signal). In addition, according to Eq. (3.7) noise intensifies continuously as a result of pumping over the photons of regular component in fluctuational one in consequence of tetraphoton parametric amplification.

And it is not the only reason of destabilization. When going over to the far diffraction zone where the linear approximation becomes inapplicable, amplification of the fluctuations is accompanied by the depletion of the soliton by itself which contributes to pumping of the parametrically amplified vacuum noise because total number of photons must remain constant.

As a result of such irreversible processes the soliton is gradually spreaded and finally it is completely degraded. However, the natural question of principle has already arisen which we have not discussed for a while. Strictly speaking, the results of calculations of $\langle N(x)\rangle$ and $\langle \Delta N^2(x)\rangle$ do not give a complete basis for making the conclusion whether the soliton is really spreading or simply the uncertainty in x appears, while the broadening of $\langle N(x)\rangle$ is the consequence of averaging over the ensemble of solitons which start the propagation at the same time and then acquire a quantum spread in the x coordinate, i.e., different time delays? In this case a quantum–mechanical averaging makes it impossible to distinguish between these two unlike processes of evolution.

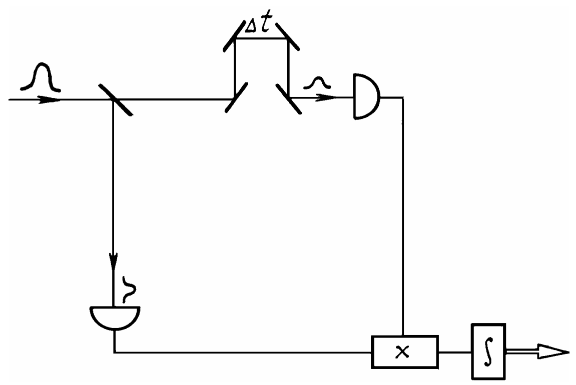

FIG. 3. Simplified diagram of a correlator. Pulse has been delayed before photodetecting in one of the channels for the time Δt with respect to the time of soliton propagation through another channel.

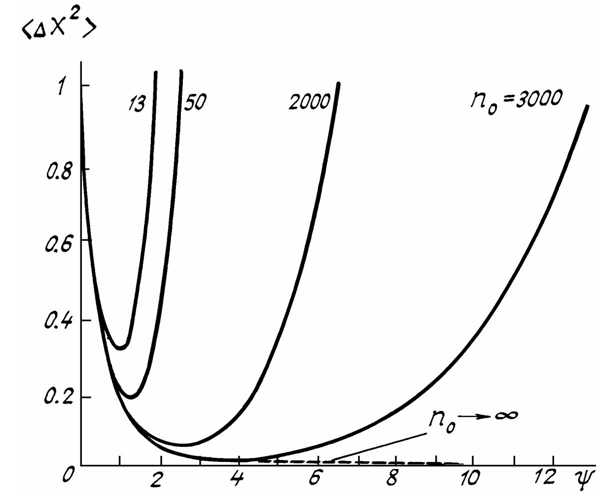

FIG. 4. Curves characterizing the evolution of the maximum squeeze ratio in the process of nonlinear propagation, i.e., with increase of Φ for different n_0 = 13, 50, 2000, and 30, 000. Dashed curve corresponds to the limit $n_0 \Rightarrow \infty$, i.e., to the ideal case without the phase fluctuations.

However, this situation is not hopeless. We can clarify the salient points by calculation of the intensity correlation function

$$K(\Delta x) = \int \langle N(x)\, N(x + \Delta x)\rangle \mathrm{d}x . \qquad (3.8)$$

Physically such a correlation function is realized, for example, in measurements of ultrashort pulsewidths. A generalized diagram is shown in Fig. 3. Measurements are independent of absolute time of the pulse arrival and are determined only by the time delay. Hence, if the soliton is not spread in the process of nonlinear propagation, the correlation function after the soliton has passed through the fiber will be the same as upon entering it (at $t = 0$) since the uncertainty in x must have no effect in this case. If the pulse is spreaded then $K(\Delta x)$ must broaden according to the degree of this spreading.

Thus

$$\langle N(x)N(x + \Delta x)\rangle \equiv \langle\psi|\varphi^+(x)\varphi^+(x + \Delta x)\varphi(x + \Delta x)\varphi(x)|\psi\rangle +$$

$$+ N(x)\,\delta(\Delta x) = \pi^{-1/2}\Delta p^{-1} \exp(-n_0)\sum_n (n_0^n/n!) \times$$

$$\times \iint \exp\{-[(p - p_0)^2 + (p' - p_0)^2])\, 2\Delta p^2 +$$

$$+ in[(x_0\,(p' - p) + t(p'^2 - p^2))]\} \times$$

$$\times F_n(x, \Delta x, p - p')\, \mathrm{d}p'\mathrm{d}p + N(x)\,\delta(\Delta x) , \qquad (3.9)$$

where the matrix element is

$$F_n(x, \Delta x, p - p') =$$

$$= \langle n, p' \,|\varphi^+(x)\, \varphi^+(x + \Delta x)\, \varphi(x + \Delta x)\, \varphi(x)|n, p\rangle .$$

Unfortunately, the direct calculation of F_n is very difficult and do not result in analytic solution in general.

For this reason we will use the following indirect estimate.

Let us go over to the new variables $p_1 = (p - p')/2$ and $p_2 = (p + p')/2$ and integrate the obtained relation over p_2. As a result omitting the subscript on p_1 we obtain

$$K(\Delta x) = n_0\delta(\Delta x) + 2\exp(-n_0)\sum_n (n_0^n/n!) \times$$

$$\times \int \exp\left\{-\left[(\Delta p^{-2} + 4t^2n^2\Delta p^2)p^2 + i2n(2p_0t + x_0)p\right]\right\} \times$$

$$\times \left[\int F_n(x, \Delta x, 2p)\,\mathrm{d}x\right]\mathrm{d}p\,. \quad (3.10)$$

Without loss of generality we may take $x_0 = p_0 = 0$. In what follows, the integration over the pulse p means that actually the external integrand describes approximately the soliton spectrum. The validity of such an approximation has been confirmed by the results of calculation of $\langle N(x)\rangle$ and of characteristic time of spreading given by Eqs. (3.3) – (3.5). Hence, this spectrum gets narrower with time, and $K(\Delta x)$ must correspondingly broaden. It means that the soliton is still spreaded! Otherwise $K(\Delta x)$ would remain constant with increase of t.

PHASE AND FREQUENCY FLUCTUATION. SQUEEZED STATES

Processes considered so far refer only to the evolution of amplitude characteristics. However, spreading and increasing the intensity noise do not close all variety of accompanying phenomena. Thus, amplification of the phase fluctuations in the process of nonlinear propagation of the soliton occurs much rapidly and is manifested perceptible at the initial stage, i.e., in the near–field zone.

It was shown in Refs. 12 and 13 that in quasistatic approximation of the given channel which is valid for small t when the dispersion spreading considered above have yet had practically no effect, the variance of the phase fluctuations of the soliton is increased according to the law

$$\langle\Delta\Theta^2\rangle = \left[1 + 4\Psi^2(t)\right]/\,4n_0\,. \quad (4.1)$$

Here $\Psi(t) = t(cn_0)^2/2$ is the doubled nonlinear phase, i.e., the nonlinear run–on of the phase in the absence of dispersion. Similar relation can be obtained and based on the estimates made in Ref. 11 with the help of the Hhartree approximation that also ignores the dispersion spreading.

If recording of pulses is made with a quadratic detector, the quantum uncertainty in phase by itself has no direct effect on measurement. However, it is the reason of at least two undesirable consequences, to say nothing of interference experiments, where the phase stability has primary importance.

First, the spread in the phase unavoidably leads to corresponding frequency destabilization. Within the framework of classic approach analogous phenomenon has been considered in Ref. 15. The propagation of the fundamental soliton with its periodical amplification for the compensation of losses in fiber has been analyzed in this paper. Resulting spontaneous noise is amplified along with the soliton. The noise is accumulated and increased, and the first symptom of its presence is the random deviation of the carrier frequency accompanying with the changes of the propagation velocity that is very undesirable in the information communication links, for which, properly, the optical solitons are intended. Thus some limiting mean free path length has arisen which limits a range of action of the information channel.

So, the variance of the frequency fluctuations in classic approximation is [15]

$$\langle\Delta\omega^2\rangle_{\rm cl} = \mathrm{e}^{\gamma t} - 1)\,A/\,3\,n_0 \simeq A\gamma t/\,3\,n_0\,, \quad (4.2)$$

where γ is the increment of an amplification which compensates for the losses and $A = (n_0 - 1)\,|c|^{1/2}/2$ is the normalized amplitude of the soliton (see Eq. (2.17)).

Simple relation (4.2) convenient for practical calculations can be generalized with an account of quantum fluctuations. It can be made by virtue of the correspondence principle[16] valid under conditions of linear amplification. However, it should be taken into account that additional "noising" due to losses (in the case of their complete compensation the increment must be doubled) as well as vacuum fluctuation (the factor of 1/2). As a result we obtain

$$\langle\Delta\omega\rangle_{\rm qu} \simeq A(2\gamma t + 1/2)\,/\,3n_0\,. \quad (4.3)$$

According to the estimates of the Ref. 15 the random deviations of frequency must be manifested for distances exceeding 1000 km.

Secondy the unpleasant consequence of the growth of the phase uncertainty is its destructive effect on production of the squeezed states.

As is well known, quantum squeezed states are associated with the possibility of suppression of the shot noise of photodetecting and corresponding improvement of the limiting characteristics of various systems using photons as carriers of information (see, for example, Refs. 10 and 17–20). One of the most promising ways of producing such states are the optical solitons.[3–10]

Let us introduce the quadrature component

$$X = \varphi\mathrm{e}^{-i\varphi} + \varphi^+\mathrm{e}^{i\varphi}\,, \quad (4.4)$$

where φ is the adjustable phase parameter which optimizes the degree of suppression of the variance of quadrature fluctuations:

$$\langle\Delta X^2\rangle \equiv \langle X^2\rangle - \langle X\rangle^2 \equiv 1 + (\langle\varphi^2\rangle\exp(-i2\varphi) +$$

$$+ \langle\varphi^+\varphi\rangle - \langle\varphi\rangle^2\exp(-i2\varphi) - |\langle\varphi\rangle|^2 +$$

$$+ \text{complex conjugate term})\,. \quad (4.5)$$

A criterion for the squeezed state is the fulfilment of the condition $\langle\Delta x^2\rangle < 1$, i.e., the decrease of the variance of the fluctuation down to the value than in vacuum.

For the fundamental soliton we have

$$\langle\psi|\varphi^2(x)|\psi\rangle \simeq \left[(2\alpha_0^2\exp(-n_0)/|c|q_2^{1/2}\right] \times$$

$$\sum_n \left\{(n_0^n(n+1)^2/\left[(n(n!)\,(n+1)!\right]^{-1/2}\right\} \times$$

$$\text{r}\,\exp\left\{itc^2(n+1)^2/2 + \left[(i2p_0(x - x_0 - p_0t) -\right.\right.$$

$$\left.\left.- (x - x_0)^2\Delta p^2\right]/q^2\right\}\int p\,\mathrm{sh}^{-1}(2\pi p/|c|) \times$$

$$\times \exp\left\{\left\{-\left[\Delta p^{-2} + i4t + 4t^2n(n+2)\Delta p^2\right]p^2 +\right.\right.$$

$$+ i2(n+1)(x - x_0 - 2p_0 t)p\}/q_2\}dp\ , \quad (4.6)$$

where

$$q_2 = 1 + i2t\Delta p^2\ .$$

The derivation of this relation is presented in Appendix 4. Let us note only that the sign of the approximation is referred to condition $n_0 \gg 1$.

Analyzing Eq. (4.6), we can conclude that the absolute value of $<\varphi^2>$ decays with t much more steeply than $<\varphi^+\varphi>$ which is determined according to Eqs. (3.1) and (3.2). This is connected first of all with the phase term $\exp[itc^2(n+1)^2/2]$ entering in Eq. (4.6) . Really,

$$\sum_n [n_0^n \exp(-n_0)/n!]\ \exp[(itc^2(n+1)^2/2] \simeq$$

$$\simeq q^{-1/2}\exp[itc^2(n_0+1)^2/2q]\ ,\ q = 1 - itc^2 n_0\ , \quad (4.7)$$

and increase of t is accompanied by decrease of the absolute value given by Eq. (4.7). In these calculations we have taken $n_0 \simeq 1$ and used the Poisson model distribution given by Eq. (B.4).

But faster decay of $|<\varphi^2>|$ in comparison with $<\varphi^+\varphi>$ according to Eq. (4.5), reduces the maximum squeeze ratio. Thus, effective generation of squeezed states at the initial stage of nonlinear propagation[3–11] must be followed by their degradation. We can separate out at least two reasons of such a behavior which consist in the following.

Within the framework of the considered tetraphoton model of nonlinear interaction, when the soliton is passing through the fiber, the regular component pumps the parametrically amplified vacuum fluctuations. In addition, the depletion of pumping occurs due to pumping over the photons in the noise component. This fact gives rise to the degradation of squeezing. The same effect takes place, for example, in the process of triphoton parametrical amplification.[21]

Second reason is more important and is manifested at the initial stage of propagation. It consists in growth of the phase fluctuations of the soliton, i.e., the pumping, whose destructive effect is analogous to the generation of squeezed states in parametric amplifiers studied in Refs. 23–24 with the only difference that synchronous amplification has been analyzed there. In our case the interaction is asynchronous in principle due to the nonlinear run–on of the of pumping phase. One can read about this in ample detail, for example in Ref. 13.

To evaluate analytically the effect of phase fluctuation let us use the following simple model. Within the framework of quasistatic approximation of the given channel the variance of the quadrature component at the top of the soliton is[8–10]

$$<\Delta X^2(t,0)> = 1 - 2\Psi \sin^2(\varphi - \Psi/2) + 4\Psi^2\sin^2(\varphi - \Psi/2). \quad (4.8)$$

Optimum squeezing, i.e., minimisation of $<\Delta x^2>$ can be obtained by adjustment of the phase parameter $\varphi = \varphi_0$ such that

$$\tan(2\varphi_0 - \Psi) = \Psi^{-1}\ . \quad (4.9)$$

However, the quantum spread in the soliton phase makes it impossible to satisfy the last condition. Thus we can conclude that even in the optimum case we have

$$<\Delta X^2(t,0)>_{\min} \simeq 1 - 2\Psi \sin(2\varphi_0 + <\Delta\Theta^2>^{1/2} - \Psi) +$$

$$+ 4\Psi^2 \sin^2[((2\varphi_0 + <\Delta\Theta^2>^{1/2} - \Psi)/2]\ , \quad (4.10)$$

where $<\Delta\theta^2>$ is given by Eq. (4.1).

Results of numerical calculation of the minimum squeeze ratio of the quadrature quantum fluctuations are shown in Fig. 4. It can be seen that effective production of squeezed states of the optical soliton is possible only for the fixed range of variations of its mean free path length and when they became large they influence destructively. However with the use of the high–intensive solitons with $n_0 > 10^5$ the degree of suppression of the fluctuations can be very high and does not limited by the considered effects. Nevertheless, the quantum limit is important from the principal point of view.

CONCLUSION

So, we have established that quantum effects accompanying the propagation of Schrodinger's soliton in the natural nonlinear lightguide lead to its (the soliton) graduate but steady annihilation. We have also clarified the tetraphoton nature of this phenomenon. For practical application of the obtained results we will estimate the maximum possible mean free path length of the soliton in fiber.

In accordance with Eqs. (2.15), (3.1) and (3.2) the fundamental soliton being adequately described classically, i.e., with the envelope in the form of hyperbolic secant of (2.17) can exist in fiber only when $|c| < \Delta p$. Taking in the limiting case, corresponding to minimal spreading, $\Delta p \simeq |c|$ from Eq. (3.4) we have

$$t_{\lim} = 2^{1/2}/n_0 c^2 \equiv 2^{1/2} n_0 T/8\pi \simeq n_0 T/20\ . \quad (5.1)$$

Here the period of the soliton $T = 8\pi/(n_0 c)^2$ is the time required for the nonlinear run–on of the phase to reach 2π.

Taking into account the fact of practically complete annihilation of the soliton as a result of growth of the noise with double broadening of its average intensity profile, established by us, we can conclude that $t_{\lim}$ really determines maximum possible propagation path.

Furthermore, because in real situations $n_0 \gg 1$, $t_{\lim}$ is much greater than T. This means that the amplitude quantum effects will be manifested only for the very long mean free path lengths or in the media with high degree of nonlinearity, i.e., under conditions when other factors such as losses and amplification necessary for their compensation,[13,15] fiber inhomogenities,[4] and dispersion effects of the third and higher orders as well as finite time of nonlinear response[25] can have significant destabilizing effect. Nevertheless, the revealed quantum annihilation of the soliton emposes principal limitations on the maximum length of the propagation path that, undoubtedly, is important.

The above discussion have touched upon only the average intensity and amplitude noise. The phase fluctuations increase faster. And with an account of the random deviation of carrier frequency, we obtain the pattern of the complete destabilization of the soliton. This is the quantum uncertainty that is the reason of all this. It should be also noted that in the process of detecting ultrashort pulses it is necessary to account the interesting peculiarities of the photocount statistics.[26]

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APPENDIX A: DETERMINING THE AVERAGE PULSE AMPLITUDE

Appendix A explains the derivation of relation (2.15). So, in accordance with Eq. (2.7), we have

$$<\psi|\varphi(x)|\psi> = \sum_{n'}\sum_{n} a^*_{n'}a_n \int\int g^*_{n'}(p')g_n(p) \times$$

$$\times \exp(it[(E(n', p') - E(n, p))]<n', p'\ |\varphi(x)|n, p> \mathrm{d}\ \mathrm{d}p' \ . \quad (A.1)$$

The results of calculation of the matrix element $<n', p'|\varphi(x)||n + 1, p>$ gives[11]

$$<n', p', \ |\varphi(x)|\ n + 1, p> = \delta_{nn'}(2\pi)^{-1}\left[(n(n + 1)\right]^{1/2} \times$$

$$\times (n - 1)!\ n!\ |c|^{2n-1/2} \times$$

$$\times \left\{\prod_{r=1}^{n}\left[(p - p')^2 + c^2(2n - 2r + 1)^2/4\right]^{-1}\right\} \times$$

$$\times \exp\{i[(n + 1)p - np']x\} \simeq \quad (A.2)$$

$$\simeq \delta_{nn'}\ 2^{-1}|c|^{-1/2}\left[(n(n + 1)\right]^{1/2} \times$$

$$\times \exp\{i[(n + 1)p - np']\ \mathrm{sech}\left[\pi(p - p')/|c|\right] . \quad (A.3)$$

Approximate expression in Eq. (A.3) appears in going to the limit $n \to \infty$ and really it can be used when $n > 10 - 500$ depending on the ratio of the parameters $|c|$ and $p - p'$.

By substituting Eq. (A.3) into Eq. (A.1) and introducing the new variables

$$p_1 = (p - p')/ 2,\ p_2 = (p + p')/2 \ . \quad (A.4)$$

With an account of Eq. (2.13) we obtain

$$<\psi|\varphi(x)|\psi> = \sum_{n}\left[\ |c|^{-1}n(n + 1)\right]^{1/2} \times$$

$$\mathrm{r}\ a^*_n a_{n+1}\ \pi^{-1/2}\ \Delta p^{-1}\ \exp[itc2n(n + 1)/4] \times$$

$$\times \int\int \exp\{(-[p_1^2 + (p_2 - p_0)^2/\Delta p^2]\} +$$

$$+ i\{[2p_1(n + 1/2) + p_2)\ (x - x_0) - t[4p_1p_2(n + 1/2) -$$

$$- p_1^2 - p_2^2\}\ \mathrm{sech}\ (2\pi p/|c|)\ \mathrm{d}p_2\ \mathrm{d}p_1 \ . \quad (A.5)$$

Integrating over p_2 and omitting the subscript on p_1 we will obtain Eq. (2.15).

APPENDIX B: DETERMINATION OF THE SHAPE OF THE PULSE ENVELOPE

This section is devoted to the derivation of relations (3.1) and (3.2) and their transformation to a form convenient for practical calculations.

In accordance with Eq. (2.7), we have

$$<\psi|\varphi^+(x)\ \varphi(x)|\psi> = \sum_{n'}\sum_{n} a^*_{n'}a_n \int\int g^*_{n'}(p')g_n(p) \times$$

$$\times \exp\{it[E(n', p') - E(n, p)]\}<n', p'|\varphi^+(x)\varphi(x)\ |n, p> \mathrm{d}p\ \mathrm{d}p' \ . \quad (B.1)$$

and[11]

$$<n', p'\ |\varphi^+(x)\ \varphi(x)|\ n, p> = \delta_{nn'}(2\pi)^{-1} \times$$

$$\times c^{2(n-1)}\ (n!)^2 \exp(in(p - p')x) \times$$

$$\times \prod_{j=1}^{n-1}\left[(jc)^2 + (p - p')^2)^{-1}\right| \quad (B.2)$$

$$\simeq \delta_{nn'}[n^2 \exp\{in(p - p')x\}/2|c|] \times$$

$$\times (p - p')\ \mathrm{sh}^{-1}[(\pi\ (p - p')/|c|] \ . \quad (B.3)$$

Approximate expression in Eq. (B.3) is valid when $n \gg 1$.

Further procedure of transformation is analogous to the procedure described in Appendix A: by introducing the variables of the type given by Eq. (A.4) and by integrating over p_2 we obtain Eqs. (3.1) and (3.2) which, however are not very convenient for practical calculations. Hence, we will transform them using the condition $n_0 \gg 1$. Now the Poisson distribution can be replaced by the Gaussian (see, for example Ref. 14):

$$\exp(-n_0)n_0^n/n! \simeq (2\pi n_0)^{-1/2}\exp[(-(n - n_0)^2/2n_0] \ , \quad (B.4)$$

and summation can be replaced by integration over n between the infinite limits and after integrating we will obtain

$$<N(x)> \simeq 4n_0|c|^{-1}\int_0^\infty p[(U + n_0 - V^2/n_0)\cos(U/V) -$$

$$- 2V\sin(U/V)]\ U^{-5/2}\ \mathrm{sh}^{-1}(2\pi p/|c| \times$$

$$\times \exp\{[(n_0(1 - U) - V^2/n_0]/2U - p^2/\Delta p^2\}\mathrm{d}p \ , \quad (B.5)$$

$$U = 1 + 8\ n_0\ t^2p^2\ \Delta p^2\ ,\ V = 2\ p\ n_0(x - x_0 - 2p_0t) \ .$$

In spite of apparent cumbersome form this relation is more convenient for numerical estimate due to the lack of summation over n.

APPENDIX C

Here we will derive relation (3.6). So we have

$$<\psi|\varphi^+(x)\ \varphi^+(x)\ \varphi(x)\ \varphi(x)|\psi> =$$

$$= \sum_{n'}\sum_{n} a^*_{n'}a_n \int\int g^*_{n'}(p')\ g_n(p)\exp\{it[E(n', p') - E(n, p)]\} \times$$

$$\times <n', p'|\varphi^+(x)\varphi^+(x)\varphi(x)\varphi(x)|n, p>\ \mathrm{d}p\mathrm{d}p'. \quad (C.1)$$

The matrix element can be written as

$$M \equiv <n', p'|\varphi^+(x)\ \varphi^+(x)\ \varphi(x)\ \varphi(x)|n, p> =$$

$$= \delta_{nn'}(n!)^{-1} \int\int f^*_{np'}(x'_1, x'_2, \ldots, x'_n) f_{np}(x_1, x_2, \ldots, x_n) \times$$

$$\times \langle 0|\varphi(x'_1)\ldots\varphi(x'_n)\varphi^+(x_1)\ldots\varphi^+(x_n)|0\rangle \mathrm{d}x'_1\ldots \mathrm{d}x'_n \mathrm{d}x_1 \ldots x_n \quad (C.2)$$

$$= \delta_{nn'} n(n-1) \int f^*_{np'}(x, x, x_1, \ldots, x_{n-2}) \times$$

$$\times f_{np}(x, x, x_1, \ldots, x_{n-2})\, \mathrm{d}x_1 \ldots \mathrm{d}x_{n-2}\,. \quad (C.3)$$

Substituting Eq. (2.9) into Eq. (C.3) gives (the symbol $\delta_{nn'}$ we will omit further assuming $n = n'$)

$$M = N_n^2 n(n-1)\int \exp\Bigg[i(p-p') \times$$

$$\times \sum_{j=1}^{n-2} x_j - 2|c|\sum_{j=1}^{n-2}|x - x_j| -$$

$$-|c| \sum\sum_{1\le i<j\le n-2} |x_i - x_j| \Bigg] \mathrm{d}x_1 \ldots \mathrm{d}x_{n-2}\,. \quad (C.4)$$

By virtue of the symmetry of function f_{np} the integration can be carried out only over the region $-\infty \le x_1 \le x_2 \le \ldots \le x_m \le x \le x_{m+1} \le x_{m+2} \le \ldots x_{n-2} \le \infty$ since the integrals over the other regions can be obtained only by various commutations of x and x_j which have no effect on the function f_{np}. Therefore, Eq. (C.4) can be represented in the form

$$M = N_n^2 n! \exp(i2x(p-p')) \sum_{m=0}^{n-2} \int_{-\infty}^{x} \mathrm{d}x_m \int_{-\infty}^{x_m} \mathrm{d}x_{m-1} \ldots \times$$

$$\times \int_{-\infty}^{x_2} \mathrm{d}x_1 \int_{x}^{\infty} \mathrm{d}x_{m+1} \int_{x_{m+1}}^{\infty} \mathrm{d}x_{m+2} \int_{x_{n-3}}^{\infty} \mathrm{d}x_{n-2} \times$$

$$\times \exp\Bigg[i(p-p')\sum_{j=1}^{n-2} x_j + 2|c|)\sum_{j=1}^{m} x_j - 2|c| \times$$

$$\times \sum_{j=m+1}^{n-2} x_j + 2|c|\,(n-2m-2)x + |c| \times$$

$$\times \sum_{j=1}^{n-2} (n-2j-1)(x_j) \Bigg]. \quad (C.5)$$

The integration gives

$$M = N_n^2\, n! \exp(in(p-p')x)\ |c|^{-(n-2)} \times$$

$$\times \sum_{m=0}^{n-2} \Bigg\{ m!\,(n-2-m)!) \prod_{r=1}^{m} [(n - r + i(p-p')/|c|] \times$$

$$\times \prod_{r=1}^{n-2-m} [(n - r - i(p-p')/|c|] \Bigg\}^{-1}. \quad (C.6)$$

It should be noted that the sum in Eq. (C.6) can be represented in the form

$$\sum_{m=0}^{n-2} \ldots = \prod_{r=2}^{n-1} [(r^2 + (p-p')^2/c^2)^{-1}] \sum_{m=0}^{n-2} [(m!\,(n-2-m)!]^{-1} \times$$

$$\times \prod_{r=2}^{m+1} [(r - i(p-p')/|c|] \prod_{r=2}^{n-1-m} [(r + i(p-p')/|c|]\,. \quad (C.7)$$

in addition

$$\sum_{m=0}^{n-2} [m!\,(n-m)!]^{-1} \prod_{r=2}^{m+1} [r - i(p-p')/|c|] \times$$

$$\times \prod_{r=2}^{n-1-m} [r + i(p-p')/|c|] = \sum_{m=1}^{n-1} m\,(n-m)\,. \quad (C.8)$$

But

$$\sum_{m=1}^{n-1} m\,(n-m) = n(n^2-1)/6\,. \quad (C.9)$$

Thus, taking Eq. (2.10) into account we have

$$M = (12\pi)^{-1}(n^2-1)\,(n!)^2|c|\ \exp[in(p-p')x] \times$$

$$\times \prod_{r=2}^{n-1} [r^2 + (p-p')^2/c^2]^{-1}\,. \quad (C.10)$$

As $n \to \infty$, finite product in Eq. (C.10) becomes infinite and, in its turn, can be written in the form of hyperbolic sine:

$$\pi x \prod_{j=1}^{\infty} (1 + x^2/j^2) = \mathrm{sh}\ \pi x\,. \quad (C.11)$$

As a result we have

$$\langle n, p'|\varphi^+(x)\ \varphi^+(x)\ \varphi(x)\ \varphi(x)|n, p\rangle \simeq$$

$$\simeq [n^2(n^2-1)\,(p-p')/12]\,[1 + (p-p')^2/c^2] \times$$

$$\times \mathrm{sh}^{-1}[\pi(p-p')/|c|]\ e^{in(p-p')x}\,. \quad (C.12)$$

By substituting this relation into Eq. (C.1), by going over to new variables of the form given by Eq. (A.4), and by integrating over p_2 we obtain Eq. (3.6).

APPENDIX D

Here the necessary explanations of the derivation of relation (4.6) are given. So, we have

$$\langle\psi|\varphi^2(x)|\psi\rangle =$$

$$= \sum_n a^*_{n'} a_{n+2} \int\int g^*_{n'}(p')\, g_{n+2}(p) \langle n, p'|\varphi^2(x)|n+2, p\rangle \times$$

$$\times \exp\ it\,[E(n, p') - E(n+2, p)]\ \mathrm{d}p\ \mathrm{d}p'\,. \quad (D.1)$$

The matrix element can be written as

$$<n, p'|\varphi^2(x)|n+2, p> = [(n+2)(n+1)]^{1/2} \int f^*_{np'}(x_1, \ldots, x_n) \times$$

$$\times f_{n+2,\, p}(x_1, \ldots, x_n, x, x) dx_1 \ldots dx_n = (n+2)(n+1)^{1/2} N_n N_{n+2} n! \times$$

$$\times \sum_{m=0}^{n} \int_{-\infty}^{x} dx_m \int_{-\infty}^{x_m} dx_{m-1} \ldots \int_{-\infty}^{x_2} dx_1 \int_{x}^{\infty} dx_{m+1} \times$$

$$\times \int_{x_{m+1}}^{\infty} dx_{m+2} \ldots \int_{x_{n-1}}^{\infty} dx_n \exp\Bigg[i2px + i(p - p') \sum_{j=1}^{n} x_j +$$

$$+ (|c|/2) \sum_{j=1}^{n} (n - 2j + 1)\, x_j +$$

$$+ (|c|/2) \sum_{j=1}^{m} (n - 2j + 3)\, x_j + (|c|/2) \times$$

$$\times \sum_{j=m+1}^{n} (n - 2j - 1)\, x_j + |c|\,(n - 2m)\, x \Bigg] =$$

$$= [(n+2)(n+1)]^{1/2} N_n N_{n+2} n! \exp\{x[n(p-p') + i2p]\} \times$$

$$\times \sum_{m=0}^{n} \prod_{r=1}^{m} \{[\,|c|\,(n - r + 1) + i(p - p')]r\}^{-1} \times$$

$$\times \prod_{r=1}^{n-m} \{[\,|c|(n - r + 1) - i(p - p')]\, r\}^{-1}. \qquad \text{(D.2)}$$

The sum in Eq. (D.2) can be represented in the form

$$\sum_{m=0}^{n} \ldots = \prod_{r=1}^{n} [(r^2 + (p - p')^2/c^2]^{-1} \times \sum_{m=0}^{n} [(m!(n - m)!]^{-1} \times)$$

$$\times \prod_{r=1}^{n} [(r - i(p - p')/|c|] \prod_{r=1}^{n-m} [(r + i(p - p')/|c|] =$$

$$= (n + 1)/\prod_{r=1}^{n} [(r^2 + (p - p')^2/c^2]\,. \qquad \text{(D.3)}$$

Thus we obtain

$$<n, p'|\varphi^2(x)|n+2, p> = (2\pi)^{-1}(n+1)^2\, [(n+2)/n]^{1/2} \times$$

$$\times \exp(ix[(n+2)p - np']) \prod_{r=1}^{n} [1 + (p - p')/r^2c^2]^{-1}\,. \qquad \text{(D.4)}$$

In going over to the limit $n \to \infty$ (when $n_0 \gg 1$) we derive

$$<n, p'|\varphi^2(x)|n+2, p> \simeq 2^{-1}(n+1)^2\, [(n+2)/n]^{1/2} \times$$

$$\times \exp(ix[(n+2)p - np'])(p - p')/|c|\,]\, \mathrm{sh}[\pi(p - p')/|c|\,]. \qquad \text{(D.5)}$$

By substituting Eq. (D.5) into Eq. (D.1) and by integrating over one of the variable we obtain Eq. (4.6).

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