ON BAROCLINICITY EFFECT IN THE FORMATION OF VORTEX MOTIONS IN THE ATMOSPHERE AND OCEAN

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Qualitative physical analysis of the vortex motion and heat influx (balance) equations, quantitative estimates from the observational data, results of mathematical (numerical) modeling, and generalization of the experience on routine operation with synoptic aerological data lead to a conclusion that a baroclinic factor (geostrophic advection of the temperature, moisture, and salinity) plays an important role in the formation, evolution, and motion of synoptic vortices. The conclusions that follow from this analysis offer the explanation (interpretation) of salient features in the formation, evolution, and motion of tropical cyclones in the atmosphere and synoptic vortices (rings) in the ocean.

Heat and moisture transport and transformation, formation of clouds and precipitation, and finally weather and climate changes are closely related with vortex motions observed in the atmosphere. A role of vortex rings in the ocean is equally important.

Vortices of various dimensions are observed in air and water mantles of the Earth. Of special interest are synoptic vortices with horizontal extension of the order of 10^2–10^3 km in the atmosphere and 10^1–10^2 km in the ocean.

Without pretension to complete understanding of numerous peculiarities of the vortex structure and motion, here we pursue a well-defined goal: to pay attention to additional effects contributing to the formation and evolution of vortices and affecting their structure and motion.

In spite of the fact that an abundance of works is devoted to the study of vortices, all attempts to establish the parameters and/or criteria to discriminate between undeveloping perturbations (vortices) and developing ones, in Dobryshman's[1] opinion, have failed.

Initial system of equations used to analyze the formation and evolution of vortices in the atmosphere and ocean comprises equations of state, motion, continuity, and transport of heat, water vapor, and moisture in the atmosphere and salinity in the ocean, as well as the radiative transfer equation.

Let us write down the equations of fluid motions

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + lv + F_x, \tag{1}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - lu + F_y, \tag{2}$$

where $\mathrm{d}/\mathrm{d}t = \partial/\partial t + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$ is the operator of total (individual) derivative; u, v, w are the projections of the fluid velocity onto the x, y, and z axes of the Cartesian coordinate system (with the z axis directed upward); ρ is the air or water density; p is the pressure; F_x and F_y are the projections of the friction force acting on a unit mass; $l = 2\omega\ \sin\varphi$ is the Coriolis parameter.

Already in Ref. 2 it was pointed out that instead of Eqs. (1) and (2), it is advantageous to use the equations for the vertical component of the vortex fluid motion $\Omega_z = \partial v/\partial x - \partial u/\partial y$ and for its divergence $D = \partial u/\partial x + \partial v/\partial y$, because in this case a small difference between two large quantities (the pressure gradient and the Coriolis force) entering Eqs. (1) and (2) can be eliminated.

To achieve the formulated goal – qualitative physical analysis of conditions of vortex formation in the atmosphere and ocean – it is suffice to write down the equation for Ω_z. It is obtained by differentiation of Eq. (2) with respect to x and Eq. (1) with respect to y and their subsequent subtraction

$$\frac{\mathrm{d}\Omega_z}{\mathrm{d}t} = -\underbrace{\frac{1}{\rho^2}\left(\frac{\partial\rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial\rho}{\partial y}\frac{\partial p}{\partial x}\right)}_{\mathrm{I}} - \underbrace{(\Omega_z + l)\,D}_{\mathrm{II}} + \underbrace{\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)}_{\mathrm{III}} - \underbrace{\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right)}_{\mathrm{IV}} + \underbrace{\beta\, v_N}_{\mathrm{V}}, \tag{3}$$

where v_N is the meridional component of the fluid velocity and $\beta = 2\omega\ \cos\varphi$ is the Rossby parameter (φ is the latitude, a is the Earth's radius, and $\omega = 7.29\cdot10^{-5}$ s^{-1} is the Earth's angular velocity).

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The atmosphere. Fairly comprehensive analysis of Eq. (3) was made in Ref. 3 with the estimate of the order of magnitude and comparison with the observational data. Here, we only note that when we estimate the contribution of term II (divergent term), which is considered to be the main term of Eq. (3) by some authors, term III (engendered by the turbulent friction) should be always taken into account, because these two terms have opposite signs and are of the same orders of magnitude. The divergent term cannot be the decisive factor in vortex formation for pure logical reasons: before the vortex formation, it is equal to zero, whereas after cyclone formation, for example, the air flows are becoming convergent ($D < 0$) and the mass inflow to the vortex center is observed. Due to this inflow, the pressure at the cyclone center increases and the vortex decays (because the pressure gradient between the center and the periphery decreases). At the same time, in accordance with Eq. (3), the vortex for $D < 0$ amplifies with time under the effect of the divergent term ($\mathrm{d}\Omega_z/\mathrm{d}t > 0$). Thus, the divergent term of Eq. (3) does not play a decisive role; moreover, it must be offset (in case of cyclone amplification) by other factors. Term IV of Eq. (3) becomes most important near high elevations (mountains) of the Earth's surface and oceanic bottom, where the derivatives $\partial\omega/\partial x$ and $\partial\omega/\partial y$ are large (the vertical velocity significantly varies in the horizontal direction).

In the present paper, we primarily aim to analyze conditions of formation and subsequent development of a vortex due to term I in the right side of Eq. (3) caused by baroclinicity of a medium – dependence of its density not only on the pressure but also on the temperature and moisture in the atmosphere or on the temperature and salinity in the ocean.

Undoubtedly, this term of Eq. (3) has long been known. However, it is very difficult to formulate any rules of vortex development for this form of presentation of term I.

It became possible to formulate the rules and simultaneously to estimate the baroclinic term from observational data when Matveev had reduced it to the form

$$-\frac{1}{\rho^2}\left(\frac{\partial\rho}{\partial x}\frac{\partial p}{\partial y}-\frac{\partial\rho}{\partial y}\frac{\partial p}{\partial x}\right)\equiv\frac{l}{T_\mathrm{v}}\left(u_\mathrm{g}\frac{\partial T_\mathrm{v}}{\partial x}+v_\mathrm{g}\frac{\partial T_\mathrm{v}}{\partial y}\right). \quad (4)$$

In the derivation of this formula, the equation of state of the humid air

$$\rho = p/(R\,T_\mathrm{v}) \quad (5)$$

was used and the components of the geostrophic wind velocity

$$u_\mathrm{g} = -\frac{1}{l\rho}\frac{\partial p}{\partial y}; \quad v_\mathrm{g} = \frac{1}{l\rho}\frac{\partial p}{\partial x}. \quad (6)$$

were substituted for the components of the pressure gradient. Here, $T_\mathrm{v} = T(1 + 0.61q)$ is the virtual temperature, T is the air temperature, q is the mass fraction of water vapor, and R is the universal gas constant of dry air. It should be emphasized that Eq. (4) is exact in the sense that no assumptions were used to derive it.

According to Eqs. (4) and (3), a new cyclonic vortex is formed or the existing cyclonic vortex is amplified in case of cold and/or drier (with smaller values of q) air advection, whereas a new anticyclonic vortex is formed or the existing anticyclonic vortex is amplified in case of warm and/or wetter air advection.

To prove the importance of baroclinic factor in the formation and development of synoptic and larger vortices, Refs. 3 and 5–12 were published for 40 years after Ref. 4 was published. In these works, numerical estimates of Eq. (4) were performed from the observational data (including continuous records of the meteorological parameters), the experience on an analysis of synoptic and aerological data accumulated over a period of many years was used to establish empirical regularities, and numerical models were constructed that described the formation and evolution of vortices in a baroclinic medium. The main conclusion drawn from these investigations is that the vortex synoptic motions are formed in the regions (zones) with significant (exceeding by an order of magnitude the average values of the parameters) horizontal gradients of the air temperature and humidity. In these regions, the baroclinic factor plays the decisive role in the formation of vortices. Already in 12–24 h, due to the effect of the baroclinic term only, the vortex strengths became comparable to those observed naturally.

Now we dwell on a more sophisticated analysis of phenomena important from scientific and applied viewpoints, namely, tropical cyclones (TCs) in the atmosphere and synoptic vortex rings in the ocean.

From numerous data collected on the tropical cyclones, it is important to indicate here that 1) TC is formed when cold air flows over a warmer water surface (with T higher than 26–27°C), most often in the intertropical convergence zone; 2) after passage of TC, a cold water wake is formed in the ocean; 3) in the process of TC formation and development, its effect extends for distances being many times greater than its radius; 4) the wind velocity and other meteorological parameters are asymmetrical about the TC center; 5) TC trajectory is often very complex in form: it includes loops, backward motion, and stop; 6) as TC moves over the land, it starts to fill up. The inclusion of the baroclinic factor allows us to interrelate and to interpret these and other data.

When the cyclone of radius R moves with the velocity u_c and the temperature of a water layer with depth h_w is decreased by ΔT_w, the ocean looses and the cyclone receives the heat

$$\varepsilon = c_\mathrm{w}\,\rho_\mathrm{w}\,h_\mathrm{w}\,2R\,u_\mathrm{c}\,\Delta T_\mathrm{w}\,\tau. \quad (7)$$

Here, $c_w = 4.1868 \cdot 10^3$ J/(kg·K) is the specific heat of water, $\rho_w \approx 10^3$ kg/m^3 is the water density, and τ is the transit time of the TC above the oceanic surface. For $h_w = 100$ m, $\Delta T_w = 1$°C, $R = 200$ km, $u_c = 10$ m/s, and $\tau = 7$ days, we obtain $\varepsilon = 1.013 \cdot 10^{21}$ J, which coincides with the estimate reported in Ref. 1.

The heat flux that enters TC for 1 s through a unit area (1 m^2) of the oceanic surface (which is the lower boundary of TC) is

$$Q = \varepsilon/(\pi R^2 \tau) = 2c_w \rho_w h_w \Delta T_w u_c/(\pi R). \quad (8)$$

For the above realistic parameters, $Q = 13.3$ kW/m^2. This value is very large, it exceeds almost 10 times the solar radiation flux at the upper boundary of the atmosphere – the solar constant (1.37 kW/m^2).

The heat extracted from the ocean, on the one hand, increases the temperature of TC (explicit heat), on the other hand, goes into evaporation of sea water, thereby increasing the water vapor mass (latent heat). Because the water vapor is saturated above the oceanic surface, the temperature increment ΔT and the mass fraction increment of the water vapor Δq are related by the Clausius–Clapeyron equation

$$\frac{\Delta q}{q} = \frac{L}{R_v} \frac{\Delta T}{T^2}, \quad (9)$$

where L is the specific latent heat of evaporation and R_v is the gas constant of water vapor.

Due to the heat flux coming from the ocean to TC, the explicit heat and latent heat are increased by $c_p \Delta T$ and $L \Delta q$, respectively. On account of Eq. (9) and formula $q = 0.622 E(T)/p$, their ratio assumes the form

$$\beta = \frac{L \Delta q}{c_p \Delta T} = \frac{0.622}{c_p} \frac{L^2}{R_v p_0} \frac{E(T_0)}{T_0^2}. \quad (10)$$

Here, p_0 and T_0 are the air pressure and temperature above the oceanic surface, c_p is the specific heat of air at constant pressure, $E(T_0)$ is the saturated water vapor pressure at T_0.

Below, the values of β at $p_0 = 1000$ hPa are given.

T_0, °C	40	30	20	10	0
β	5.76	3.62	2.18	1.25	0.58

For T_0 typical of low latitudes (from 20 to 32°C), β varies from 2 to 4. From the net energy influx from the ocean, the portion equal to $L\Delta q/(c_p \Delta T + L \Delta q) = \beta/(1 + \beta)$ goes to evaporation and the portion equal to $c_p \Delta T/(c_p \Delta T + L \Delta q) = 1.(1 + \beta)$ goes to the temperature increase. For $\beta = 3$, the first portion is equal to 3/4 and the second portion is equal to 1/4.

The explicit heat and water vapor from the oceanic surface are distributed over the entire layer due to vertical motions and turbulent exchange (practically up to the upper tropospheric boundary –the tropopause). The heat spent for evaporation of sea water is subsequently released in the process of cloud formation in the atmosphere. Thus, we may consider that practically all heat extracted from the ocean goes to the temperature increase in a cyclone.

Temporal behavior of the temperature, humidity, heat, and moisture spatial distributions are described by the equations of heat, water vapor, and liquid water content influxes. They are added to the system of dynamic equations to construct numerical models (in our works, the method of invariants is widely used).

Now we write down the heat influx equation in the following form:

$$c_p \frac{\partial T}{\partial t} = \underset{\mathrm{I}}{c_p w(\gamma - \gamma_a)} + \underset{\mathrm{II}}{\partial Q_z/\partial z} - \underset{\mathrm{III}}{c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right)} +$$

$$+ \underset{\mathrm{IV}}{(\partial Q_x/\partial x + \partial Q_y/\partial y)}, \quad (11)$$

where $\gamma = -\partial T/\partial z$ is the vertical temperature gradient, γ_a is the dry- or wet-adiabatic gradient of T (in nonsaturated wet or cloud air, respectively), and Q_x, Q_y, and Q_z are the components of the turbulent heat flux.

In Eq. (11), convection (I), advection (III), turbulence (II), and condensation (IV) heat influxes are considered. In the last term, γ_a is replaced by γ_{wa}. Terms I and II describe the vertical transport and redistribution of the heat coming from the ocean. Terms III and IV describe the interaction between TC and surrounding medium in the horizontal direction. We do not pose the problem of numerical model construction. Here, we want only to estimate the main terms of Eq. (11) within the framework of a qualitative physical analysis.

First, we estimate the average (over the air column) temperature increase $\Delta T'$ due to the heat influx from the ocean. Because the heat Q enters the air column of unit cross sectional area for 1 s, we can write

$$m c_p \Delta\theta' = Q \Delta t. \quad (12)$$

Here, $m = (p_0 - p_H)/g$ is the mass of the air column between the oceanic surface level and the tropopause height, p_0 and p_H are the corresponding pressures, g is the acceleration due to gravity, Δt is the time interval for which $\Delta T'$ is determined, $\Delta\theta'$ is the increment of the potential temperature θ related with $\Delta T'$ by the formula

$$\Delta\theta' = \Delta T' (1000/\bar{p})^{0.286}, \quad (13)$$

and $\bar{p}$ is the air pressure at the average (by air mass) altitude level. Assuming $1000/\bar{p} = 2$ and considering Eqs. (9) and (13), we can write Eq. (12) in the form

$$\Delta T' = 7.7 \cdot 10^2 \frac{h_w \Delta T_w u_c}{(p_0 - p_H) R} \Delta t, \quad (14)$$

where R is in km, $(p_0 - p_H)$ is in hPa, h_w is in m, u_c is in m/s, and Δt is in h.

For $h_w = 100$ m, $\Delta T_w = 10C$, $u_c = 10$ m/c, $R = 200$ km, and $p_0 - p_H = 900$ hPa, the temperature of the air column is increased by $\Delta T' = 4.3°C$ for $\Delta t = 1$ h. Even when h_w, ΔT_w, and u_c are halved, the temperature of the air column is increased, on average, by 0.53°C for 1 h or by 12.8°C for 24 hours.

Because there is no significant temperature increase in TC (it increases only by several degrees at the center of TC during its lifetime), a mechanism of temperature decrease with the same rate should exist. This very mechanism is the cold influx (advection).

Warm (cold) advection is caused by resulting air mass transport (external to TC) and salient features in distributions of the velocity, temperature, humidity, and cloudiness within TC.

An analysis of the synoptic data reported in Refs. 13 and 14 and an estimate of term III performed in Ref. 15 for some specific cases indicate that the cold air transport (intrusion) spreading over most troposphere is observed not only in the process of formation but also in the process of subsequent development of TC.

Radial or tangential advection of T is engendered by the peculiarities of the internal structure of TC, as a rule, asymmetric about its center.

The temperature difference and tangential advection, which is primarily responsible (along with the external advection) for breaking of the symmetry of TC, are engendered by nonuniform heat influx from the ocean. A much larger amount (from 2 to 3 time larger according to the experimental data of Ref. 16) of heat goes into the frontal zone of the TC than into its rear zone, because it passes over the cooled water surface. The temperature difference between the front and rear zones of TC in the lower troposphere is 1–5°C (see Ref. 17). The advection of T caused by this temperature difference is positive (warm advection) to the left of the direction of TC motion and negative (cold advection) to the right of the direction of TC motion.

Taking into account that the radial and tangential temperature differences reach several degrees, the radial component of the velocity is 5–20 m/s, and the tangential component of the velocity is 30–80 m/s, we arrive at a conclusion that the temperature of a cyclone is decreased due to the advection as many degrees as it is increased due to the heat influx from the ocean (convection factor), that is, as a rule, by several degrees per hour.

These estimates of T variations due to the advection are used below to estimate temporal variations of the vortex Ω_z due to baroclinic term I in Eq. (3) written in the form of Eq. (4).

The second factor in the left side of Eq. (4) – the geostrophic advection of T_v with the opposite sign – is no less (by its absolute value) than the advection of T, because u_g and v_g, especially at low latitudes, are greater (by their absolute values) than u and v. We cannot but consider that the advection of T_v includes, the advection of T and moisture q that are of the same signs: the warm advection is connected with the advection of larger q whereas the cold advection – with the advection of smaller q.

Thus, there are strong grounds to believe that the second factor in the right side of Eq. (4) is of the order of several degrees per hour. It should be noted that at temperate latitudes in frontal zones, where synoptic vortices are only formed, the maximum temperature difference due to the advection is also equal to 5–6°C/h (see Ref. 3).

Taking $\varphi = 10°N$, $T_v = 300$ K, and $(\partial T_v/\partial t)_{adv} = -(4–8)°C$, we obtain for the rate of vortex change with time due to baroclinicity the estimate

$$(\partial\Omega_v/\partial t)_{brcl} \approx (0.94 - 1.88) \cdot 10^{-10}\ s^{-2}.$$

In 7 days (from which 2 days fall on the depression stage) a vortex of strength $(5.7–11.3)\cdot 10^{-5}\ s^{-1}$ is formed in a moving air mass due to advection of T_v.

The tangential wind velocity component at distances 200–300 km from the vortex center reaches 17–34 m/s under these conditions. This estimate is valid for the cyclone as a whole. Considering that the heat influx from the ocean and the advection of T_v differ significantly (up to 2–3 times) in different parts of the cyclone, we arrive at a conclusion that the vortex can be formed in several days due to baroclinic factor (4) and the wind velocity in different parts of the vortex can reach 50–90 m/s.

Nonuniform advection of T_v breaks the symmetry of distribution of the meteorological parameters and leads to different rates of vortex variations with time (in particular, to the right and to the left of the direction of TC motion due to the tangential component of advection) and hence to a very complicated form of trajectories of TC motion (it can be intersected several times, can form loops, and so on).

By now, a large body of data has been accumulated that confirm our concept of the formation and development of TC due to the baroclinic factor. Most often TCs are formed in the intertropical convergence zone (ICZ) when cold air flows intrude over warm air. According to the data published in Ref. 18, up to 85% of all typhoons observed in the West Pacific (on average, about 30 hurricanes in a season) are formed in the ICZ when the cold air intrudes from the Southern Hemisphere into the Northern Hemisphere. In this case, from 4 to 3 days before the formation of TC, at temperate latitudes of the Southern Hemisphere, cyclogenesis intensifies (roughly at the same longitude at which the typhoon is formed). The ICZ itself is located in the monsoonal trough, is displaced toward higher latitudes from the central position, and has large extent and depth.

It is well known (see, for example, Ref. 19) that the ICZ is formed when a cold air intrudes into low latitudes: to the south of 37°N in the Northern Hemisphere or to the north of 22°S in the Southern Hemisphere. The ICZ starts to diverge without cold air influx.

According to Ref. 20, 48% of TCs are formed on cold fronts with cold advection by the definition; 42% of TCs are formed within the ICZ when it intensifies due to cold advection.

Kotel'nikova and Petrova[14] point out that air masses in 40° longitude belts on both sides of TC origin and in latitude belt between the subtropical maxima of the Northern and Southern Hemispheres take part in tropical cyclogenesis.

Convincing arguments in support of the fact that cold advection (baroclinicity) is the decisive factor in the formation and development of the TCs are presented by Grei.[21] In accordance with his data, the pressure first starts to decrease above an extended water area with a radius of 650–900 km. As the cyclone amplifies further, the wind and the vortex strengthen, whereas the pressure falls first on the periphery and only then the pressure decrease spreads toward the vortex center. It is clear that advection of T_v spreads over large water areas. The vortex is amplified due to this advection first on the periphery, where the cold surrounding air penetrates first.

Now we dwell on some exotic cases of the formation of TC reported by Dobryshman.[1] In the South-East Pacific (in the Southern Hemisphere) to the east of 160°E, typhoons are observed very seldom: only 10 typhoons for 10 years (1980–1989). However, most of these typhoons (7 from 10) were formed here in 1982–1983, during most intense El Nino with high oceanic surface temperature (OST) which spreaded over abnormally large water area of the East and Central Pacific. Even the pair of typhoons observed simultaneously in February 1989 was formed between 140 and 160°W under conditions of positive anomaly of the OST. Because at that time a vast negative anomaly of the OST was observed near South America, the temperature difference and advection of T_v were as great as in other regions of TC formation.

Most TCs are formed at certain distances from the equator. However, some cases are well known when TCs were formed in the immediate vicinity of the equator, for example, typhoon Sara (03.21–04.04, 1958) formed at 1°40′N, which regenerated twice and changed sharply its velocity and direction of motion. From the first glance, formation of such TCs cannot be explained by the effect of baroclinicity, because in Eq. (4) $l \to 0$ when $\varphi \to 0$. However, on account of Eqs. (5) and (6), formula (4) for the components of the pressure gradient assumes the form

$$\frac{l}{T_v}\left(u_g\frac{\partial T_v}{\partial x}+v_g\frac{\partial T_v}{\partial y}\right)=-\frac{R}{p}\left(\frac{\partial T_v}{\partial x}\frac{\partial p}{\partial y}-\frac{\partial T_v}{\partial y}\frac{\partial p}{\partial x}\right).$$

Here, the right side is independent of latitude. For this reason, the baroclinic term may be significant even at very small φ.

Empirical rules are in agreement with this concept of the role of the baroclinic factor. Thus, the thermal potential of TC formation, introduced by Gray,[22] comprises the deviation of the water temperature from 26°C to depths of 60 m and the vertical gradients of the equivalent potential temperature between the sea surface level and 500 hPa pressure level and of the relative air humidity between the pressure levels 700 and 500 hPa. As each parameter increases, the heat flux from the ocean to the atmosphere, T_v, and as a result, cold influx in TC are increased.

The diurnal potential $(\Omega'_z - \Omega''_z)$ introduced by Gray[22] and the individual potential $(\Omega'_z-\Omega''_z)(D''-D')$ introduced by Petrosyants and Semenov[23] are closely related with the examined effects (here, Ω'_z and D' are the curl and the divergence of the wind velocity at a pressure level of 850 hPa and Ω''_z and D'' are the same parameters at a pressure level of 200 hPa). It should be only noted that anticyclonic circulation $(\Omega''_z < 0)$ and positive divergence $(D'' > 0)$ in the upper troposphere (at a pressure level of 200 hPa) are the consequences rather than the necessary conditions for TC development. By invoking the static equation which can be used to calculate the vertical distribution of p with high accuracy even at very large velocities and acceleration, we established that in the cyclone, the horizontal pressure gradient inverts and the cyclonic circulation is changed by anticyclonic circulation at the level z^*. This level is between 2 and 5 km when ΔT_v = 8–10°C and between 10 and 13 km when ΔT_v = 4–6°C (ΔT_v is the difference of virtual temperatures at the TC center and on its periphery).

As to the curl Ω'_z and convergence D at low levels $z < z^*$, they are the greater (D' by its absolute value), the larger are the difference ΔT_v and the influx (advection) of cold air from the surrounding medium into the cyclone.

Ocean. Let us write down the equation of state of sea water in its general form

$$\rho = \rho(p, T, c),$$

where c is the water salinity.

From here we obtain

$$\frac{\partial\rho}{\partial s}=\frac{\partial\rho}{\partial p}\cdot\frac{\partial p}{\partial s}+\frac{\partial\rho}{\partial T}\cdot\frac{\partial T}{\partial s}+\frac{\partial\rho}{\partial c}\cdot\frac{\partial c}{\partial s}\,,\qquad s = x,\ y. \quad (15)$$

On account of Eq. (15), as applied to the ocean, baroclinic term I in vortex equation (3) can be written as

$$\frac{1}{\rho^2}\left(\frac{\partial\rho}{\partial x}\frac{\partial p}{\partial y}-\frac{\partial\rho}{\partial y}\frac{\partial p}{\partial x}\right)\equiv$$

$$\equiv-\frac{l}{\rho}\left[\frac{\partial\rho}{\partial T}\left(u_g\frac{\partial T}{\partial x}+v_g\frac{\partial T}{\partial y}\right)+\frac{\partial\rho}{\partial c}\left(u_g\frac{\partial c}{\partial x}+v_g\frac{\partial c}{\partial y}\right)\right], \quad (16)$$

where u_g and v_g, as in Eq. (1), are the projections of geostrophic velocity defined by Eqs. (6).

We note that term I in Eq. (3) is nonzero only when we consider the dependence of p on T and c (a baroclinic medium). At the same time, whatever the dependence of ρ on p may be, it has no effect on temporal behavior of the vortex, in accordance with Eqs. (15) and (16). Because $\partial\rho/\partial T < 0$ and $\partial\rho/\partial c > 0$, according to Eqs. (3) and (16), a new cyclonic vortex is formed due to baroclinic factor or the existing cyclonic vortex is amplified due to cold advection and/or advection of water with higher concentration of salt, whereas the anticyclonic vortex is formed due to warm advection and/or advection of water with lower concentration of salt. The decisive role, according to our estimates, plays the dependence of water density on the temperature.

Consideration of baroclinic factor (16) provides explanations for some essential features of the formation and evolution of synoptic vortices near warm streams in the ocean, like Gulf Stream and North Atlantic, Curacao, Kuril, and other streams.

A very important feature of synoptic vortex rings in the ocean is that the rings observed to the left of the stream (as a rule, to the north or west of it) are anticyclonic, whereas the rings formed to the right of the stream (most often, to the south or east of it) are cyclonic (here we speak about the Northern Hemisphere). As a rule, there are no exceptions to this rule according to numerous data generalized in Refs. 24 and 25.

There are some other contributing factors for this clearly defined classification of rings with opposite rotations. However, the role of baroclinic factor (16) is evident. Really, Gulf Stream divides cold (with lower content of salt) slope water in the east and north and warm (with higher content of salt) water of the Sargasso Sea in the east and south. The temperature gradient across the stream reaches fairly large values: from 8 to 9°C to the left of the stream and from 17 to 18°C to the right of it at a depth of 300 m. In some cases, the horizontal temperature gradient to the left and to the right of the streams increased to 0.2–0.5°C/km. The data of satellite measurements in the zone of interaction between the streams Curacao and Ojacao (to the east of Japan) indicated that very often the surface oceanic temperature was changed by 5–8°C at distances 4–10 km (see Ref. 26).

Thus, to the left of the stream the vortex is formed when warm water passes through cold water, that is, the advection heat influx is observed due to which, according to Eqs. (3) and (16), anticyclonic vortex is formed ($\partial\Omega_z/\partial t < 0$ and $\Omega_z(t) < 0$). On the other hand, the cold advection of lower T takes place to the right of the stream which is accompanied by cyclogenesis ($\partial\Omega_z/\partial T > 0$ and $\Omega_z(t) > 0$).

We note that the temperature field (and the salinity) is nonuniform not only in the transverse direction, but also in the longitudinal direction of the stream. This means that not only the first term comprising $u_g\partial T/\partial x$, but also the second term comprising $v_g\partial T/\partial y$ contributes to the baroclinic term. Because Gulf Stream, Curacao, and other streams are geostrophic, the true velocity of motion is close to the geostrophic velocity that enters Eq. (16).

Most often the formation of vortices is associated with meanders. However, the meander is only the region with a particular increased or decreased temperature. The vortex is formed due to the effect of baroclinicity (advection) only when the water mass in the meander starts to penetrate into the medium with higher or lower temperature.

If the matter had been only in the meanders, both cyclones and anticyclones would have been formed on each side of the stream.

We also note that the concept of vortex formation only when the meanders are separated from the jet leads to the conclusion about the vortex sign opposite to the observed one. Because the stream velocity is maximum at the jet axis and decreases on the periphery, it is evident that such separation would be accompanied by cyclonic circulation to the left of the stream and anticyclonic circulation to the right of it.

It is very important to stress that the cyclone temperature is lower and the anticyclone temperature is higher than the temperature of the surrounding medium. This testifies that in the process of formation and subsequent development the cyclone is filled with cold water, whereas the anticyclone – with warm water.

The vortex ring formation is a long process; it lasts from several weeks to several months. This fact along with the fact that the vortex ring moves with water contained in it allows us to conclude that the synoptic vortices cannot be thought of as waves and they are not formed when the wave stability breaks.

Other more important peculiarities are the motion and lifetime of vortices. The cyclonic rings of Gulf Stream persist 6–12 months and anticyclonic rings – about 4 months, on average (their lifetime changes from several days to a year). Then these rings start to move to the west and south-west with an average velocity of 3–4 km/day practically parallel to the stream itself, but in the reverse direction. Some investigators try to associate this motion of rings with large-scale streams. However, not only in the upper water layer, but also at a depth of 250 m to the north of Gulf Stream, a stream persists from the data of modeling of streams in the global ocean whose direction coincides with that of Gulf Stream. Only at a depth of 1000 m in northern part of Gulf Stream the eastern stream is observed with very low velocities. As to the Sargasso Sea where the Gulf Stream cyclones are formed, here the velocity is very small at all depths. In addition, it sharply changes its direction from point to point.

Because the synoptic vortices are most strong in the upper layers of the ocean (the rotational velocity in the Gulf Stream rings reaches 3 m/s in the uniform water layer and the average velocity is 1.5 m/s; it decreases with depth very fast: at depths between 1000 and 2000 m it reduces to 0.1 m/s), it follows from the data on large-scale streams presented above that they cannot be the main reason for motion of vortices to the west. The dominating role in this motion is played by the same baroclinicity (geostrophic warm advection and advection of salinity), due to which these rings were formed, and the β-effect. Really, in the western part of the anticyclone, the warm advection (warm water from the southern part enters here) and the vortex ring amplification are observed due to rotational motion, whereas in the eastern part of the anticyclone – the cold advection and decay of the same vortex ring. The motion of the ring in the direction reverse to that of the main stream is a result of this process.

Analogous pattern is typical of cyclones: here in its eastern part the vortex ring decays due to the warm advection and in its western part it is formed due to the cold advection; as a result, the ring moves to the west.

Under the effect of friction forces, the flows in the cyclone converge toward its center, whereas they diverge from the center of the anticyclone. As a result, new water masses come from the stream, namely, warm water enters into the anticyclone and cold water – into the cyclone. This supports (regenerates) the ring for a long time.

Not only cyclones, but also anticyclones after separation from the Gulf Stream move to the west and south–west with an average velocity of 3–4 km/day a short distance from the Stream. At first, the temperature contrast between the ring and the surrounding medium is high not only in the layer of main thermocline but also in the upper uniform water layer (including its surface). As the ring passes several hundreds of kilometers (on average, 500 km) in the eastern and south-eastern directions, the temperature of the ring becomes equal to that of the stream and warm (cold) advection vanishes. As a result, the ring decays, most often in 4–6 months after formation (its lifetime varies from several days to 12 months).

In connection with the problem at hand, we cannot but dwell on a series of investigations performed by Marchuk and Sarkisyan and their scholars and followers in the last 40 years (from numerous works, we mention here only Refs. 27–30). In their basic researches, the effect of baroclinicity on the formation of stream field and oceanic dynamics as a whole has received much attention along with other factors. However, in accordance with the problem formulation in Refs. 27–30 the baroclinicity is manifested only together with the effect of bottom relief (the abbreviation JEBER fort the joint effect of baroclinicity and relief is widely used) or with the β-effect.

Terms describing JEBER appear in the vortex ring equations only for streams averaged over the vertical water columns or when the oceanic bottom rises. The results of modeling testify that JEBER plays an important role in the formation of rings and integral mass transport. However, because the density is a function of the pressure, is there any reason why the term proportional to Jacobian $J(H, p)$ cannot be nonzero also in a barotropic medium? We think that the effect first considered by Sarkisyan is more common in nature: it takes into account the joint effect of the density variations (of any medium) and horizontal relief of the bottom on vortex rings of the total stream and the surface of the ocean.

As to baroclinic term (16), as far as we know it was not taken into account in all calculations on the dynamics of the ocean made to date, though the baroclinic factor by itself plays an important (we cannot exclude that even the decisive) role in the formation and development of such important objects as synoptic vortices.

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