# DEPENDENCE OF THE QUALITY OF RECONSTRUCTION OF A PHASE SURFACE BY AN ADAPTIVE MIRROR ON A NUMBER OF SERVODRIVES AND CONFIGURATION OF THEIR DISPOSITION

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*Influence of a flexible mirror construction on the accuracy of reconstructing a preset phase surface is studied using numerical simulation method. Estimates of this type of a phase corrector for using it as an active element of an adaptive system have been made. An optimal configuration and a number of servodrives of a mirror intended for compensation for thermal blooming effect are determined for a wide range of the beam and propagation path parameters.*

## 1. INTRODUCTION

One of the problems that can arise when designing an adaptive optical system is the development of an active element (a wave–front corrector) which could meet the general requirements to optical devices and satisfy the conditions of the problem being solved using such a system. In fact, the construction of an adaptive mirror is mainly determined by the distortions this mirror is to compensate for. As was shown in Ref. 1, the atmospheric turbulence can be compensated for with a mirror having 40–60 servodrives while thermal blooming can be corrected for using only 6–8 servodrives.[2,3] Since in Ref. 3 compensation for thermal defocusing has been analyzed for beams with the intensity varied within a narrow range it is impossible to estimate the efficiency of the mirrors used to compensate for strong nonlinear distortions. At the same time there are some theoretical investigations of compensation for thermal blooming[4,5] where a correcting profile was described with Zernike polynomials. It was shown that a number of these polynomials must be increased with a radiation intensity increase to control without the loss of efficiency. That means that under conditions of high nonlinearity a correcting phase surface has a sufficiently complicated shape and one would expect a larger number of servodrives to be necessary for its shaping. Thus the problem on determining the optimal number of degrees of freedom for an adaptive corrector cannot be considered as solved.

In this paper optimization of a number and configuration of disposition of servodrives of an adaptive mirror intended for use in a system of correction for the laser–beam thermal distortions has been made based on numerical simulations. To this end, we have estimated when varying the mirror construction the approximation accuracy for surfaces of different complexity and determined the effect of the corrector on the efficiency of compensation for thermal lens.

## 2. A MODEL

In the numerical experiments a mirror model was used in the form of a flexible thin square plate hinged at its center.[6] The plate deformation $W(x, y)$ was described with a biharmonic equation[7]

$$D\left(\frac{\partial^4 W}{\partial x^4} + 2\,\frac{\partial^4 W}{\partial x^2\,\partial y^2} + \frac{\partial^4 W}{\partial y^4}\right) = f(x, y)\ , \qquad (1)$$

where $D = Eh^3(12(1 - \sigma^2))$ is the cylindrical rigidity; $E$ is the Young modulus; $h$ is the plate thickness; $\sigma$ is the Poisson coefficient; $f$ is the transversly distributed load (in the calculations we used the values of all coefficients characteristic of a copper plate). The conditions of hinging center are as follows:

$$W(x_0, y_0) = 0,\ \ D\left(\frac{\partial^2 W(x_0, y_0)}{\partial n^2} + \sigma\,\frac{\partial^2 W(x_0, y_0)}{\partial \tau^2}\right) = 0\ , \quad (2)$$

where $x_0$, $y_0$ are the center coordinates; $\partial/\partial n$ and $\partial/\partial \tau$ are the derivatives with respect to a normal and tangent, respectively. The equation was solved numerically using the method of finite elements.[8] As compared to the previous papers[3,9] a number of nodes of the calculational grid along each of the coordinates was increased to 13. The dimensions chosen enabled us to vary a number of servodrives within a sufficiently wide range and to choose different configurations of their attachment.

Using this method we have constructed numerical models for six mirrors depicted schematically in Figs. 1 *a–e*. The mirrors had the following peculiarities: (*a*) a model with a minimum number of control coordinates (as shown in Ref. 3 fewer number of servodrives does not provide for compensation even for moderate thermal nonlinearity); (*b*) a number of servodrives is the same as in the case (*a*) but the distance between the drives and the center of the mirror is smaller, i.e., density of the drives disposition in the central portion of the corrector is higher, (*c*) further increase of servodrives density at the central portion; (*d*) and (*e*) variations of disposition geometry of control points, without changing their number; and (*f*) sharp increase of the number of control coordinates due to an increased number of drives at the mirror periphery. The density in the central portion remained constant.

The models of flexible plates were used to determine the accuracy of reconstructing the surface described by Zernike polynomials and in the problem on compensating for the stationary thermal blooming. The approximation has been performed by matching the mirror surface to the phase profile at the points of the servodrives fixing. In correction for the effect of thermal lens the beam radius $r_0$ was 1/10 of the length of the mirror side $d_m$ (the grid on which the quasioptics problem was solved did not allow us to change this ratio). As is seen from Fig. 1, the following parameters

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of the corrector were varied: density of servodrives in the region covered by the beam, their number, and configuration of their fixing. In so doing, the models constructed made it possible to estimate separately the effects of the aforementioned factors on both the precision of shaping the assigned profile with a mirror and the efficiency of the correction for thermal lens effect.

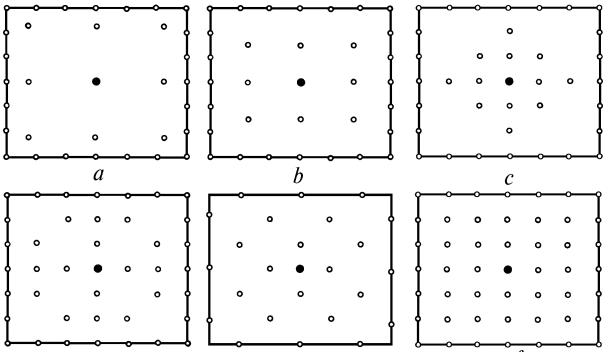


*FIG. 1. Models of adaptive mirrors.*

The surface to be approximated was assigned with the polynomials given in Ref. 10 (the representation is in a polar system of coordinates): $Z_1 = 2\, r\cos\Theta$ (tilt), $Z_2 = \sqrt{3}\,(2\, r^2 - 1)$ (defocusing), $Z_3 = \sqrt{6}\, r^2\sin(2\Theta)$ (astigmatism), $Z_4 = \sqrt{8}\,(3\, r^2 - 2\, r)\sin\Theta$ (coma), $Z_5 = \sqrt{6}\, r^3\sin(3\Theta)$, $Z_6 = \sqrt{5}\,(3\, r^4 - 2\, r^2 + 1)$ (spherical aberration), $Z_7 = \sqrt{10}\,(4\, r^4 - 3\, r^2)\sin(2\Theta)$, $Z_8 = \sqrt{10}\, r^4\sin(4\Theta)$, $Z_9 = \sqrt{12}\,(10\, r^5 - 12\, r^3 + 3\, r)\sin\Theta$, $Z_{10} = \sqrt{12}\,(5\, r^5 - 4\, r^3)\sin(3\Theta)$, $Z_{11} = \sqrt{12}\, r^5\sin(5\Theta)$. We changed the numbering since the polynomials obtained by rotating the system of coordinates were omitted. The accuracy of shaping was characterized by a relative rms error ε:

$$\varepsilon = \left\{ \frac{\int\int (\varphi - W)^2 f \, \mathrm{d}x \, \mathrm{d}y}{\int\int \varphi^2 f \, \mathrm{d}x \, \mathrm{d}y} \right\}^{1/2}, \qquad (3)$$

where φ is the assigned profile of phase distribution, $W$ is the bending of the mirror surface, $f = \exp(-(x^2 + y^2)/r_f^2)$ is the weighting function, and $r_f$ is the radius of the weighting function.

Correction has been calculated for a path with the length $0.5\, z_d$, $z_d$ is the diffraction length. The path was divided into two portions: a portion occupied with a distributed thermal lens (the length $z_{NL}$) and a portion of linear propagation with the length $z_L$. Nonlinearity of the medium was determined by the parameter $R_v$ (Ref. 3) which is proportional to the radiation intensity. A beam in the plane $z_0 = z_{NL} + z_L$ was characterized by a focusing criterion which is proportional to the power incident on the aperture of given dimensions

$$J = \frac{1}{P_0} \int\int \rho(x, y)\, I(x, y, z_0)\, \mathrm{d}\, x\, \mathrm{d}\, y\,, \qquad (4)$$

where $P_0$ is the total power, $I$ is the intensity, $\rho = \exp(-(x^2 + y^2)/r_0^2)$ is the weighting function, and $r_0$ is the initial radius of the beam.

The control was performed using the algorithm for a modified phase conjugation[3] which had some peculiarities in application to solving the problem under study. Thus, the mirror was used not for phase approximation during the control but for shaping the surface determined from the correction. That is, an optimal phase surface $U_{\rm opt}(x, y)$ was determined which then was approximated with the mirror. Such an approach enabled us, on the one hand, to directly estimate the accuracy of shaping sufficiently complicated profiles and to determine the reduction of correction quality caused by the mirror and, on the other hand, to simplify and shorten the calculations.

## 3. RESULTS

The accuracy of reconstructing the polynomials with mirrors *b*, *c* (Fig. 1) is illustrated in Fig. 2. Depicted in the figures are the values of the rms error ε obtained for different radii of the weighting function $r_f$. Similar plots for the remaining models are not included into this paper since the behavior of the curves is quite similar to the results presented here. In particular, when the mirror (*a*) is used the rms error is somewhat larger than that for the mirror (*b*) for all the polynomials. The models *c–f* (Fig. 1) give, in fact, equal values ε for $r_f = 1/10\, d_m$ (i.e., in the region occupied with a beam in the quasioptics problem).

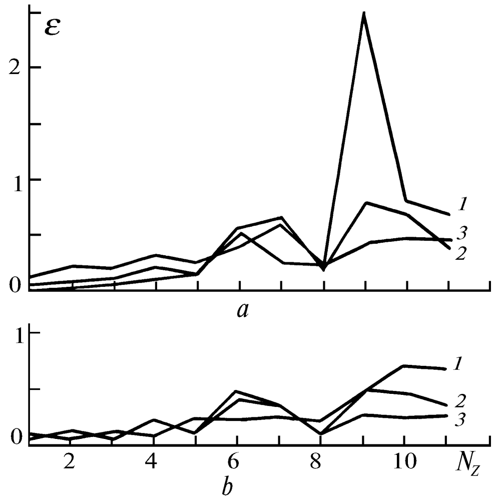


*FIG. 2. Accuracy of Zernike polynomials reconstruction with mirrors b (a) and c (b) from Fig. 1: ε is the rms error, $N_z$ is the number of the Zernike polynomial to be reconstructed, $r_f = 1/10\, d_m$ (curve 1), $r_f = 1/5\, d_m$ (curve 2), and $r_f = 1/2\, d_m$ (curve 3).*

From a comparison of data represented in Fig. 2*a* and *b* as well as from description of the relevant results for other models one can draw a conclusion that the correctors (*a*) and (*b*) (Fig. 1) in their central portion shape the complex surfaces (polynomials with $N_z \geq 9$) with lower accuracy than the remaining mirrors do. The errors ε at the center are almost equal for the models *c–f*. Since the number of polynomials sufficient for compensation for thermal nonlinearity is approximately known[5] one can assume that the use of mirrors (*a*) and (*b*) is reasonable for $z_{NL} \approx 0.5 z_d$ and $R_v \approx -20$. We may also expect that more mirrors of more complicated shapes must be used to correct a thin thermal lens with larger values of the parameter $R_v$.

The efficiency of compensation for thermal blooming is shown in Tables I–IV where the resulting values of the focusing criterion $J$ and the rms error $\varepsilon(r_f)$ obtained for the radius of the weighting function $r_f = r_0$ ($r_0$ is the initial radius of the beam) are tabulated.

*TABLE I. Resulting values of the focusing criterion J. Parameters used:* $R_v = -20$, $z_{NL} = 0.5$, $z_L = 0$. *Initial value* $J_0 = 0.19$ *and for an ideal corrector* $J_{\text{ideal}} = 0.41$.

| Servodrives configuration | *a* | *b* | *c* | *d* | *e* | *f* |
|---|---|---|---|---|---|---|
| $J$ | 0.38 | 0.39 | 0.40 | 0.41 | 0.41 | 0.40 |
| $\varepsilon(r_f = r_0)$ | 0.10 | 0.05 | 0.09 | 0.09 | 0.09 | 0.09 |

*TABLE II. Resulting values of the focusing criterion J. Parameters used:* $R_v = -30$, $z_{NL} = 0.5$, $z_L = 0$. *Initial value* $J_0 = 0.11$ *and for an ideal corrector* $J_{\text{ideal}} = 0.29$.

| Servodrives configuration | *a* | *b* | *c* | *d* | *e* | *f* |
|---|---|---|---|---|---|---|
| $J$ | 0.24 | 0.26 | 0.25 | 0.24 | 0.26 | 0.26 |
| $\varepsilon(r_f = r_0)$ | 0.80 | 0.75 | 0.40 | 0.44 | 0.44 | 0.40 |

As is seen from Table I, compensation for the effect of a distributed thermal lens of a moderate lens power can be done with all mirrors and equally efficiently. The resulting values of the criterion $J$ are close to the values $J_{\text{ideal}}$ obtained with an ideal corrector, the errors $\varepsilon$ being small.

With the increase of nonlinearity (Table II, $R_v = -30$) $\varepsilon$ for the models (*a*) and (*b*) becomes much larger than that for the remaining ones but the efficiency remains approximately equal for all the correctors. Precision of the approximation for this case is illustrated by Fig. 3 where a section of the phase profile by the *XOZ* plane and corresponding bending of the mirror (configurations *b* and *c*) are presented. The error is seen to decrease when the model (*b*) is replaced by the model (*c*) (Fig. 1) but the surface to be reconstructed is sufficiently smooth and apparently just this fact explains the situation when the presence of errors does not result in the reduction of the field concentration.

Much more complicated configurations of phase surfaces are observed for the parameters $z_{NL} = 0.1\, z_d$, $z_{NL} = 0.4\, z_d$, and $R_v = -90$ (Fig. 4). Under these conditions the mirrors (*a*) and (*b*) do not reconstruct characteristic features of the phase and hence the resulting values of $J$ decrease (Table III). The remaining models provide for approximately the same concentration of the field in the recording plane.

Increase of the thin thermal lens power (Table IV, $R_v = -110$) leads to further increase of $\varepsilon$ for the configurations (*a*) and (*b*) (Fig. 1). As a consequence, the resulting values of $J$ decrease that is most noticeable for the mirror (*a*) which does not provide the growth of the focusing criterion during the beam control.

Thus it should be pointed out in conclusion that the efficiency of correction for thermal nonlinearity with an adaptive mirror is determined, first of all, by the density of disposition of servodrives in the region occupied with a beam. This conclusion is supported not only by the increase of $J$ when going from the model *b* to the model *c* (Fig. 1) but also by the fact that the resulting values of the criterion for the mirror (*f*), where the number of drives at the periphery has been substantially increased as compared to that for the remaining models, do not increase.

*TABLE III. Resulting values of the focusing criterion J. Parameters used:* $R_v = -90$, $z_{NL} = 0.1$, *and* $z_L = 0.4$. *Initial value* $J_0 = 0.07$ *and for an ideal corrector* $J_{\text{ideal}} = 0.42$.

| Servodrives configuration | *a* | *b* | *c* | *d* | *e* | *f* |
|---|---|---|---|---|---|---|
| $J$ | 0.24 | 0.25 | 0.33 | 0.33 | 0.34 | 0.33 |
| $\varepsilon(r_f = r_0)$ | 0.52 | 0.48 | 0.18 | 0.17 | 0.18 | 0.17 |

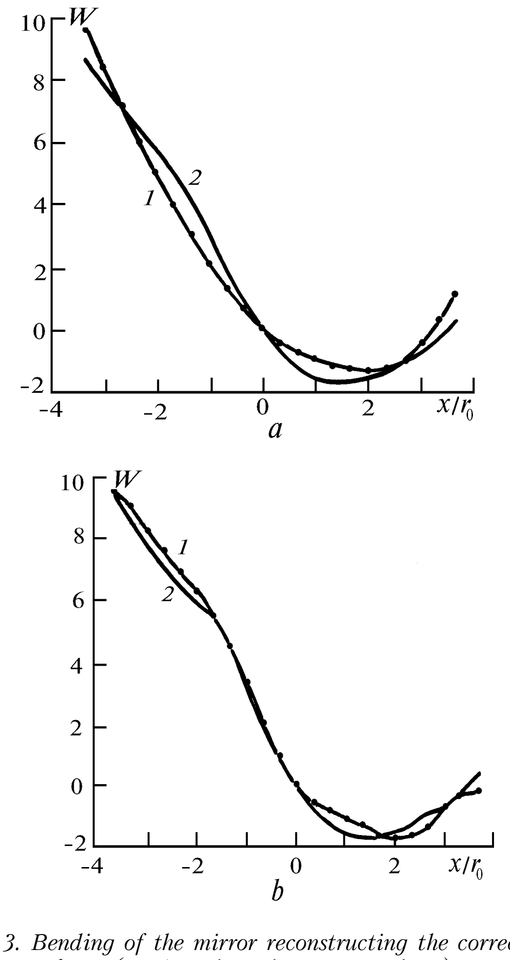


*FIG. 3. Bending of the mirror reconstructing the correcting phase surface (section by the ZOX plane). Curve 1 represents the profile of the phase surface, while curve 2 shows the profile of the mirror. (a) is for servodrive configuration b from Fig. 1 and (b) for the configuration c from Fig. 1. Parameters used:* $z_{NL} = 0.5$, $z_L = 0$, *and* $R_v = -30$.

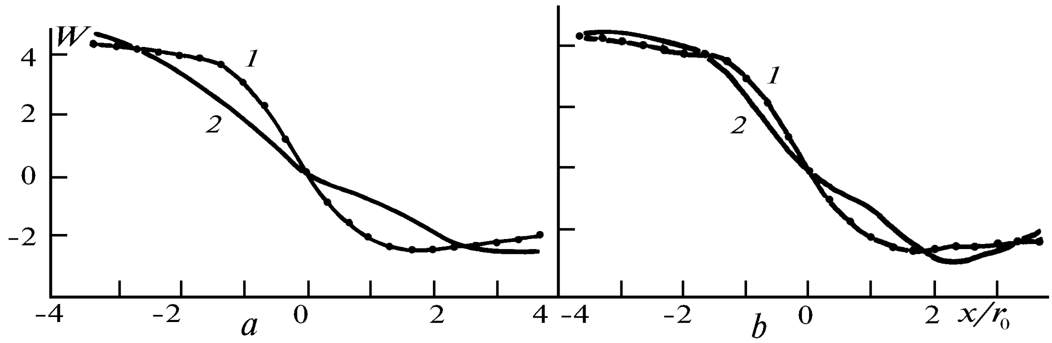


*FIG. 4. Bending of the mirror reconstructing the phase surface (section by the ZOX plane). Curve 1 represents the profile of the phase surface and curve 2 represents the mirror profile. (a) is for servodrive configuration b from Fig. 1 and (b) for the configuration c from Fig. 1. Parameters used:* $z_{NL} = 0.1$, $z_L = 0.4$, *and* $R_v = -90$.

*TABLE IV. Resulting values of the focusing criterion J. Parameters used:* $R_v = -110$, $z_{NL} = 0.1$, *and* $z_L = 0.4$. *Initial value* $J_0 = 0.05$ *and for an ideal corrector* $J_{ideal} = 0.42$.

| Servodrives configuration | *a* | *b* | *c* | *d* | *e* | *f* |
|---|---|---|---|---|---|---|
| $J$ | 0.04 | 0.21 | 0.38 | 0.34 | 0.34 | 0.34 |
| $\varepsilon(r_f = r_0)$ | 1.95 | 0.72 | 0.31 | 0.48 | 0.47 | 0.31 |

It seems likely that the efficiency cannot be increased by changing only the geometry of drives disposition without changing their density in the central portion (comparison of the models *c* and *d*, Fig. 1).

The above conclusions concern, first of all, the cases of propagation of beams of high intensity ($|R_v| \geq 90$). When beams of a moderate power ($|R_v| = 20$, 30) propagate through the medium the surface of the correcting phase is relatively smooth. Therefore all the aforementioned models including the simplest ones (*a*) and (*b*), give nearly equal results on correction for thermal blooming (Tables I and II).

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