MODAL CORRECTOR OF THE LOWEST-ORDER PHASE ABERRATIONS

V.P. Kandidov, I.V. Larionova, and V.V. Popov

M.V. Lomonosov State University, Moscow

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The possibility of the formation of first- and second-order phase aberrations with the help of a continuous elastic mirror, controlled by a system of concentrated load along its contour, is analyzed. The response functions of the mirror actuators and the base control modes are calculated using the theory of thin plates. Significant attention is devoted to determining the accuracy with which the lowest-order phase aberrations are reproduced by the mirror. The computational results are in good agreement with the experimental data obtained on an operating model of the mirror with electromagnetic actuators.

On atmospheric paths the lowest-order aberrations of the wavefront predominate in the nonlinear distortions of light beams. In Ref. 1 it is shown that aberrations of up to third order make the main contribution to the phase distortions of a beam propagating under conditions of wind refraction. The lowest-order aberrations can be efficiently compensated by using simple phase correctors with modal control at a small number (~ 10) of coordinates. Thus the possibility of suppressing the thermal self-action of a beam under conditions of wind refraction with the help of an adaptive mirror with three modal-control channels was examined in Ref. 2.

Modal control of the phase of a beam requires correctors with a nonlocal response function. Examples of such correctors are continuous elastic mirrors with fast actuators, fabricated in the form of piezoelectric stacks[3] and mirrors with discrete[4] and continuous[5] bimorph actuators. However such correctors have a low sensitivity (of the order of 10^{-2} μm/V) and a relatively small dynamic range, which restricts their use to visible radiation. In the infrared region compensation of nonlinear distortions requires controllable mirrors with a range of displacement of the reflecting surface of up to 100 μm. Such correctors are also of interest in laser technology for controlling beam position and size.

A modal corrector for controlling aberrations of up to third order inclusively can be built using flexible elastic mirrors which are deformed by loads applied along their contour. In this case the range of control can be significantly expanded by using electromechanical actuators to form the loads. In spite of the fact that such mirrors operate more slowly they can be used for adaptive phase control in the case of the propagation of quasi continuous-wave radiation under conditions of irregular wind refraction.[6]

In this paper we study a model of an elastic continuous mirror intended for modal phase control based on first- and second-order aberrations. The response function, the base modes for an operating prototype of a circular mirror clamped at the center and controlled with the help of six concentrated loads along its contour are calculated based on the theory of thin plates.

1. In modal control the deflection of the. reflecting surface of the mirror $w(r, \theta)$ is the superposition of the base forms of deflection $w_i(r, \theta)$:

$$w(r,\theta) = \sum_{i=1}^{I} U_i w_i(r,\theta), \tag{1}$$

where U_i are the coordinates of modal control and I is the number of such coordinates.

It is convenient to choose as the basis $w_i(r, \theta)$, $i = \overline{1, I}$, the system of Zernike polynomials $Z_i(r, \theta)$, which are usually employed to describe optical aberrations. The base forms $w_i(r, \theta)$ are formed as linear combinations of the response functions of the actuators $\Psi_k(r, \theta)$:

$$w_i(r,\theta) = \sum_{k=1}^{K} b_{ik}\Psi_k(r,\theta), \tag{2}$$

where K is the number of actuators, $K \geq 1$.

The response function of the k-th actuator $\Psi_k(r, \theta)$ is the deflection of the mirror with a unit displacement of the k-th actuator and zero displacements of the other actuators. The coefficients $b_{i,k}$, $k = \overline{1, K}$, form the basis vector $\vec{b}_i$ whose elements are proportional to the controlling displacements, forming the i-th form $w_i(r, \theta)$ of deflection of the mirror. Thus the collection of response functions $\Psi_k(r, \theta)$, $k = \overline{1, K}$, is the most important characteristic of the mirror, and determines the possibility and accuracy with which the chosen basis of modal control is formed with its help.

We shall study a controllable mirror in the form of an elastic, circular, thin plate whose center is clamped (Fig. 1). Rods, which are displaced by means of pushers with the help of actuators and restoring springs perpendicular to the plane of the mirror, are evenly spaced along the contour of the plate. As a result the mirror is deformed by a system of concentrated forces and moments acting on its contour.

We shall calculate the response function $\Psi_k(r, \theta)$ by a variational method using the approximations of the theory of thin plates,[7] which is applicable if the thickness h of the plate is much smaller than its radius R_0 ($h \ll R_0$). Let their six rods be on the contour of the mirror: $K = 6$. Then assuming that the rods are points where the rods are attached:

$$\Psi_k \left[R_0, (j-1)\frac{\pi}{3}\right] + l_2 \frac{\partial \Psi_k}{\partial r} \left[R_0, (j-1)\frac{\pi}{3}\right] = y_0 \delta_{jk}, \quad j=\overline{1,6}, \tag{3}$$

where y_0 is a unit displacement on the k-th actuator and l_2 is the distance from the point of application of the force to the edge of the mirror.

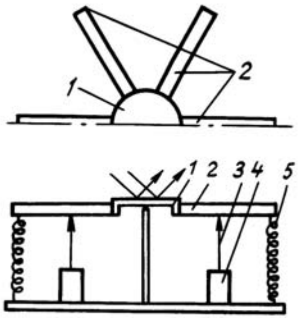


FIG. 1. Construction of the adaptive mirror: 1 — reflecting plate; 2 — rods; 3 — pusher; 4 — stepping motor; 5 — restoring spring.

Since the elasticity of the holding system at the center is not known a priori we shall first assume that it is absolutely rigid. In this case the form of the deflection at the point $r = 0$ must satisfy the following boundary conditions:

$$\Psi_k(0, \theta) = 0; \tag{4}$$

$$\frac{\partial \Psi_k}{\partial r}(0, \theta) = 0. \tag{5}$$

The total energy of elastic deformation of the mirror and the system clamping the actuators has the form

$$\varepsilon = \frac{D}{2} \int_0^{R_0}\int_0^{2\pi} \left[\nabla^4 \Psi_k - 2(1-\nu) \frac{\partial^2 \Psi_k}{\partial r^2} \left(\frac{1}{r}\frac{\partial \Psi_k}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Psi_k}{\partial r^2}\right) + 2(1-\nu)\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Psi_k}{\partial \vartheta}\right)\right]^2 r\,dr\,d\theta + \sum_{l=1}^{6} \frac{K_l X_l^2}{2} + \frac{\eta}{2}\left[\frac{\partial \Psi_k}{\partial r}(0,0)\right]^2;$$

$$D = \frac{Eh^3}{12(1-\nu^2)}, \tag{6}$$

where E is Young's modulus; ν is the Poisson ratio of the material of the elastic plate; K_l is the stiffness of the of the l-th spring; X_l is the deformation of the l-th spring; and, η is the holding factor at the center of the mirror.

We shall approximate the response function of the k-th actuator $\Psi_k(r, \theta)$ by the series

$$\Psi_k(r,\theta) = \sum_{n=1}^{3}\sum_{m=0}^{3} C_{nm}\left[\frac{r}{R_0}\right]^{n+1} \cos\left[m\left(\theta - (k-1)\frac{\pi}{3}\right)\right]. \tag{7}$$

According to the variational method the coefficient C_{nm} in the series (7) is determined by run. minimizing the energy (6) with respect to the set of coefficients C_{nm} satisfying the geometric boundary conditions (3)–(5).

The contour lines of deflection for the response function of the mirror actuator are shown in Fig. 2. One can see that the response function is substantially nonlocal.

The basic vectors $\vec{b}_i$, $i = \overline{1, I}$ were calculated by the method of the least squares. In so doing, taking the distribution of the intensity of the light beam on the mirror into account, a weight in the form of a Gaussian function with a characteristic scale $a = 0.3R_0$ was employed. Thus the problem of minimizing the following functional of the discrepancy between the mode sought $w_i(r, \theta)$ and the corresponding polynomial $Z_i(r, \theta)$ was solved:

$$F(\vec{b}_i) = \int_0^{R_0}\int_0^{2\pi} \left[Z_i(r,\theta) - \sum_{k=1}^{6} b_{ik}\Psi_k(r,\theta)\right]^2 \times \exp^{-r^2/a^2} r\,dr\,d\theta. \tag{8}$$

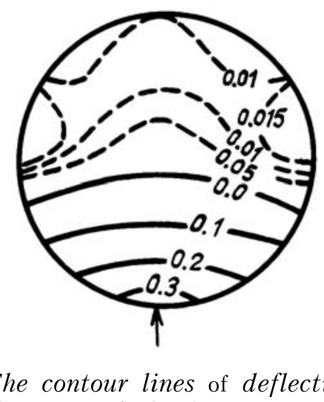


FIG. 2. The contour lines of deflection for the response function of the lower actuator, whose location is marked by the arrow.

The components of the vectors $\vec{b}_i$, $i = \overline{1, 5}$, forming the base modes of the mirror, with which the reflected beam acquires tilts, focusing, and astigmatism of the phase, are presented in Table I.

TABLE I.

Type of aberration	Displacements of the actuators					
	b_{11}	b_{12}	b_{13}	b_{14}	b_{15}	b_{16}
1	0	-3.3	-3.3	0	3.3	3.3
2	3.8	1.9	-1.9	-3.8	-1.9	1.9
3	2.4	2.4	2.4	2.4	2.4	2.4
4	-3.6	1.8	1.8	-3.6	1.8	1.8
5	0	-3.1	3.1	0	-3.1	3.1

The cross sections of the form of deflection of the mirror for these modes $w_i(r, \theta)$, $i = \overline{1, 5}$, are shown in Fig. 3. One can see that the second-order modes w_i, $i = \overline{3, 5}$, are close to the corresponding Zernike polynomials. The tilt modes w_i, $i = \overline{1, 2}$, differ substantially from $Z_($, $i = \overline{1, 2}$, because we have assumed that the center of the mirror is rigidly held.

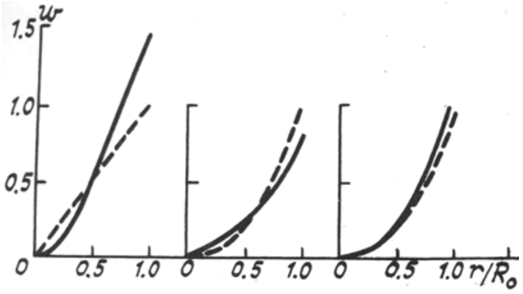


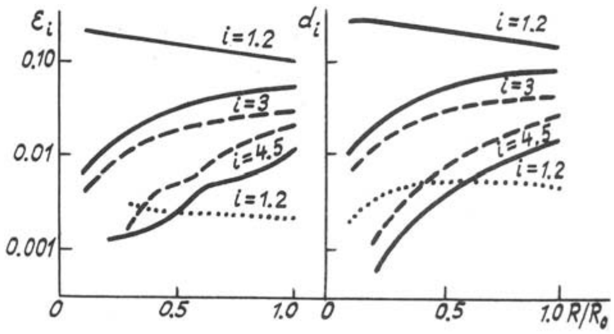
FIG. 3. Sections of deflection of the mirror by the plane θ = const in reproducing tilts (a), focusing (b), and astigmatisms (c). The solid lines correspond to the base modes w_1, $i = \overline{1, 5}$, and the broken lines correspond to the Zernike polynomials Z_i, $i = \overline{1, 5}$.

2. It is possible to form a given basis, in particular, in the form of a system of Zernike polynomials $Z_i(r, \theta)$ with finite accuracy on a mirror with a limited number of actuators. The standard deviation ε_i of the mode $w_i(r, \theta)$ on a correctable aperture of radius R is equal to

$$\varepsilon_i = \left\{ \int_0^R \int_0^{2\pi} \left[Z_i(r,\theta) - Cw_i(r,\theta) \right]^2 r dr d\theta \Big/ \int_0^R \int_0^{2\pi} Z_i^2(r,\theta) r dr d\theta \right\}^{1/2}. \quad (9)$$

Since the modes $w_i(r, \theta)$ were determined to within a constant factor we introduced a normalization coefficient C in Eq. (9). The value of this constant is calculated by minimizing ε_i.

The dependence of ε_i on the radius R of the working aperture of the mirror for the five lowest-order aberrations is shown in Fig. 4a (solid curves). The rms error $\varepsilon_{1,2}$ for the tilt modes increases monotonically as the radius R of the working aperture decreases; this is caused by significant deviations of the form of deflection at the center of the mirror, held rigidly on the axis, from a tilted plane (Fig. 3a). Conversely, for focusing, the error ε_3 decreases as the radius R decreases; this is evidently attributable to the increase in the relative contribution of the moment load and therefore to the fact that the form of the deflection of the mirror is more nearly parabolic. In forming astigmatisms the discrepancies $\varepsilon_{4,5}$ also decrease as R decreases. For a Gaussian beam with $a = 0.3R_0$, the working aperture may be taken to be equal to the visible spot, whose radius $R \simeq 0.6 R_0$. Then on the working aperture the error in the formation of phase aberrations is equal to 3.5% for focusing, 0.5% for astigmatisms, and about 13% for tilts.



a b

FIG. 4. The normalized standard deviation ε_i, $i = \overline{1, 5}$, of the base modes w_i, $i = \overline{1, 5}$ from the corresponding Zernike polynomials Z_i, $i = \overline{1, 5}$ (a) and the relative weight d_i, $i = \overline{1, 5}$, of the extraneous aberrations (b) as a function of the radius R of the working aperture of the mirror. i = 1, 2 – tilts; I = 3 – focusing, I = 4, 5 – astigmatisms. The solid lines correspond to the starting model of the mirror, the dotted lines correspond to a "semirigid" mirror, and the dashed lines correspond to the improved mirror.

The error ε_i arises owing to the presence of extraneous polynomials Z_p, $p \neq i$ in the base mode $w_i(r, \theta)$. The relative weight of the extraneous aberrations in the base mode $w_i(r, \theta)$ is given by the following relation;

$$d_i = \left[\sum_{\substack{p=1 \\ p \neq i}}^{P} \alpha_{ip}^2 \Big/ \sum_{p=1}^{P} \alpha_{ip}^2 \right]^{1/2}, \quad (10)$$

where α_{ip} is the coefficient in the expansion of the mode $w_i(r, \theta)$ in a system of the Zernike polynomials:

$$\alpha_{ip} = \int_0^R \int_0^{2\pi} w_i(r,\theta) Z_p(r,\theta) r dr d\theta \Big/ \int_0^R \int_0^{2\pi} Z_p^2(r,\theta) r dr d\theta. \quad (11)$$

The weight of the extraneous aberrations d_i, $i = \overline{1, 5}$, as a function of the radius R of the working aperture is shown in Fig. 4b. The dependence $d_i(R)$, $i = \overline{1, 5}$, is similar to the errors ε_i obtained for the corresponding modes. The relative weight d_i of the extraneous aberrations in the base modes $w_{1,2}$, corresponding to tilts of the mirror, increases approximately up to 25% as R decreases. The main extraneous aberration in $w_{1,2}$ is the coma. As indicated above this is due to the assumption that the center of the mirror is rigidly held down. In the base focusing mode w_3 the weight d_3 increases as R increases, reaching 7% at $R = R_0$. The largest extraneous aberration in w_3 is the spherical aberration, and its sign is opposite to that of the focusing. The weight of the extraneous aberrations in the formation of the astigmatisms $w_{4,5}$ does not exceed 1.5%.

The increase in the angular divergence of the radiation reflected from the mirror is related with the error in the reproduction of the lowest-order phase aberrations. The quality of tilt control and control of the beam focusing can be characterized by the power $P(\theta_0)$ of the radiation propagating after the corrector in a given solid angle θ_0:

$$P(\theta_0) = \int_0^{2\pi} \int_0^{\theta_0} I(\theta,\varphi) d\theta d\varphi. \quad (12)$$

The radiant intensity of the radiation $I(\theta, \varphi)$ is determined by the amplitude of the light field $E(x, y, 0)$ of the reflected beam. For a collimated beam with a Gaussian profile incident normally on the aperture of the mirror we have

$$E(x,y,0) = E_0 \exp\left\{-\frac{x^2+y^2}{2a_0^2}\right\} \exp\left\{i\varphi(x,y)\right\};$$

$$a_0 = 0.3R_0. \quad (13)$$

In the case of an ideal corrector, whose deflection corresponds to the Zernike polynomials, the phase $\varphi(x, y)$ is equal to

$$\varphi_i^0(x,y) = 2kZ_i(x,y); \quad (14)$$

in the case of the mirror studied here

$$\varphi_i(x,y) = 2kw_i(x,y). \quad (15)$$

Let θ_0 be the angle at which the power $P^0(\theta) = \exp^{-1}P$, where P is the total power, for a Gaussian beam reflected from an ideal corrector. For the same beam reflected from the mirror under study it is not difficult to find the power $P^*(\theta_0)$ in the solid angle θ_0 with the help of the parabolic theory of diffraction and the relations (12), (13), and (15). As the calculations show, in the case of focusing by the mirror the power $P^0(\theta_0)$ is equal to 99.8% of the power $P^0(\theta_0)$ of a beam with an ideal parabolic front, and in the case of tilting the percentage is 66%.

3. Interferometric investigations of the operating model of the mirror showed that the deviation of the real base forms of deflection from the corresponding Zernike polynomials over the entire aperture of the mirror is about 5% for tilts, 21% for focusing, and 15% for astigmatisms. It is obvious that the tilts are reproduced by the model of the mirror better than expected in the calculations, and therefore the assumption that the center of the mirror is rigidly clamped was not confirmed.

The experimental results showed that the holding factor of screw-type clamping at the center of the mirror is equal to $\eta = 0.16\ D$. This made it possible to calculate the response function, the basic vectors, and modes for a "semirigid" model of the mirror which better describes the experimental model. For the "semirigid" model a computed error in reproducing the tilts of the mirror does not exceed 0.3% (for a working aperture of any radius, see Fig. 4a). The relative weight of the extraneous aberrations in the base modes corresponding to the tilts of the mirror does not exceed 0.5% for this case (Fig. 4b).

It should be noted that the significant error obtained in the experiment for second-order aberrations is connected with the fact that the required tolerances are exceeded in the fabrication of the mirror plate.

The appearance of spherical aberration accompanying the formation of focusing on the mirror is explained by the fact that the restoring springs generate a strong moment load whose sign is opposite to that of the moment force of the pushers. This can be eliminated by placing the restoring springs between the mirror and the pushers. The rms error in reproducing the focusing by a mirror with this improved construction is 1.5–2 times lower depending on the radius of the working aperture (Fig. 4a) and the relative weight of the spherical aberration decreases from 7% to 4% at $R = R_0$ (Fig. 4b). The astigmatisms are not reproduced as well as by the initial construction. Nonetheless it is preferable to change the position of the restoring springs, since all second-order aberrations are reproduced with an accuracy which is not less than 3% on a working aperture with radius $R = 0.6R_0$.

4. The theoretical analysis performed above and interferometric measurements confirmed that first- and second-order phase aberrations could be corrected with the help of a flexible mirror, controlled by loads applied to its contour. The use of rods moved by

electromechanical actuators for obtaining the loads makes it possible to combine a wide dynamic range of phase control with the required accuracy in positioning the reflecting surface.

The order of the correctable aberrations can be increased with the help of a system of actuators which create independent force and moment loads.

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