STATISTICAL CHARACTERISTICS OF LASER RADIATION TRANSMITTED THROUGH THE TURBULENT ATMOSPHERE FOR SAMPLING TIMES OF THE ORDER OF THE INTENSITY CORRELATION TIME

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Results of experimental investigations into statistical characteristics of the laser radiation transmitted through the turbulent atmosphere are presented. For sampling times comparable with the intensity correlation time, the probability distribution of photocounts for non-Gaussian radiation field has been derived. With the help of the inverse Laplace transform, the distribution of the integral radiation intensity has been reconstructed. The feasibility of describing the integral intensity distribution by the lognormal law has been established.

Efficiency of laser systems for optical communication and detection and ranging is principally limited by the effect of field fluctuations at a reception point, caused by inhomogeneity of the refractive index of the atmosphere and fluctuations of signals and noise in a receiving train. Therefore, optimization and evaluation of the efficiency of laser systems call for the study of fluctuations of laser radiation and noise aimed at the construction of their statistical models. To record the laser radiation, a photomultiplier tube (PMT) is used. In this case, the main statistical characteristic is the emission probability of n photoelectrons for the sampling time T, $P(n, T)$.

The probability distribution of photocounts (PDP) was studied in ample detail in the case of the Gauss field in the receiving plane.[1] As a result, a number of approximate solutions were constructed as functions of the relations between T and τ_c, the correlation time of the laser radiation intensity fluctuations in the atmosphere. In a number of cases of laser system operation, the field distribution in the receiving plane is substantially non-Gaussian. In this connection, only a few results are known for asymptotes $T \ll \tau_c$ and $T \gg \tau_c$. Thus, for example, introducing the condition of lognormality of the intensity distribution

$$\omega(I) = \frac{1}{\sqrt{2\pi}\,\sigma I} \exp\left\{-\left[\ln\frac{I}{I_0} + \frac{\sigma}{2}\right] / \, 2\sigma^2\right\}^2 , \qquad (1)$$

where I_0 is the radiation intensity without turbulence and

$$\sigma^2 = \left\langle\left(\ln\frac{I}{I_0}\right)^2\right\rangle - \left\langle\ln\frac{I}{I_0}\right\rangle^2 ,$$

the PDP for $T \ll \tau_c$ can be described by the Diament – Teich distribution

$$P(n, \langle n\rangle, \sigma) = \frac{M^n\, e^{-M}}{n!} \frac{\exp[-0.5\,\sigma^2\,(M - n)^2]}{[1 + \sigma^2 M]^{1/2}} , \qquad (2)$$

where for each n, the parameter M should be determined by solving the transcendental equation of the form

$$\ln M = \ln\langle n\rangle + \sigma^2(n - M - 0.5).$$

For $T \gg \tau_c$ the PDP obeys the Poisson distribution.

In this paper, the results of experimental laboratory investigations of the PDP for sampling times of the order of the correlation time of the intensity are presented. This allows us to ensure the stationarity and to control the basic parameters of the laboratory path to the degree that cannot be achieved in an outdoor experiment.

The block diagram of the experimental setup is shown in Fig. 1. The radiation of an LG – 38 laser with a wavelength of 0.63 μm passes through a short path with intense turbulization of air, made by a heater, and enters a photodetector. With the help of the neutral and polarization filters, the radiation intensity is coarsely and finely regulated in wide limits, and with the help of the collimator and diaphragms (∅ 0.4 mm), the regime of "point samplingB is set. The optical radiation is detected with the help of the FEU – 147 – 3 photomultiplier operating in the regime of photon counting. From the output of the photomultiplier, single-electron pulses are standardized in the amplifier-shaper (AS) and are applied at the Cr 6 photon counter.

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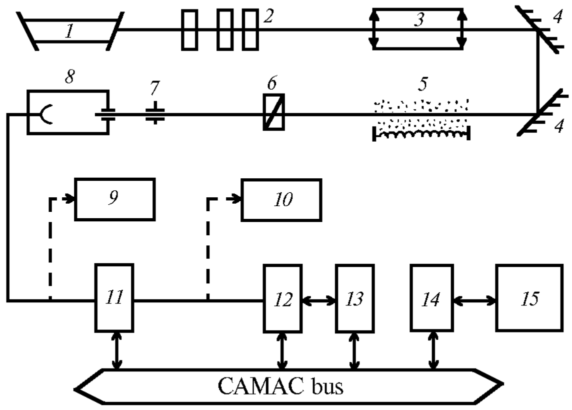

FIG. 1. Block diagram of the experimental setup: 1) laser, 2) neutral filters, 3) collimator, 4) reflector, 5) heater, 6) polarization filter, 7) diaphragm, 8) PMT, 9) oscillograph, 10) wavemeter, 11) amplifier-shaper, 12) counter, 13) timer, 14) CRATE controller, and 15) computer.

At the end of each sampling interval of duration T, determined by a strobe pulse of the timer (SPT), the number of photocounts n, accumulated in the counter, was transmitted to the computer, where the unity was added in the preset array to the nth address of a memory. After that the counter was switched to the initial state to count the number of pulses of the next sample. This process was repeated until the number of samples be sufficient to construct the histogram of photocount distribution. Further the histogram was statistically processed and probabilities and various moments of distribution were calculated. The relative variance of photocounts β_n^2, the coefficient of asymmetry K_{as}, and the excess coefficient K_{ex} were used to estimate numerically the parameters of distributions.

To study the dependence of the PDP on the sampling time, the method is commonly used in which with the increase of T, the radiation intensity I is correspondingly decreased in order the mean value of photocounts remain constant: $\langle n \rangle = \text{const}$. This method works well for small $T/\tau_c \ll 1$ (where $\tau_c \simeq 1$–100 ms). However, as the sampling time approaches the correlation time of the intensity, the duration of the experiment considerably increases and a large number of measurements cannot be performed. Moreover, the system operation with the sampling time $T \approx \tau_c$ is characterized by extremely low radiation flux, when the dark flux density of electrons of the PMT becomes comparable with the density flux of photoelectrons and small variations of the radiation intensity may engender the sharp change of the signal – to – noise ratio. Therefore, we took advantage of another approach that was free of the above-indicated disadvantages, namely, for fixed I we studied the distribution as a function of I.

The investigations were conducted for three different states of the induced turbulence, which, judging from the parameter β_n^2 (for $T \ll \tau_c$ and small intensity fluctuations, $\beta_n^2 \approx \beta_0^2$, where β_0^2 is the scintillation index), corresponded to the weak and intermediate turbulence in the real atmosphere. For each state of the turbulent path, the PDP was determined for 10 sampling times. To eliminate statistical errors and to ensure sufficient accuracy of investigations, the PDP was determined 10 times for each sampling time. The number of counts in each experiment on the determination of the PDP was $\approx 10^6$, which ensured sufficient statistics of averaging.

Each experiment on determination of the PDP was preceded by recording of a series of counts in the order of their arrival for subsequent calculation of the autocorrelation function. The standard deviation of the correlation time of the intensity fluctuations did not exceed 7 % for each state of the turbulent path. In connection with this, we can state that the condition of the path stationarity was reasonably met.

From the experimental PDP for the non–Gaussian field, the dependence of the relative variance of photocounts β_n^2 on the sampling time was first established. Using the least–squares method, we succeeded in the description of this dependence for a wide interval of sampling times, including $T \approx \tau_c$, by the analytical function of the form

$$\beta_n^2(z) = \exp\,[az + b + c/(d + z)], \tag{3}$$

where $z = T/\tau_c$, and a, b, c, and d are the coefficients determined from the experimental conditions.

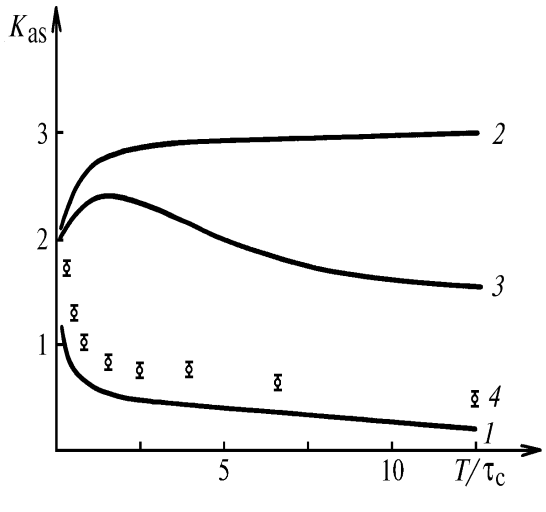

FIG. 2. Dependence of the asymmetry coefficient on the sampling time: 1) K_{as} for the Poisson distribution, 2) Diament-Teich distribution, 3) truncated Diament-Teich distribution, and 4) experimental distribution with $\tau_c = 0.62$ ms, $\sigma^2 = 0.49$.

As show the experimental results, the PDP for the non–Gaussian field and $T \approx \tau_c$ differed considerably from the known approximate distributions. In Fig. 2 the dependence of the experimental coefficients of asymmetry on the sampling time is shown for the stationary turbulence with $\sigma^2 = 0.49$. For comparison, the coefficients of asymmetry are also given here for: a) Poisson and b) Diament-Teich distributions, determined from the expressions for the moments of the field energy and their relations with factorial moments of photocounts, and c) Diament-Teich distribution calculated for the probability of counts taken for the corresponding distribution[2] (truncated Diament-Teich distribution, see below).

As seen from the curves, these distributions are unsuitable for describing the experimental results. From an analysis of these results, we can draw the conclusions important for future investigations: for $T \approx \tau_c$, the finiteness should be considered of experimental averaging in the determination of the distribution moments. To compare with theoretical curves, truncation should be used, that is, the summation in the determination of moments n_{max} should be limited by the channel, in which n_{max} is the maximum number of counts for the sampling time of this experiment.

Because the known distributions are unsuitable for the description of the experimental PDP at $T \approx \tau_c$, it is necessary to find new distributions describing the experimental results. This can be done, for example, using the Mandel formulas[3]

$$P(n, T) = \int_0^\infty \frac{(\eta\, U)^n}{n!} \exp(-\eta U)\, \omega(U)\, dU, \qquad (4)$$

where $U = \int_0^T I(t)\, dt$ is the energy released on the detector for the sampling time. To solve equation (4), the intensity distribution should be specified. It is well known that for $T \ll \tau_c$ the intensity distribution is unambiguously determined by the photocount distribution derived by the inversion of formula (4). For the sampling time $T \approx \tau_c$, the photocount distribution should specify the integral intensity distribution. Then according to equation (4), the probability of the event that we do not record even a single photocount during time T (for the specified average number of photocounts $<n>$) is equal to

$$P(0, <n>) = \int_0^\infty \exp(-<n>\, x)\, \omega(x)\, dx, \qquad (5)$$

where $\omega(x)$ is the probability distribution of the normalized energy detected by the detector, and $x = U/<U>$. Thus, the quantity $P(0, <n>)$ can be considered as the Laplace transform of the function $\omega(x)$, and if the dependence $P(0, <n>)$ is known (for example, from the experiment), the corresponding $\omega(x)$ can be found, if we take the inverse Laplace transform of the function $P(0, <n>)$

$$\omega(x) = \frac{1}{2\pi i} \int_{b-i\infty}^{b+i\infty} P(0, <n>) \exp(<n>\, x)\, dn. \qquad (6)$$

Expanding $\omega(x)$ in the Laguerre functions, after a number of manipulations in accordance with Ref. 3, we derive a solution to the inverse problem in the form

$$\omega(x) = \sum_{m=0}^{\infty} a_n(x)\, P(n, <n>), \qquad (7)$$

where

$$a_n(x) = 2 <n> (-2)^n \sum_{m=0}^{\infty} \binom{m}{n} l_n(2<n>x) =$$

$$= 2 <n> (-2)^2 \sum_{k=0}^{\infty} \binom{n+k}{n} l_{n+k}(2<n>x)$$

and $l_n(y)$ are the Laguerre functions.

In Fig. 3, the distributions $\omega(x)$ are shown by dots, obtained by solving the inverse problem. The results are presented for one state of turbulence and different sampling times. Solid curve corresponds to the lognormal distribution, with the parameter σ^2 being a function of the sampling time.

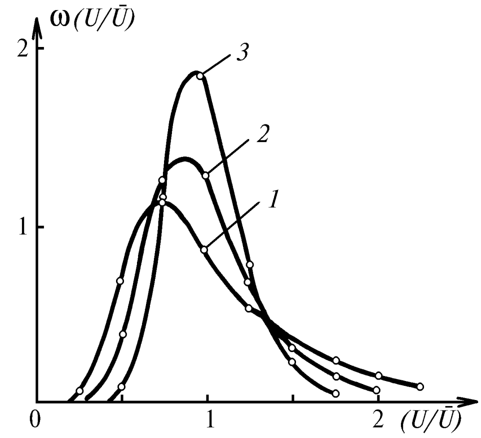

FIG. 3. Distribution of the integral intensity at $\tau_c = 0.68$ ms. *The dots were calculated from the photocount distribution by taking the inverse Laplace transform; solid curves were calculated for the lognormal distribution: 1)* $\sigma_U^2 = 0.18$, $T = 0.125$ ms; *2)* $\sigma_U^2 = 0.09$, $T = 0.125$ ms; *3)* $\sigma_U^2 = 0.05$, $T = 4$ ms.

In this case, the parameter of the lognormal distribution was the variance of the integral intensity σ_U^2 rather than the intensity variance σ^2 used earlier. The dependence of the integral intensity variance on the sampling time may be evaluated from Eq. (3). As seen from Fig. 3, the lognormal distribution with the parameter σ_U^2 describes fairly accurately the integral intensity distribution.

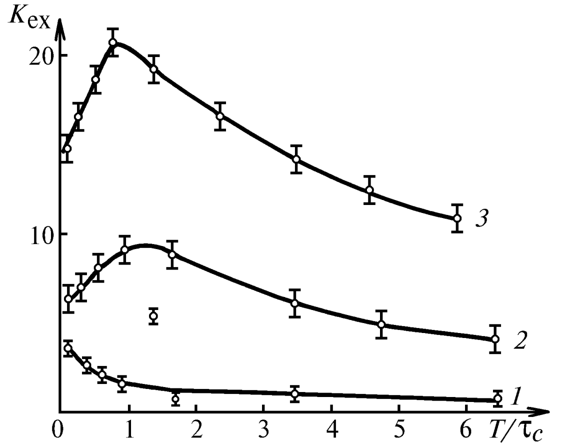

FIG. 4. Dependence the excess coefficient on the sampling time for three different states of the turbulence: τ_c = 0.68 *(1)*, 2.48 *(2)*, and 3.63 ms *(3)*.

The results of many experiments, a part of which is shown in Fig. 3 allow us to state that the general behavior of the integral intensity distribution does not depend on the sampling time; only the distribution parameter depends on the sampling time.

Another important conclusion which can be drawn from these results is the feasibility of using the Diament – Teich distribution to describe the PDP in a wide range of sampling times, including $T \approx \tau_c$. For this, it will suffice to substitute the variance of the instantaneous intensity by the variance of the integral intensity in this distribution.

In Fig. 4, the dependence of the excess coefficients of the experimental distribution on the sampling time for three different states of the turbulent path is shown. The solid curve corresponds to the coefficients of the truncated Diament – Teich distribution, in which the integral intensity variance is used as a parameter of the distribution. As seen from the curves, even for the fourth moments this distribution describes well the experimental results. Coincidence of the distributions for the fourth moments indicates the high accuracy of description of the PDP of the laser radiation transmitted through the turbulent atmosphere by the Diament – Teich distribution with the parameter σ_U^2 (T/τ_c) in a wide range of sampling times for the lognormal intensity distribution.

REFERENCES

1. G.N. Glazov, *Statistical Problems of Lidar Sensing of the Atmosphere* (Nauka, Novosibirsk, 1987), 312 pp.
2. Z.T. Azamatov, M.R. Bedilov, A.K. Kasimov, et al., "*Statistical characteristics of the laser radiation transmitted through the turbulent atmosphere for the sampling time comparable with the intensity correlation time,B* Preprint No. 5, Tashkent State University, Tashkent (1994), 17 pp.
3. S.A. Akhmanov, Yu.E. Dbyakov, and A.S. Chirkin, *Introduction into Statistical Radar Systems and Optics* (Nauka, Moscow, 1981), 640 pp.