ON THE CALCULATION OF RADIATIVE TRANSFER THROUGH A BOUNDED SCATTERING MEDIUM

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Algorithm for calculating the fluxes of radiation propagated through the scattering medium has been presented. The comparison with data obtained by the Monte Carlo method has been made.

The main methods for calculating the radiation fluxes in bounded media are numerical ones and require computers and much computation time.[1] At the same time, the solution of a number of applied problems calls for the development of efficient analytical methods. Now a number of methods has been developed, e.g., the FA method,[2] the multiple reflection method (MRM),[3,4] and the modified δ—Eddington method.[5] Since this problem is of great interest, the algorithm for one of the above-mentioned methods, namely, the MRM is discussed below as well as some computational results obtained by this method.

The accuracy of the calculation of radiative transfer characteristics depends on a number of important factors one of which is the approximation of a real scattering phase function.[1] When developing the analytical methods one of the most generally applicable ways of approximation is the parameterization of a volume scattering phase function via the integral parameters. The choice of specific representation of the scattering phase function strongly affects the calculational accuracy. By way of example we may point out Ref. 6.

The radiative transfer problem is solved here for a bounded disperse medium. The general methodology was described in detail in Ref. 4. Let us assume the model of a scattering medium in the form of a rectangular parallelepiped and choose the Cartesian coordinates in such a way that its origin coincides with one of the parallelepiped vertices and its axes are directed along the parallelepiped edges. Let us now specify the optical dimensions of the volume as τ_{x0}, τ_{y0}, and τ_{z0}. The scattering phase function $\chi(\theta)$ is represented via the six integral parameters with the normalization condition

$$\gamma + \beta + \sum_{i=1}^{4} \mu_i = 1 , \qquad (1)$$

where η, β, and μ characterize the scattering in the directions $\pm x$, $\pm y$, and $\pm z$ and θ is the scattering angle. The way of determining the integral parameters has been described in Refs. 3 and 4. When the scattering phase function is axisymmetric this representation is analogous to Ref. 7. Absorption is taken into account by means of the photon survival probability Λ.

Let a parallel flux of monochromatic radiation with the intensity $I_0 = 1$ be incident on one of the side boundaries of the volume in the direction $+x$. In this case, following Ref. 4, the fluxes $I_1(\tau, \Lambda, \theta)$, $I_2(\tau, \Lambda, \theta)$, and $I_3(\tau, \Lambda, \theta)$ leaving the bounded medium in the direction along the $\pm x$, $\pm y$, and $\pm z$ axes and the absorbed flux $I_\Lambda(\tau, \Lambda, \theta)$ are given by the relations

$$I_1(\tau, \Lambda, \theta) = \frac{[1 - R^2(\tau_y, \tau_z, \Lambda, \theta)]\exp[-K(\tau_y, \tau_z, \Lambda, \theta)\tau_{x_0}]}{1 - R^2(\tau_y, \tau_z, \Lambda, \theta)\exp[-2K(\tau_y, \tau_z, \Lambda, \theta)\tau_{x_0}]} ;$$

$$I_2(\tau, \Lambda, \theta) = \frac{\{1 - \exp[-2K(\tau_y, \tau_z, \Lambda, \theta)\tau_{x_0}]\} R(\tau_y, \tau_z, \Lambda, \theta)}{1 - R^2(\tau_y, \tau_z, \Lambda, \theta)\exp[-2K(\tau_y, \tau_z, \Lambda, \theta)\tau_{x_0}]} ; \qquad (2)$$

$$I_3(\tau, \Lambda, \theta) = I_s(\tau, \Lambda, \theta) + I_\mathrm{K}(\tau, \Lambda, \theta) =$$

$$= \frac{[1 - R(\tau_y, \tau_z, \Lambda, \theta)]\{1 - \exp[-K(\tau_y, \tau_z, \Lambda, \theta)\tau_{x_0}]\}}{1 + R(\tau_y, \tau_z, \Lambda, \theta)\exp[-K(\tau_y, \tau_z, \Lambda, \theta)\tau_{x_0}]} ,$$

where the variable coefficients $K(\tau_y, \tau_z, \Lambda, \theta)$ and $R(\tau_y, \tau_z, \Lambda, \theta)$ take the following form:

$$K(\tau_y, \tau_z, \Lambda, \theta) = \sqrt{P(\tau_y, \tau_z, \Lambda, \theta)[1 - \Lambda(\eta - \beta)]} ;$$

$$R(\tau_y, \tau_z, \Lambda, \theta) = \frac{K(\tau_y, \tau_z, \Lambda, \theta) - P(\tau_y, \tau_z, \Lambda, \theta)}{K(\tau_y, \tau_z, \Lambda, \theta) + P(\tau_y, \tau_z, \Lambda, \theta)} , \qquad (3)$$

and the function $P(\tau_y, \tau_z, \Lambda, \theta)$ determines the scattering along the x axis.

Let us consider in detail the algorithm for solving the problem of radiative transfer. It should be emphasized that this analytical approach allows the solution to be obtained for a medium of arbitrary configuration with the scattering phase function anisotropic in all directions. To simplify the derivations it is expedient to introduce simplifications usually employed in the scattering theory: $\tau_{y_0} = \tau_{z_0} = \tau_c$ and an axisymmetric scattering phase function $\mu_i = \mu$, $i = 1, 2, 3, 4$. Then the function $P(\tau_c, \Lambda, \theta)$ is given by the formula

$$P(\tau_c, \Lambda, \theta) = P_0(\Lambda, \theta) - 4\mu[2\mu\Lambda + P_0(\Lambda, \theta) -$$

$$- P_1(\tau_c, \Lambda, \theta)] F_1(\tau_c, \Lambda, \theta)/P_1(\tau_c, \Lambda, \theta) , \qquad (4)$$

where

$$P_0(\Lambda, \theta) = 1 - \Lambda(\eta + \beta) ; \qquad (5)$$

$$P_1(\tau_c, \Lambda, \theta) = P_0(\Lambda, \theta) - 4\mu^2\Lambda F_0(\tau_c, \Lambda, \theta)/P_0(\Lambda, \theta) ; \qquad (6)$$

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$$F_0(\tau_c, \Lambda, \theta) = 1 - \frac{[R_0(\Lambda, \theta) + 1]\{1 - \exp[-K(\Lambda, \theta)\,\tau_c]\}}{K_0(\Lambda, \theta)\,\tau_c\{1 + R_0(\Lambda, \theta)\exp[-K_0(\Lambda, \theta)\,\tau_c]\}}. \quad (7)$$

The coefficients $K_0(\Lambda, \theta)$ and $R_0(\Lambda, \theta)$ are calculated from formulas (3) in which $P(\tau_c, \Lambda, \theta)$ is replaced by $P(\Lambda, \theta)$. The function $F_1(\tau_c, \Lambda, \theta)$ is calculated in analogy with Eq. (7) with $K_0(\Lambda, \theta)$ and $R_0(\Lambda, \theta)$ replaced by $K_1(\tau_c, \Lambda, \theta)$ and $R_1(\tau_c, \Lambda, \theta)$. The values $K_1(\tau_c, \Lambda, \theta)$ and $R_0(\tau_c, \Lambda, \theta)$ are calculated from formula (3) with $P_1(\tau_c, \Lambda, \theta)$ in place of $P(\tau_c, \Lambda, \theta)$. The absorbed energy is found from the formula

$$I_K(\tau, \Lambda, \theta) = I_3(\tau, \Lambda, \theta)(1 - \Lambda)\{\eta + \beta + 4\mu \times$$

$$\times [1 - 2\mu + 2\mu F_0(\tau_c, \Lambda, \theta)/P_0(\Lambda, \theta)] \times$$

$$\times F_1(\tau_c, \Lambda, \theta)\}/P(\tau_c, \Lambda, \theta)\,. \quad (8)$$

This algorithm makes it possible quite accurately to determine the fluxes of radiaton propagating through the bounded scattering volume. The results of calculations of the radiation fluxes for the conservative medium in the form of a cube with the scatting phase function for cloud C_1 (Ref. 8) are shown in Fig. 1 (in this case $\eta = 0.8475$, $\beta = 0.0115$, and $\mu = 0.03525$). Depicted in this figure are also the data obtained by the Monte Carlo method under the same conditions.[2]

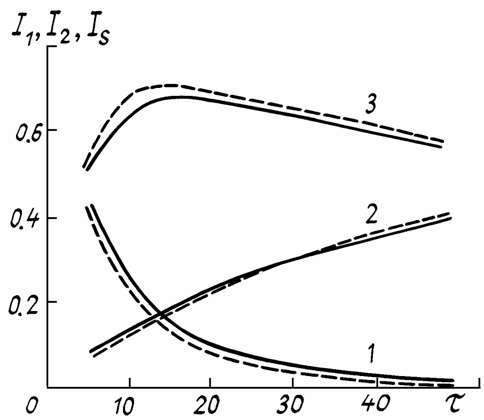

FIG. 1. The values of the transmitted, reflected, and exited radiation fluxes through the side boundaries as functions of optical dimensions of the medium: 1) $I_1(\tau, \Lambda, \theta)$*, 2)* $I_2(\tau, \Lambda, \theta)$*, and 3)* $I_3(\tau, \Lambda, \theta)$*. The solid curves show the results of calculations from Eq. (2) and the dashed curves show the results of calculations by the Monte Carlo method.*

The choice of such a model of scattering medium allows us to show in the same plot the dependences of the values of radiation fluxes on the optical depth for a wide range of both longitudinal and transverse optical dimensions of the medium. The comparison of the results shows that with increase of τ the calculational accuracy becomes higher.

Figure 2 shows the results obtained for the absorbing medium as a plot of $I_2(\tau, \Lambda, \theta)$ vs the optical dimensions for different values of the photon survival probability Λ. We can conclude based on the satisfactory agreement with the exact data[2] that in our analytical method the absorption has been accounted for sufficiently correctly.

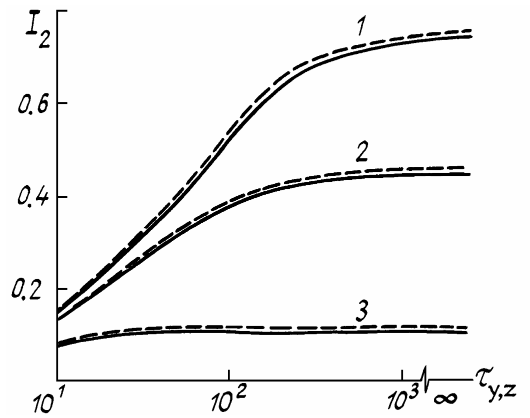

FIG 2. The values of the reflected radiation fluxes as functions of optical dimensions of the medium: 1) $\Lambda = 0.999$*, 2)* $\Lambda = 0.99$*, and 3)* $\Lambda = 0.9$*,* $\tau_{x0} = 200$*. The solid curves show the results of calculations from Eq. (2) and the dashed curves show the results of calculations by the Monte Carlo method.*

It should be noted in conclusion that not all of the available methods provide a correct comparison of the radiation fluxes transmitted through the bounded scattering medium since the results depend strongly on the configuration of this medium.

REFERENCES

1. J. Lenobl', *Radiative Transfer through the Scattering and Absorbing Atmospheres* (Gidrometeoizdat, Leningrad, 1990), 264 pp.
2. R. Davies, J. Atmos. Sci. **35**, 1712–1725 (1978).
3. B.A. Savel'ev and S.B. Mogil'nitskii, Izv. Vyssh. Uchebn. Zaved., Ser. Fiz., No. 8, 82–85 (1982).
4. B.V. Goryachev, M.V. Kabanov, and B.A. Savel'ev, Atm. Opt. **3**, No. 2, 125–132 (1990).
5. J.H. Joseph and W.J. Wiscombe, J. Atmos. Sci. **33**, 2452–2459 (1976).
6. W.E. Meador and W.R. Weaver, J. Atmos. Sci. **37**, 630–643 (1980).
7. C.M. Chu and S.W. Churchill, J. Phys. Chem. **59**, 855–863 (1955).
8. D. Deirmendjian, *Electromagnetic Scattering on Spherical Polydispersions* (Elsevier, Amsterdam; American Elsevier, New York, 1969).