INVESTIGATION OF THE ATMOSPHERIC AEROSOL BY THE METHOD OF INVERSION OF THE DAYTIME SKY BRIGHTNESS ANGULAR DISTRIBUTION FUNCTION

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Recieved November 27, 1989

A simplified procedure for determining the optical characteristics of atmospheric aerosol from measurements of the daytime sky brightness angular distribution function and the results are presented.

The results of spectrozonal reconstruction of the index of refraction and the size spectrum of aerosol particles, present in the atmosphere when the brightness angular distribution function is measured, are discussed.

It is shown that the data obtained adequately represent the general case.

Measurements of the sky brightness angular distribition function at the solar almucantars in seven or eight sections of the spectrum and wavelengths A ranging from 412 to 2195 nm were performed in the town of Ryl'sk during the morning on August 1 and 10, 1987 at solar zenith angles z_0 ranging from 65° to 77°. Before and after the measurements the brightness of a reference screen was measured at each almucantar and in each spectral channel. In addition, the optical thickness of extinction of solar radiation by the atmosphere ("Bouguer thickness") was determined by the "exoatmospheric brightness" method.

A filter photometer, built at the Astrophysical Institute of the Academy of Sciences of the Kazakh SSR was used to measure the brightness angular distribution function. The characteristics of the photometer were as follows: the photometer had eight spectral channels; the angle of the field of view of the photometer was equal to one degree; and, the expected total rms error in the measurements of the brightness angular distribution function was equal to 5–7%.

The spectral-brightness angular distribution function $\mu_H(\theta, \lambda)$ were determined from data obtained by simultaneous measurements of the spectral brightness of the daytime sky at the solar almucantar

$$B(\lambda) = E_0(\lambda)\, m_0 \mu_H(\theta, \lambda) \exp[-m_0 \tau_{exp}(\lambda)] \quad (1)$$

and the spectral brightness of a reference screen

$$B_s(\lambda) = E_0(\lambda) \frac{\rho_s}{\pi} \exp[-m_0 \tau_{exp}(\lambda)], \quad (2)$$

where $E_0(\lambda)$ is the exoatmospheric spectral solar illumination intensity; $m_0 \simeq \sec z_0$ is the atmospheric mass in the direction toward the sun; and, ρ_s is the albedo of the screen.

From the relations (1) and (2) we obtain

$$\mu_H(\theta, \lambda) = \frac{\rho_s B(\lambda)}{\pi m_0 B_s(\lambda)}.$$

The brightness angular distribution functions were measured, following the program of the joint Soviet-American aerosol experiment, for the purpose of determining, by inversion methods, the optical and microphysical characteristics of atmospheric aerosol, namely, the optical thickness and the altitude-averaged spectral scattering phase functions and the size spectrum of the aerosol particles.

Of the existing fast methods for inverting the brightness angular distribition function in which there is no need to perform repeated iterations of the results of the numerical solution of the transfer equation, as done, for example, in Ref. 1, we chose the method proposed in Ref. 3. Because the method of Ref. 3 was tested using model data with precise initial conditions, it was necessary to test the stability of this method in the presence of noise by analyzing data obtained by in situ measurements distorted by errors.

The essence of the method of Ref. 3 consists of the following (in what follows the indices λ are dropped):

1. The optical thicknesses τ_H and the skewness Γ_H of the brightness angular distribution functions, distorted by multiple effects, are calculated from measurements of the brightness angular distribition function $\mu_H(\theta)$:

$$\Gamma_H = \frac{\int_0^{\pi/2} \mu_H(\theta) \sin\theta \, d\theta}{\int_{\pi/2}^{\pi} \mu_H(\theta) \sin\theta \, d\theta}; \tau_H = 2\pi \int_0^{\pi} \mu_H(\theta) \sin\theta \, d\theta. \quad (3)$$

0235-6880/90/08 753-06 $02.00

2. The optical thickness τ_1 sought is estimated, to a first approximation, using the following formula (proposed by V.A. Smerkalov):

$$\tilde{\tau}_1 = \frac{\ln\left[1 + \tau_H\left(1 - \frac{q}{m_0}\right)\right]}{1.1 + \ln\left[1 + \tau_H \exp\left(-\frac{18\tau_H}{m_0^3}\right)\right]} \quad (4)$$

The characteristic of the q-albedo of the underlying surface appearing in. Eq. (4) is estimated according to the type of ground cover. Then the formula $\tilde{\Gamma}_1 \simeq 1 + \frac{\tau_H}{\tilde{\tau}_1}(\Gamma_H - 1)$ is used to estimate to a first approximation, the skewness of the single-scattering phase function.

3. Three values of the optical thickness $(\tau_1)_i$ are given: $(\tau_1)_1 = 0.7\tilde{\tau}_1$; $(\tau_1)_2 = \tilde{\tau}_1$, and $(\tau_1)_3 = 1.5\tilde{\tau}_1$. For this family of thicknesses $(\tau_1)_I$ a family of distorted optical thicknesses is calculated:

$$(\tau_H)_i = (\tau_1)_i + (\tau_2)_i + (\tau_q)_i,$$

where τ_2 is the optical thickness for multiply scattered light and τ_q is the optical thickness of the scattered light, determined by the effect of an underlying surface with albedo q.

The components τ_2 and τ_q were calculated based on the algorithms of the analytical model of the field of the scattered radiation of the atmosphere[4] $\tau_2 = \tau_1(e^{\nu} - 1]$,

$$\nu = \frac{\tau_1(m_0 + \tau_1^2) + (0.25 m_0 \tau_1)^3\left[1 + \frac{0.3}{\tau_1^2}\sqrt{\Gamma_1 - 1}\right]}{2\tau_1 + 0.43 m_0};$$

$$\tau_g = \frac{2\tau_1 q}{m_0}\left[1 + \frac{\tau_1 q}{\sqrt{\Gamma_1} + 0.2\sqrt{\tau_1}}\right] \times$$

$$\times \exp\left[\frac{(\tau_1 m_0)^2}{4.8 + \tau_1 m_0} - \frac{\sqrt{\Gamma_1} - 1}{m_0}\right].$$

As shown in Ref. 4, the errors in the calculations of the brightness angular distribution function based on this model ($\sigma_\mu = 5.2\%$) are comparable to their measurement errors (~5–7%). This justifies the use of the approximate model[4] for solving problems of this kind without using the laborious methods for solving the transfer equation.

4. The dependence of τ_H on τ_1 is approximated, within the adopted limits of variation of $(\tau_1)_i$ by the polynomial

$$(\tau_H)_i = (\tau_1)_i^2 + b(\tau_2)_i + c.$$

The values of the coefficients a, b, and c are determined by solving the obtained system of three algebraic equations. The value of τ_H obtained earlier from the formula (3) is substituted into the formula

$$\tau_1 = \frac{-b \pm \sqrt{b^2 - 4a(c - \tau_H)}}{2a} \quad (5)$$

and the optical thickness τ_1 is determined (the minus sign in front of the radical is used if $b < 0.006$).

5. More accurate values of τ_q and $\tau_2 = \tau_H - \tau_1 - \tau_q$ are found for the value of τ_1 obtained. The the formula (see Ref. 3)

$$\mu_1(\theta) = \mu_H(\theta) - \frac{\tau_q}{4\pi} - \frac{\tau_2}{4\pi}\frac{W(\theta)}{K}\sqrt{\gamma_H(\theta)} \quad (6)$$

is used to determine the values of the directed atmospheric scattering coefficients $\mu_1(\theta)$. In the formula (6)

$$W(\theta) = \frac{1}{\gamma_H(60^\circ)} + \frac{3\pi(\Gamma_H - 1)\left(\sqrt{\gamma_H(\theta)} - 1\right)}{(\Gamma_H + 1)\left[4 + \gamma_H(\theta)\right]};$$

$$K = \frac{1}{2}\int_0^\pi W(\theta)\sqrt{\gamma_H(\theta)}\,\sin\theta\, d\theta; \quad \gamma_H(\theta) = 4\pi\frac{\mu_H(\theta)}{\tau_H}.$$

6. The following are determined from the values found for τ_1 and $\mu_1(\theta)$:

– the optical thickness of the aerosol $\tau_a = \tau_1 - \tau_R$ (τ_R is the molecular scattering thickness);

– the directed aerosol scattering coefficients $\mu_a(\theta) = \mu_1(\theta) - \mu_R(\theta)$, where $\mu_R(\theta) = 0.7629\frac{\tau_R}{4\pi}(1 + 0.9324\cos^2\theta)$;

– the aerosol scattering phase functions $\gamma_a(\theta) = 4\pi\frac{\mu_a(\theta)}{\tau_a}$; and,

– the backscattering skewness Γ_1 and the aerosol scattering skewness Γ_a.

In solving inverse problems of this form it is usually required that the solution be nonnegative. In this case, based on the physical meaning of the problem, it is usually required that the solution be significantly positive, namely, $(\gamma_a)_{min} \geq \frac{1}{3\pi}$ imposed. To this end $(\gamma_a)_{min}$ is found and the difference $\Delta = \frac{1}{3\pi}(\gamma_a)_{min}$ is determined. .If $\Delta \leq 0$, no corrections are made in the solution, and if $\Delta > 0$, corrections are made. For this $\delta\mu_a = \frac{\tau_a\Delta}{4\pi}$ is determined, and the solution is corrected for it

$$\mu_a^*(\theta) = \frac{\mu_a(\theta) + \delta\mu_a}{1 + \Delta}$$

and more accurate values of the aerosol scattering phase function are obtained:

$$\gamma^*_a(\theta) = 4\pi \frac{\mu^*_a(\theta)}{\tau_a} .$$

More accurate values of the skewnesses Γ^*_1 and Γ^*_a are calculated from the corrected values of $\mu^*_1(\theta) = \mu^*_a(\theta) + \mu_R(\theta)$.

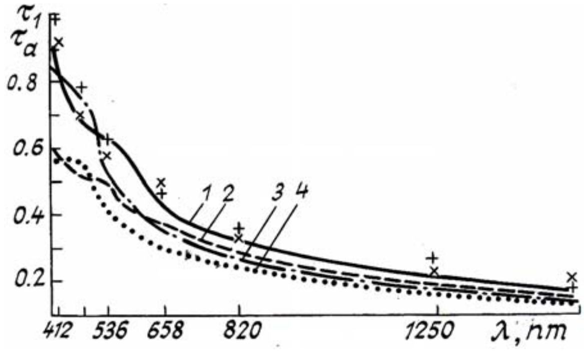

FIG. 1. The spectral behavior of the optical thicknesses $\tau_1(\lambda)$ and $\tau_a(\lambda)$. 1, 2 and + refer to August 1; 3, 4 and × refer to August 10; 1, 3 — τ_1; 2, 4 — τ_a; +, × — τ_{exp}.

L.K. Ushakova at the Academician E.K. Fedorov Institute of Applied Geophysics developed, based on the algorithms presented above, a FORTRAN-IV computer program "Atmos", for calculating the aerosol light-scattering characteristics.

As a result of the calculations the values of the characteristics τ_H, τ, τ_a, Γ_H, Γ_1, Γ_a, $\mu_H(\theta)$, and $\mu_a(\theta)$ as well as the values of the aerosol light-scattering phase functions $\gamma_a(\theta)$, normalized to the condition $\frac{1}{2}\int_0^\pi \gamma_a(\theta)\sin\theta d\theta = 1$, were determined in seven or eight sections of the spectrum for August 1 and 10.

Figure 1 shows the results of the determination of the spectral behavior of the atmospheric τ_1 and aerosol τ_a thicknesses as well as the values of τ_{exp} (crosses).

Figure 2 shows, as an example, the results of the calculation of the angular characteristics of light scattering $\mu_1(\theta)$ and $\mu_a(\theta)$ for λ = 820, 1250, and 1560 nm. Table I gives a sample of the computer print out of the obtained results.

It should be noted that the method of Ref. 3 gives the single-scattering (pure scattering) optical thickness τ_1. This makes it possible to determine a very important characteristic — the optical thickness of the absorption of light by aerosol τ_{aa} by comparing the obtained values of τ_1 with the experimental values $\tau_{exp} = \tau_1 + \tau_{aa} + \tau_{O_3}$. The calculations show that the absorption of light by the aerosol on both August 1 and 10, 1987 was equal to the background level, constituting on the average over the spectrum τ_{aa} = 0.046.

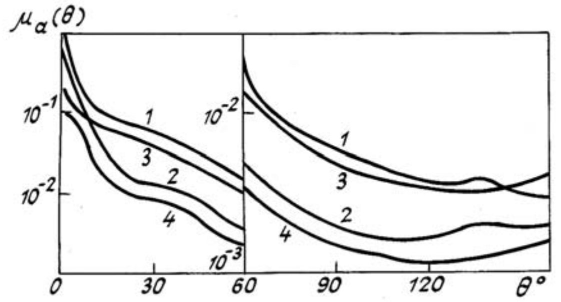

FIG. 2. Directed-scattering coefficient $\mu_a(\theta, \lambda)$; 1, 2) August 1; 3, 4) August 10; 1, 3 — λ = 820 nm; 2 — λ = 1250 nm, 4 — λ = 1560 nm.

TABLE 1. August 10, 1987, Ryl'sk. λ = 0.82 μm; q = 0.4; m_0 = 3.69; τ_H = 0.26; τ_1 = 0.162; τ_a = 0.144; τ_R = 0.019; Γ_H = 2.895; Γ_1 = 3.639; Γ_a = 4.668.

θ^0	$\mu_H(\theta)$	$\mu_1(\theta)$	$\mu_a(\theta)$	$\gamma_a(\theta)$
2.0	0.2540	0.21552	0.21326	18.77795
4.0	0.1460	0.11657	0.11431	10.06521
6.0	0.1130	0.08741	0.08516	7.49845
8.0	0.0975	0.07401	0.07177	6.31907
10.0	0.0882	0.06608	0.06385	5.62196
15.0	0.0740	0.05417	0.05198	4.57671
20.0	0.0664	0.04791	0.04577	4.03007
30.0	0.0564	0.03981	0.03781	3.32955
40.0	0.0421	0.02854	0.02673	2.35367
50.0	0.0316	0.02056	0.01894	1.66762
60.0	0.0235	0.01460	0.01315	1.15800
70.0	0.0175	0.01029	0.00899	0.79157
80.0	0.0140	0.00782	0.00661	0.58236
90.0	0.0118	0.00628	0.00511	0.44964
100.0	0.0110	0.00572	0.00452	0.39759
110.0	0.0104	0.00530	0.00400	0.35241
120.0	0.0100	0.00502	0.00358	0.31507
130.0	0.0098	0.00491	0.00328	0.28894
140.0	0.0103	0.00523	0.00342	0.30105
150.0	0.0115	0.00607	0.00408	0.35900
160.0	0.0125	0.00677	0.00463	0.40771

Representing on a logarithmic scale the values of τ_a presented in Fig. 1 shows that the aerosol thickness on August 1 varied over the spectrum approximately as $\lambda^{-1,78}$ while on August 10 it varied as $\lambda^{-2.04}$. In the case of a Jung's aerosol distribution

$$\frac{dN}{d\ \lg r} = \frac{A}{r^{\nu^*}}$$

it is found that on August 1 the distribution parameter ν^* was equal to 3.78 while on August 10 ν^* = 4.04.

Therefore, on both August 1 and 10 the aerosol consisted of relatively small particles, and the large particles were apparently removed from the atmosphere by the daily rains.

This is also indicated by the fact that the spectral average of the skewness of the aerosol scattering Γ_a was below the usual value: on August 1 $\bar{\Gamma}_a = 5.75$ while on August 10 $\bar{\Gamma}_a = 5.22$.

More detailed information about the microstructure of the aerosol was obtained by inverting the aerosol scattering phase functions in the microphysical characteristics of the aerosol by the spectrozonal method developed at the Institute of Applied Geophysics.[5]

The index of refraction and the size spectrum of the small particles $\left(\rho = \dfrac{2\pi r}{\lambda} \le 3.2\right)$ were determined by inverting the scattering phase functions for scattering angles $\theta \ge 75°$. This part of the phase function is more sensitive to a change in the index of refraction and the size spectrum of the small particles.

The size spectrum and the index of refraction of the large particles was determined by inverting the scattering phase function for angles $\theta \le 90°$.

The results of inversion showed that on August 1 and 10 the atmospheric aerosol had close microphysical characteristics.

Figure 3 shows, as an example, the size spectrum and the indices of refraction of small, medium, and large particles of atmospheric aerosol. They were determined by inverting the aerosol phase functions for $\lambda = 0.82$ and 1.64 μm based on data for August 10, 1987.

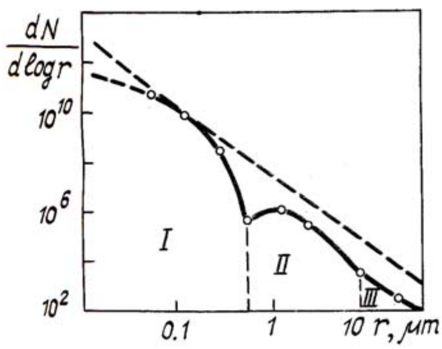

FIG. 3. The aerosol particle size distribution function dN/dlogr; I – $m = 1.53–0.007i$, II – $m = 1.43 – 0.005i$, and III – $m = 1.34$.

Figure 3 also shows (dashed line) the Jung particle-size distribution with the average value $\nu^* = 3$. One can see that the content of average and large particles on August 10 in the aerosol was less than is usually observed.

When the aerosol phase functions, obtained for other pairs of wavelengths (412–824 and 658–1316 nm) were used the microphysical characteristics of the aerosol were found to be comparatively close to the data presented in Fig. 3: the discrepancies in the number density of optically active particles did not exceed 30% and the values of the real part of the index of refraction of the particles differed by not more than 4.5%, while the values of the imaginary part differed by not more than a factor of two. These discrepancies can apparently serve as a lower estimate of the possible errors in the determination of the microphysical characteristics by the methods employed in this work.

In solving inverse problems it is usually conjectured that particles of all sizes have the same index of refraction. The spectrozonal method employed in this work permits determining the index of refraction of the particles separately for different size fractions of the aerosol. For this reason, it was of interest to examine the adequacy of the results obtained and the correspondence of the results with the meteorological conditions at the time of the measurements.

At the time of the aerosol experiment in Ryl'sk it rained every day, and on August 1 and 10 the relative humidity was high. Under these conditions particles characteristically grow by the condensation mechanisms (see, for example, Ref. 2); in these mechanisms, as the particles grow their index of refraction approaches the index of refraction of water ($n = 1.33$).

Changes of precisely this type in the index of refraction of the particles were obtained by solving the inverse problem. The real part of the index of refraction of particles of the first mode (submicron particles, $r < 0.85$ μm) has the value $n_{\rm I} = 1.53$; for the second mode $n_{\rm II} = 1.43$; and, spectrozonal method is used to solve inverse problems it is also assumed that on the section of the spectrum $\lambda_1 - \lambda_2$, where $\lambda_2 = 2\lambda_1$, the complex index of refraction of particles does not change as the wavelength changes.

On the whole the results obtained make it possible to draw the following conclusions:

1. Using the engineering methods, developed at the Institute of Applied Geophysics, for fast processing of measurements of the daytime sky brightness angular distribution function it is possible to reconstruct the altitude-averaged optical and microstructural characteristics of the atmospheric aerosol. Combining the photometric apparatus with a computer makes it possible to obtain the optical and microstructural characteristics of the aerosol on-line while measuring the brightness angular distribution functions.

2. The method for inverting the brightness angular distribution functions[3] was tested under the following assumptions:

– the underlying surface is a Lambertian surface;

– the albedo of the underlying surface is known accurately enough; and,

– the condition that the atmosphere is azimuthally uniform is satisfied, etc.

Under the real conditions of the experiment (especially, owing to the proximity of a city) it is hardly possible to count on these conditions being satisfied. It is obvious that there have occurred cases when the conditions of inversion are disturbed; in the general case, such disturbances can lead to incorrect (unphysical) solutions. The results of the processing showed that the method of Ref. 3 is very stable with respect to possible disturbances of the conditions of inversion.

3. The errors occurring in the reconstruction of the optical thickness of the atmosphere τ_1 owing to the approximate character of the algorithms of Refs. 3 and 4 rapidly diminish as the optical thickness τ_1 diminishes (in the red and IR sections of the spectrum, for high-altitude measurements). This favorably distinguishes the method of inversion suggested in Ref. 3 from the "Bouguer" methods for determining the optical thickness τ_{exp}. The errors in determining τ_{exp} by Bouguer's method are connected with the errors in measuring the transmission of the atmosphere σ_p by the relation $\sigma\tau_{exp} = \sigma_p/\tau_{exp}$. As τ_{exp} decreases the error in determining the optical thickness $\delta\tau_{exp}$ increases rapidly. Thus, choosing for the field conditions $\sigma_p = 2\%$, we fine that as τ_B decreases from 0.1 to 0.05 the error is determining τ_{exp} increases from 20 to 40%. The error in determining τ_1 by the method of inversion of the brightness angular distribution function, connected with the approximate character of the algorithms of Refs. 3 and 4, under these conditions decreases to several percent. In addition, unlike the "long-term" Bouguer method, the method of inversion used in Ref. 3 is a "short-term" method, it does not involve prolonged measurements, and it can be used when the atmosphere is optically unstable. This is why we chose this method to analyze the measurements.

4. In connection with the uncertainty in the initial conditions of inversion (the nonorthotropic nature of the underlying surface is neglected, the albedo of the underlying surface is not given accurately, the scattering phase; functions are not measured accurately, etc.), it is pointless to use the computationally non complicated methods of the type described in Refs. 1 and 6 for solving inverse problems of this type.

It is our duty to thank N.A. Zaitseva for rendering assistance in organizing the measurements and N.V. Smolenskaya, for assistance in performing the measurements.

REFERENCES

1. V.S. Antyufeev, A.I. Ivanov, G. Sh. Livshits, and G.A. Mikhailov, Izv. Akad. Nauk SSSR, Ser. FAO **16**, No. 2, 146 (1980).
2. A.S. Laktionov, *Equilibrium Heterogeneous Condensation* [in Russian] (Gidrometeoizdat, Leningrad, 1988).
3. V.A. Smerkalov, Tr. IPG, No. 68, 109 (1987).
4. V.A. Smerkalov, *Analytical Model of the Field of Radiation Scattered by the Free .Atmosphere of the Earth* [in Russian], Moscow, VINITI, No. :2885-B88, April 14 (1988).
5. V.A. Smerkalov, Izv. Akad. Nauk SSSR, Ser. FAO 20, No. 10, 876 (1984).
6. A.N. Tikhonov and V.A. Arsenin, *Methods for Solving Ill-Posed Problems* [in Russian] (Nauka, Moscow, 1974).