

SPATIAL STRUCTURE OF THE INTRACAVITY LIGHT FIELD OF A LASER WITH AN EXTERNAL REFLECTOR

A.P. Shelekhov

*Institute of Atmospheric Optics,
Siberian Branch of the Academy of Sciences of the USSR, Tomsk
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The problem of an intracavity lasing of the laser with an external reflector under conditions of spatial mismatch between the incident and reflected waves is discussed. A system of equations is derived for the coefficients of series expansion of the light field in the eigenfunctions of an empty resonator in the case of a spatially homogeneous active medium. The derivation was made within the framework of semiclassical theory of quantum generators. A comparison of techniques for recording the optical fields using heterodyne detection and coherent laser detection. It is shown that these techniques differ only by the shapes of the transmission functions of the detectors, which in the latter case is described by the eigenfunctions of the empty resonator.

INTRODUCTION

The detection of very weak signals by means of including the atmosphere in the channel of the feedback optical loop of a laser is one of the most effective methods of signal detection in the IR together with the heterodyne detection. In the last few years there have appeared works¹⁻³ on theoretical and experimental studies of the possibilities of the method based on the laser detection indicating that the problem is of definite interest. In my opinion, to put this method into practice it is necessary to solve the problem on response of a laser detector to the optical radiation with both random and deterministic spatial structure. The problem may be formulated as the determination of the spatial structure of the intracavity light field of a laser with an external reflector, which accounts for the spatial mismatch between the incident and reflected waves in laser detecting. The solution of this problem enables us to expand the range of the problems which may be studied by the methods based on the coherent recording of the optical field.^{4,5}

In this paper the spatial structure of the intracavity light field of the laser with an external reflector is assumed to be described by the series expansion of the light field in terms of the eigenfunctions of the empty resonator. In principle, any object, for example, a mirror, a topographic object, a ground or sea surface, particles in the atmosphere, and so on, may serve as an external reflector. The coefficients of series expansion of the light field in terms of the eigenfunctions of empty resonator satisfy the system of equations, derived for the case of a spatially homogeneous active medium in the framework of a semiclassical theory of quantum generators. The formula describing the intracavity field structure of a laser may be used for calculation of the experimentally measured quantities. The results of calculation of coherent component of the total energy flux in laser detecting of

the radiation reflected from the surface which is placed in the medium with large-scaled random inhomogeneities and some consequences from the obtained relation are presented.

THE SYSTEM OF EQUATIONS DESCRIBING THE INTRACAVITY GENERATION OF LASER WITH AN EXTERNAL REFLECTOR UNDER CONDITIONS OF SPATIAL MISMATCH BETWEEN THE INCIDENT AND REFLECTED WAVES

The laser with an external reflector is shown in Fig. 1. We assume that the time dependence of the field components is described by the harmonical oscillation with the slowly-varying amplitude

$$E(\rho, z; t) = u(\rho, z; t)e^{-i\omega t},$$

where $\rho = \{x, y\}$ and $x, y,$ and z are spatial coordinates. Here the z axis coincides with the optical axis of the laser. We represent the complex amplitudes of the light field inside and outside of laser cavity in the form of counter propagated waves

$$u_I(\rho, z; t) = u^+(\rho, z; t) + u^-(\rho, z; t),$$

$$u_{II}(\mathbf{r}, z; t) = u_r^+(\mathbf{r}, z; t) + u_i^-(\mathbf{r}, z; t),$$

where $u^+(\rho, z; t)$ and $u^-(\rho, z; t)$ are the intracavity light fields of the optical waves propagating along the z axis in positive and negative directions, respectively, $u_r^+(\rho, z; t)$ is the light field of the optical wave reflected from the external object, and $u_i^-(\rho, z; t)$ is the light field of the incident wave. The directions of wave propagation are indicated with arrows in Fig. 1.

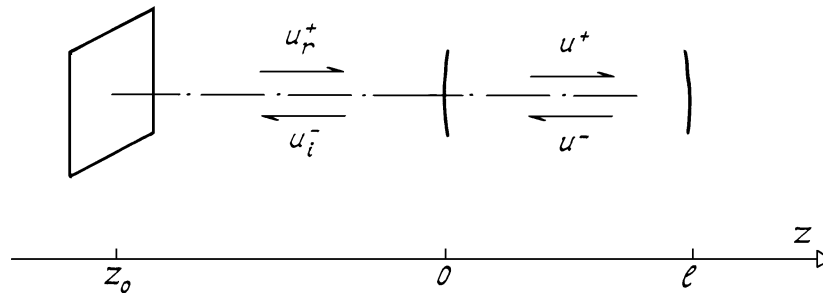


FIG. 1.

The shortened equation describing the intracavity generation of the laser for the slowly-varying amplitude has the form

$$\left\{ \frac{\partial}{\partial t} - \frac{1}{2} (\sigma_+(N) - \sigma_-) + \frac{1}{2i\omega} (\omega^2 + c^2\Delta) \right\} u_l(\rho, z; t) = 0, \quad (1)$$

where σ_- are the linear losses in active medium, $\sigma_+(N)$ is an amplification coefficient of the active medium, which depends on the relative population inversion of the working transition N , Δ is the Laplacian operator.⁷

The boundary conditions at the surfaces of the laser mirrors may be written in the form:

$$\tilde{u}_i^-(\mathbf{r}, 0; t) = \tau_1 \tilde{u}^-(\mathbf{r}, 0; t); \quad (2)$$

$$\tilde{u}^+(\mathbf{r}, 0; t) = r_1 \tilde{u}^-(\mathbf{r}, 0; t) + \tilde{u}_r^+(\mathbf{r}, 0; t), \quad (3)$$

$$\tilde{u}^-(\mathbf{r}, l; t) = r_2 \tilde{u}^+(\mathbf{r}, l; t), \quad (4)$$

where τ_1 is the transmittance of the left mirror, r_1 and r_2 are the reflectances of the left and right mirrors, and l is the cavity length. The tilde (\sim) in formulas (2), (3) and (4) implies that the values of the slowly-varying amplitudes correspond to that at the surfaces of the laser mirrors. Boundary condition (2) accounts only for the first-order interference of light reflected from the atmospheric object and from the left mirror.⁶

We set the relation between the complex amplitudes of the incident and reflected waves by means of the operator \hat{T} , which describes reflection and scattering of the optical wave by the atmospheric object

$$\tilde{u}_r^+(\rho, 0; t) = \tau_1 \hat{T} \tilde{u}_i^-(\rho, 0; t). \quad (5)$$

Relation (5) is applicable to some problems in atmospheric optics. For example, calculational methods for optical waves reflected from the rough surfaces of different types (sea waves, relief of dry land, paper surface, mat glasses, and so on) have been discussed in Ref. 7, the results of investigation of reflection of the optical waves from the specular and diffusely-reflecting surfaces and from the PC mirror placed in the large-scaled randomly inhomogeneous medium have been presented in Refs. 8, 9, and 10; the theory of scattering of optical waves by discrete scatterers has been presented in Refs. 7, 11, and 12. Since the scattering of optical radiation by the external atmospheric object is rather complex phenomenon, the field distribution of incident $\tilde{u}_i^-(\rho, 0; t)$ and reflected $\tilde{u}_r^+(\rho, 0; t)$ waves are different and therefore are spatially mismatched.

It is expedient to solve equation (1) simultaneously with Eq. (5) with boundary conditions (2), (3) and (4) with the use of series expansion of light field in terms of the eigenfunctions of the empty resonator with ideally conducting walls.^{13,14} The equation which defines the eigenfunctions of empty resonator has the form

$$\{\Delta + k_{mk}^2\} u_{mk}(\rho, z) = 0, \quad (6)$$

where $k_{mk} = \omega_{mk}/c$ and $\omega_{mk} = \omega'_{mk} + i\omega''_{mk}$ is the complex natural frequency of an empty resonator. The subscripts m and k denote the number of half-wave oscillations along the x axis and y axis. The subscript corresponding to the number of half-wave oscillations along the z axis is assumed to be fixed and therefore is omitted in notation of ω_{mk} and $u_{mk}(\rho, z)$. We also represent the solution of equation (6) in the form of a sum of counter propagated waves.

$$u_{mk}(\rho, z) = u_{mk}^+(\rho, z) e^{ik_{mk}z} - u_{mk}^-(\rho, z) e^{-ik_{mk}z},$$

where $u_{mk}^\pm(\rho, z)$ are the slowly-varying amplitudes which, as is well known, satisfy the parabolic equation^{13,14}

$$\left\{ \pm 2ik_{mk} \frac{\partial}{\partial z} + \Delta_\perp \right\} u_{mk}^\pm(\rho, z) = 0, \quad (7)$$

where Δ_\perp is the two-dimensional Laplacian operator.⁷

We write the series expansion of the intracavity light field of optical waves in terms of the eigenfunctions of the empty resonator in the form

$$u^\pm(\rho, z; t) = \sum_{m, k} e^{\pm ik_{mk}z} \beta_{mk}^\pm(t, z) u_{mk}^\pm(\rho, z), \quad (8)$$

where $\beta_{mk}^\pm(t, z)$ are the coefficients of the expansion. Based on the method of reducing the boundary-value problem to the problem of a resonator with distributed losses,¹⁵ we derive the system of ordinary nonlinear differential equation for the coefficients of the series expansion. Direct substitution of relation (8) into Eq. (1), with boundary conditions (3) and (4) leads to the boundary-value problem to be solved for $\beta_{mk}^\pm(t, z)$ and representing a system of nonlinear partial differential equations with the conditions at $z = 0$ and $z = l$

$$\beta_{mk}^+(t, 0) = r_1 \beta_{mk}^-(t, 0) + s_{mk}^+(t, 0), \quad (9)$$

$$\beta_{mk}^-(t, l) = r_2 \beta_{mk}^+(t, l), \quad (10)$$

where

$$s_{mk}^+(\tau, 0) = \int \tilde{u}_{mk}^+(\rho, 0) \tilde{u}_r^+(\rho, 0; \tau) d\rho. \tag{11}$$

Boundary conditions (9) and (10) follow from orthogonality of the eigenfunctions of the empty resonator. We use relations (9) and (10) for determining the derivative $\partial\beta_{mk}^\pm(t, z)/dz$. We approximate $\beta_{mk}^\pm(t, z)$ with the help of the smooth function of the variable z assuming that the mirror transmittance and the coupling constant between the laser and the external reflector are small values. The expansion in a power series of z/l has the form

$$\beta_{mk}^\pm(t, z) = \pm\beta_{mk}^\pm(t) + \beta_{mk}^{(1)\pm}(t) + \frac{z}{l}\beta_{mk}^{(2)\pm}(t) + \dots, \tag{12}$$

where $\beta_{mk}^\pm(t)$ is the main term of the expansion and $\beta_{mk}^{(1)\pm}(t)$ and $\beta_{mk}^{(2)\pm}(t)$ are the corrections in the first approximation in terms of the small parameters, i.e., in the mirror transmittance and the coupling constants between the laser and the external reflector. It is obvious from Eq. (12) that the derivative

$$\frac{\partial\beta_{mk}^\pm(t, z)}{\partial z} \cong \frac{1}{l}\beta_{mk}^{(2)\pm}(t).$$

After substituting expansion (12) into boundary conditions (9) and (10) followed by combining the terms of the same order, we obtain the following expression for the derivative:

$$\frac{\partial\beta_{mk}^\pm(t, z)}{\partial z} = \frac{1}{l}(1+r)\beta_{mk}^\pm(t) - \frac{1}{2l}s_{mk}^\pm(t), \tag{13}$$

where

$$r = \frac{r_1 + r_2}{2}, s_{mk}^\pm(t) = s_{mk}^\pm(t, 0).$$

Differentiating the eigenfunctions of the empty resonator with respect to the spatial coordinates defined by Eqs. (6), (7), and (13) enables one to transfer from solving Eqs. (1) – (5) to solving the system of ordinary nonlinear differential equations for the expansion coefficients $\beta_{mk}^\pm(t, z) \cong \pm\beta_{mk}^\pm(t)$. The system for $\beta_{mk}^\pm(t)$ has the simplest form for the model of the medium in which the amplification coefficient is independent of the spatial coordinates. This model is basic in the theory of single-mode generation of lasers.¹⁴ In this case we have the following system of the ordinary nonlinear differential equations:

$$\frac{\partial\beta_{mk}^\pm(t)}{\partial t} = \frac{1}{2}\left\{\sigma_+(N) - \sigma_-^{mk} + 2i\Delta\omega'_{mk}\right\} * \beta_{mk}^\pm(t) + \frac{c}{2l}s_{mk}^\pm(t), \tag{14}$$

where $\sigma_-^{mk} = \sigma_- + \frac{2l}{c}(1+r) + 2\omega''_{mk}$ are the total losses and $\Delta\omega'_{mk} = \omega - \omega'_{mk}$.

The system of equations (14) enables one to study the dynamics of coefficients in the expansion of the intracavity light field of the laser with the external reflectors of different types (specular, diffuse or alternative surface, the

system of scattering particles, and so on) and the properties of the medium on the path located between the laser and the external reflector. The modes of the empty resonator are taken as base functions, therefore it is expedient to take the spatial matching of the fields $\tilde{u}_r^+(\rho, 0; t)$ and $\tilde{u}_{mk}^+(\rho, 0)$ into account. It is obvious from formula (11) that the value $s_{mk}^\pm(t)$ is the coefficient of the expansion of the field of the incident wave $\tilde{u}_r^+(\rho, 0; t)$ in the eigenfunctions of the empty resonator $\tilde{u}_{mk}^+(\rho, 0)$. Thus the value $s_{mk}^\pm(t)$ describes the spatial matching between the field of the incident wave $\tilde{u}_r^+(\rho, 0; t)$ and the mode of the empty resonator $\tilde{u}_{mk}^+(\rho, 0)$. In general, the field $\tilde{u}_r^+(\rho, 0; t)$ has the arbitrary spatial structure. Therefore the only part of the field influencing the dynamics of the coefficients $\beta_{mk}^\pm(t)$ is that which spatially matches with the mode of the resonator $\tilde{u}_{mk}^+(\rho, 0)$.

We consider the situation when the optical wave is reflected by the surface placed in the large-scale randomly-inhomogeneous nonabsorbing medium. It follows from Ref. 8 that the relation between the complex amplitudes of the incident wave and the reflected one may be written in the form

$$\tilde{u}_r^+(\rho, 0; t) = -\tau_1 \sum_{mk} \beta_{mk} \left(t - 2\frac{|z_0|}{c}\right) \times e^{2ik|z_0|} \tilde{u}_{mk}^+(\rho, 0), \tag{15}$$

$$\tilde{u}_{r,mk}^+(\rho, 0) = \int \exp\left(\frac{ik_1}{2R_1}\rho^2\right) G(\rho, \rho_1; |z_0|) K_0(\rho_1) \times G(\rho_1, \rho_2; |z_0|) \exp\left(\frac{ik_1}{2R_1}\rho_2^2\right) \tilde{u}_{mk}^+(\rho_2, 0) d\rho_1 d\rho_2,$$

where z_0 is the coordinate of the center of the reflecting surface, R_1 is the curvature radius of the left mirror, $k_1 = kn_1$, n_1 is the refractive index of the substrate of the left mirror, $K(\rho)$ is the reflectance of the surface, $G(\rho, \rho_1, z)$ is the Green's function for the large-scale randomly-inhomogeneous medium,⁷⁻¹⁰ and $2k|z_0|$ is the regular phase run-on for the plane wave.

After substituting Eq. (15) into Eq. (14) and transforming it we obtain the system of stochastic nonlinear differential equations with the time delay of the argument which describes the generation of the laser with the external reflecting plane placed in the randomly-inhomogeneous nonabsorbing medium

$$\frac{\partial\beta_{mk}^\pm(t)}{\partial t} = \frac{1}{2}\left\{\sigma_+(N) - \sigma_-^{mk} + 2i\Delta\omega'_{mk}\right\} \beta_{mk}^\pm(t) + \frac{c}{2l} e^{2ik|z_0|} \sum_{m'k'} \gamma_{m'k'}^{mk} \beta_{m'k'}^\pm \left(t - 2\frac{|z_0|}{c}\right), \tag{16}$$

where

$$\gamma_{mk}^{m'k'} = -\tau_1^2 \int \tilde{u}_{mk}^{+*}(\rho, 0) \tilde{u}_{r, m'k'}^+(\rho, 0) d\rho$$

are the coupling constants between the laser and its external reflector for different modes representing the random complex values.

It follows from Eq. (15) that the field $\tilde{u}_r^+(\rho, 0; t)$ is the superposition of the waves $\tilde{u}_{r, m'k'}^+(\rho, 0)$ reflected from the surface and, in addition, the initial distribution of the incident wave field is specified in terms of the eigenfunctions of the empty resonator. The values $\gamma_{mk}^{m'k'}$ are all proportional to the coefficients of the expansion of the field of optical wave $\tilde{u}_{r, m'k'}^+(\rho, 0)$ in the eigenfunctions of the empty resonator $\tilde{u}_{mk}^+(\rho, 0)$. Thus, the coupling constants between the laser and external reflector $\gamma_{mk}^{m'k'}$ describe the spatial matching between the field of the reflected waves $\tilde{u}_{r, m'k'}^+(\rho, 0)$ and the empty resonator mode $\tilde{u}_{mk}^+(\rho, 0)$. The spatial mismatching between the fields $\tilde{u}_{r, m'k'}^+(\rho, 0)$ and $\tilde{u}_{mk}^+(\rho, 0)$ takes place due to such phenomena as diffraction of the optical wave from the laser aperture and the reflecting surface and scattering of the optical wave from the large-scale inhomogeneities of the medium and from the rough surface.

We will consider the regime of a single transverse mode generation in the cases of the complete spatial matching and spatial mismatching between the fields $\tilde{u}_{r, m'k'}^+(\rho, 0)$ and $\tilde{u}_{mk}^+(\rho, 0)$. In the former case it follows from the Eq. (15) that

$$\tilde{u}_{r, m'k'}^+(\rho, 0) = -K_0 \tilde{u}_{mk}^+(\rho, 0),$$

where $K(\rho) = -K_0$ is the reflectance of an ideal mirror. The coupling constants between the laser and external reflector for different modes are the real values and equal to

$$\gamma_{mk}^{m'k'} = \tau_1^2 K_0 \delta_{mm'} \delta_{kk'}$$

Thus, the system of equations (16) takes the form

$$\frac{\partial \beta_{mk}(t)}{\partial t} = \frac{1}{2} \left\{ \sigma_+(N) - \sigma_-^{mk} + 2i\Delta\omega'_{mk} \right\} \times \beta_{mk}(t) + \frac{c}{2l} \tau_1^2 K_0 e^{2ik|z_0|} \beta_{mk} \left(t - 2 \frac{|z_0|}{c} \right). \tag{17}$$

The steady-state solution of Eq. (17) is not trivial only for the single mode, for which the conditions are satisfied

$$\sigma_+(N) - \sigma_-^{\bar{m}\bar{k}} = -\frac{2}{7} \tau_1^2 K_0 \cos 2k|z_0|; \tag{18}$$

$$\Delta\omega'_{\bar{m}\bar{k}} = -\frac{2}{7} \tau_1^2 K_0 \sin 2k|z_0|,$$

where \bar{m} and \bar{k} are the subscripts corresponding to the high-quality transverse mode. The fact that only the single mode is excited under conditions of complete spatial matching between the fields $\tilde{u}_{r, m'k'}^+(\rho, 0)$ and $\tilde{u}_{mk}^+(\rho, 0)$ in

the case of the steady-state generation of the laser is because of the independency of $\sigma_+(N)$ on spatial coordinates. We note that Eq. (17) for high-quality transverse mode after transformation

$$\beta_{\bar{m}\bar{k}}(t) = \bar{\beta}_{\bar{m}\bar{k}}(t) \exp(i\Delta\omega'_{\bar{m}\bar{k}} t)$$

means different choice of the carrier frequency of the field coincide with the equation for the model⁶ without absorption, which is well studied now.

Now we consider the regime of the single-mode generation in the case of spatially mismatched fields $\tilde{u}_{r, m'k'}^+(\rho, 0)$ and $\tilde{u}_{mk}^+(\rho, 0)$. The intensity of the field of high-quality transverse mode is by several orders of magnitude greater than that of the rest of low-quality modes, therefore we may assume that relative population inversion is determined by the value $|\beta_{\bar{m}\bar{k}}|^2$, while the main contribution in the sum

$$\sum_{m', k'} \gamma_{mk}^{m'k'} \beta_{m'k'} \left(t - 2 \frac{|z_0|}{c} \right)$$

comes from the term $\gamma_{\bar{m}\bar{k}}^{\bar{m}\bar{k}} \beta_{\bar{m}\bar{k}} \left(t - 2 \frac{|z_0|}{c} \right)$. For such a regime of generation we have the following system of equations:

$$\frac{\partial \beta_{\bar{m}\bar{k}}(t)}{\partial t} = \frac{1}{2} \left\{ \sigma_+(N) - \sigma_-^{\bar{m}\bar{k}} + 2i\Delta\omega'_{\bar{m}\bar{k}} \right\} \beta_{\bar{m}\bar{k}}(t) + \frac{c}{2l} e^{2i\bar{k}|z_0|} \gamma_{\bar{m}\bar{k}}^{\bar{m}\bar{k}} \beta_{\bar{m}\bar{k}} \left(t - 2 \frac{|z_0|}{c} \right). \tag{19}$$

It is obvious from intercomparison of Eqs. (17) and (19) that for $m = \bar{m}$ and $k = \bar{k}$ they differ only by a factor of magnitude at $\beta_{\bar{m}\bar{k}} \left(t - 2 \frac{|z_0|}{c} \right)$. Therefore, from the physical viewpoint the high-quality mode generation under conditions of the spatial mismatching is equivalent to the generation of the same mode under conditions of entire spatial matching between the fields $\tilde{u}_{r, m'k'}^+(\rho, 0)$ and $\tilde{u}_{mk}^+(\rho, 0)$ but with another reflectance and equivalent phase run-on which is equal to $2k|z_0| + \arg \gamma_{\bar{m}\bar{k}}^{\bar{m}\bar{k}}$. When $m \neq \bar{m}$ and $k \neq \bar{k}$ the term with time delay of the argument in Eq. (19) describes the external source. For stationary generation the external source results in the underthreshold excitation of low-quality modes, while for complete matching between the fields $\tilde{u}_{r, m'k'}^+(\rho, 0)$ and $\tilde{u}_{mk}^+(\rho, 0)$ it does not occur. Thus, the spatial mismatching between the fields $\tilde{u}_{r, m'k'}^+(\rho, 0)$ and $\tilde{u}_{mk}^+(\rho, 0)$ is the main reason for underthreshold excitation of low-quality modes under conditions of the stationary laser generation.

THE HETERODYNE LASER DETECTION OF THE OPTICAL FIELDS

The system of equations (16) may be investigated by both numerical methods and the methods of the theory of

stochastic differential equations.^{7,16} In this paper we will investigate only the system of equations (16) with the help of the perturbation theory applicable to the problem of the heterodyne laser detection of the optical fields. The values $\gamma_{mk}^{m'k'}$ are the small parameter of the problem. We set

$$\beta_{mk}(t) = \beta_{mk,0}(t) + \beta_{mk,1}(t) + \dots, \quad (20)$$

$$\omega = \omega_0 + \omega_1 + \dots, \quad (21)$$

where $\beta_{mk,0}(t)$ and ω_0 are the terms of zeroth order in the expansion of $\beta_{mk}(t)$ and ω in small parameters $\gamma_{mk}^{m'k'}$; $\beta_{mk,1}(t)$ and ω_1 are the terms of the first order of the perturbation theory.

The value of the total flux of energy

$$p^\pm = \int |u^\pm(\rho, z; t)|^2 d\rho \quad (22)$$

is of interest from the physical viewpoint under conditions of heterodyne detection. The coherent component of the total flux of energy is its information-bearing part and in the first order of the perturbation theory in the case of quasistationary generation is expressed in the following form:

$$p_c^\pm = \beta_{\bar{m}\bar{k},0}^* \beta_{\bar{m}\bar{k},1} + \beta_{\bar{m}\bar{k},0} \beta_{\bar{m}\bar{k},1}^*. \quad (23)$$

It is convenient to introduce the complex coherent components of the total flux of energy for the theoretical considerations in the form

$$\hat{p}_c^\pm = \beta_{\bar{m}\bar{k},0}^* \beta_{mk,1}.$$

Applying the standard methods of the perturbation theory to Eq. (16) we obtain that for the amplification coefficient of the active medium, which depends solely on $|\beta_{mk}|^2$, the expression for the complex coherent component of the total energy flux takes the form

$$\hat{p}_c^\pm = \frac{c}{2l} \frac{\tau_1 e^{2ik_0 z_0}}{\partial \sigma_+(N_0)/\partial |\beta_{mk,0}|^2} *$$

$$* \int \tilde{u}_{\bar{m}\bar{k}}^{\pm*}(\rho, 0) \tilde{u}_{r,\bar{m}\bar{k}}^{\pm}(\rho, 0) d\rho, \quad (24)$$

where N_0 is the relative population inversion of an isolated laser and $k_0 = \omega_0/c$.

We will compare the heterodyne laser detection with standard heterodyne detection of optical fields with deterministic or randomly spatial field distribution. It is obvious from formula (24) that the two methods differ in the shape of the transmission function of the detector. In the case of traditional heterodyne detection the transmission function is determined by the characteristics of the input aperture and by the field of the reference wave.^{4,5} The eigenfunctions of the empty resonator represent the transmission function in the heterodyne laser detection. The shape of the eigenfunctions of the empty resonator is determined by the parameters of the resonator and by the

values of the subscripts of high-quality mode \bar{m} and \bar{k} . Thus, one can significantly change the properties of the laser detector by varying the parameters of the resonator

and the values of the subscripts of the high-quality mode. For example, the eigenfunctions of the empty steady resonator with infinite mirrors have the form^{13,14}

$$\begin{aligned} \tilde{u}_{\bar{m}\bar{k}}^{\pm}(\mathbf{r}, 0) &= \tilde{u}_{\bar{m}}^{\pm}(x) \tilde{u}_{\bar{k}}^{\pm}(y) \\ \tilde{u}_{\bar{m}}^{\pm}(t) &= \frac{1}{\sqrt{a2^m m! \sqrt{\pi}}} H_m(t/a) e^{-t^2/2a^2}, \end{aligned} \quad (25)$$

where $H_m(t/a)$ are the Hermite polynomials, a is the distribution parameter which is expressed in terms of the beam matrix elements as follows:

$$a^4 = \frac{1}{k_{\bar{m}\bar{k}}^2} \left(-\frac{AB}{CD} \right). \quad (26)$$

The beam matrix elements have the form

$$A = 1 - \frac{l}{R_1}, \quad B = l,$$

$$C = -\frac{1}{R_1} - \frac{1}{R_2} + \frac{l}{R_1 R_2}, \quad \text{and} \quad D = 1 - \frac{l}{R_1}, \quad (27)$$

where R_2 is the curvature radius of the right mirror. It follows from Eq. (25), (26) and (27) that the properties of the laser detector with the steady resonator and infinite end-mirrors for the fixed values of subscripts \bar{m} and \bar{k} are determined only by the geometric dimensions of the resonator, by the curvature radii of the right and left mirrors R_1 and R_2 , and by the length of the resonator l .

For the set of subscripts of the lowest order $\bar{m} = \bar{k} = 0$ the transmission function of the detector obeys the Gaussian distribution. The radiation pattern of such a detector has one lobe, the maximum of which is oriented along the optical axis of the resonator. We will estimate the value of the field-of-view angle of the detector. To this end, we represent the field $\tilde{u}_{r,00}^+(\rho, 0)$ in the form of the plane wave with unit amplitude which is incident on the laser detector at an angle α . Then the field $\tilde{u}_{r,00}^+(\rho, 0)$ at the surface of the left mirror may be written in the form

$$\tilde{u}_{r,00}^+(\rho, 0) = \exp \left\{ \frac{ik_1}{2R_1} \rho^2 + ik\alpha \mathbf{q}\rho \right\}, \quad (28)$$

where \mathbf{q} is the unit vector. We will define the field-of-view angle as the value of the angle α_0 , at which the square modulus of complex coherence of the component of the total energy flux in the case of detection of the optical radiation in the form of Eq. (28) decays by a factor of e^{-1} with respect to its maximum. Depending on the curvature radius, two regimes of operation of the laser heterodyne detection are identified first. One corresponds to the case of laser detector with a flat left mirror as $R_1 \rightarrow \infty$. The field-of-view angle of such a laser detector is determined by its diffraction resolution

$$\alpha_0 = 1/ka. \quad (29)$$

In the second regime of laser detection in the case of finite curvature radius of the left mirror R_1 the field-of-view angle equals to

$$\alpha_0 = \frac{a}{R_1} n_1. \quad (30)$$

The result may be interpreted as follows. The field $\tilde{u}_{r,00}^+(\rho, 0)$ may be generally represented in the form of expansion in the plane waves incident on the laser detector at different angles. It is wellknown⁵ that the superposition of the plane waves coming from the solid angle, which is determined by the diffraction resolution of the detector, is named a single spatial mode. It follows from Eq. (24) that in the first regime of operation the heterodyne laser detector records one spatial mode of incident field. Such a detection regime is the single-mode. In the second regime of operation, as it is obvious from formula (30), the detector records the radiation contained in several spatial modes of incident field. Such an operation should be named the multi-mode. We note that both single-mode and multi-mode regimes of operation in laser detecting have their own analog in traditional heterodyne detecting.⁵

Thus, the response of the laser detector to the weak optical radiation reflected by the surface, which is placed in the large-scale randomly inhomogeneous medium, is described by formula (24), while the response to the signal with arbitrary amplitude is described by system (16). To describe the response of a laser detector to the optical radiation scattered by an atmospheric object, it is necessary to start from system (14).

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