INVESTIGATION OF THE EFFECT OF THE OPTICAL CHARACTERISTICS AND ARRANGEMENT OF CLOUDS ON THE SPATIAL AND ANGULAR DISTRIBUTION OF THE LIGHT FIELD

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An algorithm for studying light fields in a cloudy atmosphere is constructed based on the small-angle modification of the method of spherical harmonics. The effect of the optical characteristics and the position of the cloud layer on the spatial and angular distribution of the light field is investigated. It is shown, based on a criterion introduced for evaluating the quality of image transmission, that as the distance between the object and the scattering layer increases, in spite of the fact that the optical transfer function of the medium degrades monotonically, the quality of image transmission can decrease or increase monotonically and can be of an extremal character.

Investigations of cloud structures performed by optical methods essentially reduce to investigation of light fields in the atmosphere in the presence of clouds. Since cloud formations are spatially limited and have the form of optically dense scattering layers, effects that are inherent to radiation transfer in such media must be taken into account when investigating clouds: the multiple character of the scattering and the effect on radiation transfer of not only the optical parameters of the clouds but also the position of the cloud layer on the path.

The linearity and invariance of optical systems makes it possible to use a linear-systems approach to describe radiation transfer in a scattering medium and to treat the medium as a separate element of the entire observation system, characterized by an optical transfer function (OTF), which is the Fourier transform of the scattering function (FSF) of the medium.

The sharply anisotropic character of the scattering in a cloudy atmosphere makes the small-angle approximation most useful for investigation of light fields in such an atmosphere (for solving the radiation transfer equations (RTE)). The solution[1] that is most widely employed, because the integral term of the RTE transforms into an integral of the convolution type, is valid only for extremely anisotropic scattering phase functions and very small viewing angles; this reduces sharply the applicability of the solution under atmospheric conditions.

It is well known that in the theory of vision there are two possible approaches to determining the FSF.[2] According to the first approach the FSF is defined as the brightness produced in the direction of the optical axis by an elementary Lambertian radiator, and it is a function of the position of this source in the object plane. According to the second approach the FSF is the angular distribution of the brightness at the point of observation from an elementary Lambertian radiator located in the object plane on the optical axis. The first approach to this problem is exact while the second approach is approximate, but on the basis of the small-angle approximation, employed in this paper (i.e., for angular dimensions of the object less than 5%), the second approach gives an error of less than 10%.[2] We note that within the limits of applicability of the small-angle approximation the light field produced by an elementary Lambertian radiator is equivalent to the light field produced by a point isotropic source. Based on the foregoing discussion, we shall choose for the FSF the angular distribution of the brightness of an isotropic point source.

In this paper a small-angle modification of the method of spherical harmonics (SH) is employed to solve the RTE.[3] The FSF is represented in the form of a series in Legendre polynomials

$$L(\hat{l},\ r) = \frac{1}{4\pi}\sum_{k=0}^{\infty}(2k + 1)C_k(r)P_k(\mu), \tag{1}$$

where $L(\hat{l}, r)$ is the brightness of the light field at the point r in the direction $\hat{l}$ from an isotropic point source of light (the symbol $\wedge$ here and below denotes unit vectors); $C_k(r)$ are the coefficients in the expansion in Legendre polynomials; P_k are Legendre polynomials; and, $\mu = (\hat{l}, \hat{r})$ is the cosine of the angle at which the brightness field is viewed.

The classical method of spherical harmonics leads to an infinite tridiagonal system of differential equations for the expansion coefficients C_k. To obtain acceptable accuracy several hundreds (and sometimes significantly more) terms must be retained in the expansion, i.e., the system must be solved with the same number of equations, which presents enormous difficulties even at the ' present level of development of computer technology. In the small-angle modification (which we employed) of the spheri-

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cal-harmonics method the coefficients C_k can depend continuously and monotonically on the number k, and this makes it possible to reduce the infinite system to a single, second-order, partial differential equation, which can be solved analytically. The solution retains all the features of the exact solution of the RTE and neglects only the variance in the paths of the scattered photons and backscattering. This makes it possible to use the solution in a wide range of viewing angles and it also removes the sharp restrictions on the anisotropy of the scattering phase function of the medium. Comparisons with the Monte Carlo method showed that the solution gives an error of not more than 10–15% for atmospheric scattering phase functions with viewing angles of 40°–50° (i.e., at the level of the variance of the Monte Carlo method itself). In addition, the series solution is more useful in engineering calculations than the solution in the form of an improper integral.[1]

In the situation studied in this paper the FSF of the medium is circularly symmetric, and the OTF can be found as a Hankel transformation of the FSF. Since $P_k(\cos\varphi) \simeq J_0(k\varphi)$ (as $\varphi - 0$, and J_0 is a Bessel function) and $\sum_{k=0}^{\infty}\frac{2k+1}{4\pi} \simeq \frac{1}{2\pi}\int_0^{\infty}\nu d\nu$ (the case of sharp scattering phase functions the number of nonzero terms in the expansion (1) will be very large) is equivalent to the optical transfer function with $k = \nu$, where ν is the spatial frequency. In what follows, to simplify the presentation we shall call C_k the optical transfer function. We note that in the arguments given in this paragraph a clear relationship with the small-angle forms of Refs. 1 and 3.

We shall study the transfer of an image of a bright object through an optically dense scattering layer (Fig. 1). We shall use the parameter $t = d/D$ to characterize the position of the layer (following Refs. 5–7); here d is the distance from the object to the center of the layer and D is the distance from the object to the point of viewing (detector).

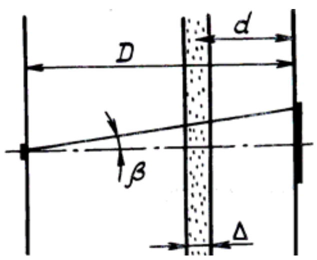

FIG. 1. The computational scheme.

For fixed D the OTF of the medium will be a function of t and according to Ref. 3 it will be described by the formula

$$C_k(t) = \frac{1}{D^2}\exp\left\{-\tau + \frac{2\Lambda\varepsilon D}{q}\cdot\frac{g^{qt}}{\ln g}\, sh\left[q\,\frac{\Delta}{2D}\ln g\right]\right\}, \tag{2}$$

where $q = \sqrt{k(k+1)}$; Δ is the thickness of the layer; $\tau = \varepsilon\Delta$ is the optical thickness of the layer; ε and Λ are, respectively, the extinction coefficient and albedo of the single-scattering layer; and, g is the parameter in the Henyey-Greenstein function, which is used to approximate the scattering phase function of the layer (g is the average cosine of the angle of the phase function).[8]

It is not difficult to derive from Eq. (2) an expression for the frequency-contrast characteristic (FCC) of the medium F_k:

$$F_k(t) = \frac{C_k(t)}{C_0} =$$

$$= \exp\left\{\Lambda\varepsilon\left[\frac{2D}{q}\frac{g^{qt}}{\ln g}\, sh\left[q\,\frac{\Delta}{2D}\ln g\right] - \Delta\right]\right\} \tag{3}$$

Analysis of the formulas (2) and (3) shows that the OTF and FCC of the medium decrease monotonically as a function of k and become narrower as the parameter t increases (Fig. 2). This result agrees with the results obtained on the basis of a different form of the small-angle approximation as well as in the experimental works.[9–12] The different behavior of the FCC obtained in Refs. 5–7 is explained by the fact that these works were concerned not with the FCC of the medium but rather the FCC of the system "medium + bounded viewing angle of the detector," and this difference can be very significant. Figure 2 shows that the FCC of the medium has a "shelf," whose size can be easily found from Eq. (3):

$$\lim_{k \to \infty} F_k(t) = \exp(-\Lambda\varepsilon\Delta).$$

We note that $F_k(t)$ becomes sharply narrower at $t \approx 0$ as t increases.

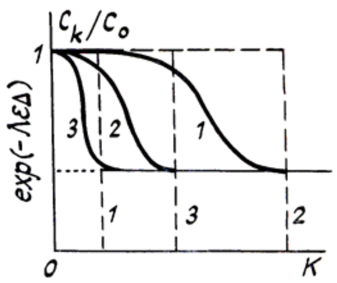

FIG. 2. The dependence of the frequency-contrast characteristic of the medium on the parameter t. Solid lines: 1) $t \sim 0$; 2) $t \simeq 0.1$; 3) $t \to 1$. The dashed lines show the spectrum of the object (for convenience, a rectangular spectrum is taken): 1) large object; 2) small object; and 3) object of average size.

The distribution of the brightness in the image is a convolution integral of the brightness distribution in the volume with the FSF of the medium (in the frequency domain the convolution transforms into a product of spectra):

$$L'(\mu, t) = \frac{1}{4\pi}\sum_{k=0}^{\infty}(2k+1)\,C_k(t)\,\vartheta_k P_k(\mu), \tag{4}$$

where ϑ_k are the coefficients in the expansion of the brightness distribution in the object in Legendre polynomials ("Legendre spectrum" of the object).

For convenience and to make it easier to construct a numerical algorithm we shall use for the test object a diffusely radiating disk of unit brightness. For a disk, i.e., for a rectangular distribution of the brightness over the object, ϑ_k have the form

$$\vartheta_k = (P_{k-1}[\cos\beta - P_{k+1}(\cos\beta)])/(2k+1), \tag{5}$$

where β is one-half the angular size of the disk.

The results of the calculation of the distribution of the brightness in the image of a disk for different positions of the layer on the object-detector path are presented in Fig. 3.

The best example of the effect of the character of the stratification of the medium on the formation of the light field is the dependence of the image transfer quality (ITQ) of a cloudy atmosphere on the position of the cloudy layer on the observation path. Several different (sometimes contradictory) viewpoints regarding the character of the dependence of the ITQ on the position of the layer have been presented in the literature.[5–7,9–13] This situation motivated this author to perform further investigations.

In choosing a criterion for image transfer quality we shall not only use the OTF of the medium, but we shall also take into account the principle of operation of the image analyzer, which is based on comparing the image with the original[14]

$$\gamma = 1/\int_{(\Omega)}(\bar{L}' - \bar{L})^2 d\Omega, \tag{6}$$

where $\bar{L} = L(\mu)/L_{\max}$; $\bar{L}' = L'(\mu)/L_{\max}$; where $L(\mu)$ and $L'(\mu)$ are the angular distributions of the brightness in the object and the image, respectively; $L_{\max}$ and $L'_{\max}$ are the maximum brightness of the object and image; and, Ω is the viewing angle of the detector.

In the limit $\Omega \to 4\pi$ according to Parseval's theorem, the formula (6) assumes the form

$$\gamma \underset{\Omega \to 4\pi}{=} 1/\sum_{k=0}^{\infty}[\bar{a}^{1}_k - \bar{a}_k]^2, \tag{7}$$

where $\bar{a}_k = a_k/L_{\max}$, $\bar{a}'_k = a'_k/L'_{\max}$, a_k is the spectrum of the object, and a'_k is the spectrum of the image. The image quality criterion γ introduced in this manner reflects both the transfer of contrast to the image (owing to the normalization of the brightnesses) and the transfer of the fine structure of the object.

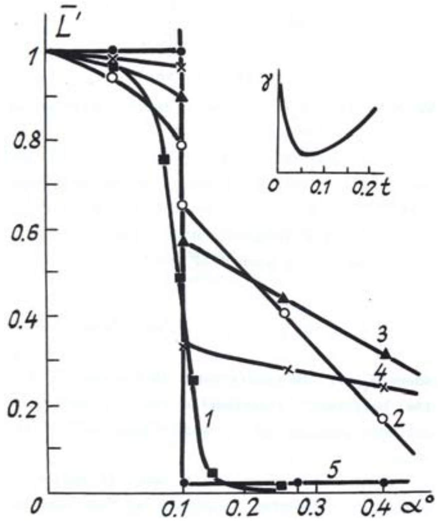

FIG. 3. The brightness distribution in the image of a disk at different positions: t = 0.001, 0.05, 0.1, 0.2, *and* 0.99 *(the curves 1–5, respectively). In the calculations* $\beta = 0.1°$; $\tau = 5$, $D = 5$; $\Delta = 0.05$; $g = 0.97$; *and,* $\Lambda = 1.0$. *The dependence* $\gamma = \gamma(t)$ *is shown in the upper righthand corner.*

We shall use the expressions (4) and (5) to determine the quantities appearing in the formulas (6) and (7): $a_k = \vartheta_k$ is the spectrum of the object and $a'_k = \vartheta_k C_k$ is the spectrum of the image; $L_{\max} = 1$; and, $L'_{\max} = L'(1, t)$. It is obvious that the brightness will be maximum at the center of the image (for $\mu = 1$). Then

$$\gamma(t) = 1/\int_{(\Omega)}[L'(\mu, t)/L'(1, t) - L(\mu)]^2 d\Omega \underset{\Omega \to 4\pi}{=} 1/\sum_{k=0}^{\infty}\vartheta_k^2\,[C_k(t)/L'(1, t) - 1]^2. \tag{8}$$

It is obvious that γ is a function of the shape and size of the object, the optical characteristics of the scattering layer, and the parameter t. Investigations of the ITQ for a scattering layer using the formula (8) were performed on a computer for a wide range of values of the parameters enumerated above. Detectors with both unbounded and bounded fields of view were modeled (in the second case the integration was performed numerically).

The calculations showed that, in both cases, as the distance between the object and the scattering layer increases (i.e., as the parameter t increases), in spite of the monotonic degradation of the FCC of the medium the ITQ can decrease or increase monotonically and it can have an extremal character (i.e., the t-effect occurs). The form of the dependence $\gamma(t)$ is determined by the ratio of the dimensions of the object and the characteristic scale of the FSF of the medium as well as the ratio of the scattered and directly transmitted (attenuated according to Bouguer's law) components in the image of the object. The dependence of the character of the behavior of $\gamma(t)$ on the dimensions of the object is apparently of greatest interest. This dependence can be qualitatively explained as follows.

If the size of the object is significantly greater than the characteristic scale of the FSF of the medium (the spectrum of such an object will be significantly narrower than the FCC of the medium — see Fig. 2), then as t increases the spectrum of the image of the object will differ increasingly more strongly from the spectrum of the original, and the ITQ will decrease monotonically ("tracing paper" effect) (Fig. 4a, the average cosine of the angle of the scattering phase function $g = 0.97$, and $\beta = 1°$).

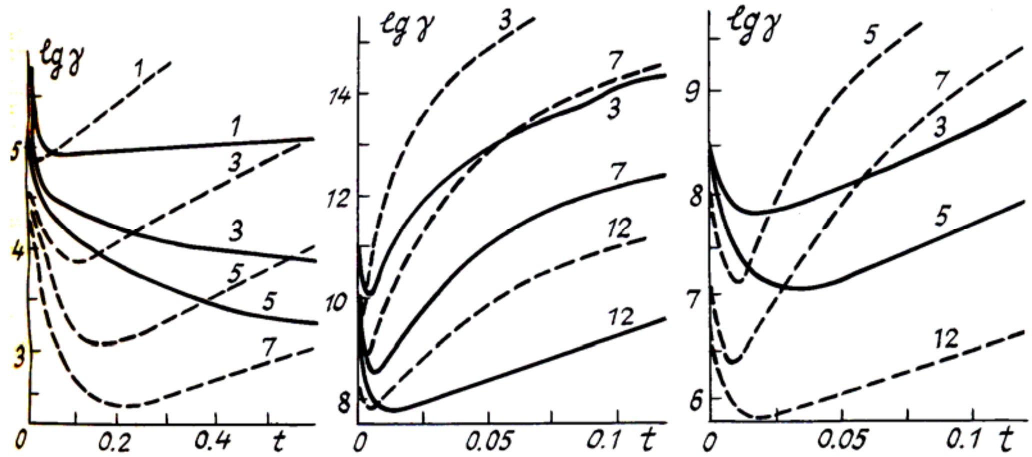

FIG 4. The dependence $\gamma = \gamma(t)$. The numbers on the curves correspond to the optical thickness τ of the layer. In all figures $\Lambda = 1.0$, $\Delta = 0.05$, and $D = 5.0$; the angle of the field of view of the detector is equal to 10 angular sizes of the object, a) $\beta = 1.0°$. The solid lines correspond to $g = 0.97$ and the dashed lines correspond to $g = 0.80$. b) $\beta = 0.001°$. The solid lines correspond to $g = 0.99$ and the dashed lines correspond to $g = 0.97$. c) $\beta = 0.01°$. The solid line corresponds to $g = 0.99$ and the dashed line corresponds $g = 0.95$.

In the other extreme case, when the object is very small compared with the characteristic scale of the FSF of the medium with the layer flush against the object and the spectrum of the object is correspondingly much wider than the FCC of the medium, as t increases only increasingly lower frequencies of the spectrum are distorted. Because the FCC contains a "shelf" the high frequencies of the spectrum of the object are reduced by the same factor, i.e., they are not distorted. For this reason, as t increases an increasingly narrower section of the spectrum becomes distorted, and this results in monotonic improvement of the ITQ of the object (Fig. 4b, $\beta = 0.001°$, $g = 0.97°$).

The transfer of an image of an object whose size is comparable to the characteristic scale of the FSF of the medium when the layer is located near the object is an intermediate case. For small t the FCC is "wider" than the spectrum of the object and the first case is obtained, i.e., the ITQ decreases as t increases. As t increases further the relation between the FCC of the medium and the spectrum of the object changes and a situation analogous to the second case is observed, i.e., as t increases the ITQ increases. Thus the dependence $\gamma(t)$ has an extremal character, i.e., we obtain the t-effect. Note that since the FCC becomes narrower more sharply for the initial changes in t, the region of minimum image quality is drawn toward small values of t.

The computational results presented in Fig. 4 reflect the phenomena that occur when the optical parameters of the layer change: broadening of the scattering phase function of the layer (decrease of g) results in broadening of the FSF of the medium, which results in a displacement of the effects described above in the direction of monotonic improvement of image quality; increase in the optical thickness of the layer τ as well as increase of the single-scattering albedo Λ (resulting in a lowering of the "shelf" on the FCC of the medium) shift the effects in the direction of monotonic reduction of image quality.

Note that the limitation of the field of view of the detector does not introduce anything fundamentally new and merely shifts the observed effects in the direction of monotonic improvement of image quality.

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