RECONSTRUCTION OF THE PRIMARY HYDROOPTICAL CHARACTERISTICS FROM THE LIGHT FIELD OF A POINT SOURCE

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In the framework of unified approach, a method is discussed for reconstruction of the primary hydrooptical characteristics from a light field produced in a medium by a point source with a wide directional pattern. The data are presented of in situ *measurements of the scattering coefficient, absorption coefficient, and scattering phase function of water of Lake Baykal.*

1. INTRODUCTION

The study of optical properties of some media is of scientific and practical interest. The most important optical characteristics are so-called primary parameters that do not depend on illumination and observation conditions. As known, among them are the absorption coefficient $\varkappa$, extinction coefficient ε (connected with the scattering coefficient by the relation $\varepsilon = \varkappa + \sigma$), and the scattering phase function $\chi(\alpha)$. In addition, one often uses the single scattering albedo $\Lambda = \sigma/\varepsilon$, differential scattering coefficient in the given direction $\sigma(\alpha) = \sigma\chi(\alpha)$, and a number of the other parameters, which, in their turn, can be obtained from the aforementioned basis set of the primary characteristics.[1] Various methods are applied for obtaining these parameters. In this paper, we consider the approach that allows one to obtain all aforementioned characteristics in a unified way, from a light field produced in a homogeneous medium by a point source with a wide directional pattern.

The propagation of radiation in a medium is described by the kinetic equation, with the coefficients being the well-known functions of the primary optical characteristics. We consider here the inverse problem: to derive the primary optical parameters knowing the spatiotemporal distribution of brightness measured in the experiment. It is easy to show[2] that in the single scattering approximation the number of photons $\dot{N}$, coming from elementary solid angle Ω per unit time to the detector tilted at angle α to the source–detector axis, is equal to

$$\dot{N} = \begin{cases} \dot{N}_0 & \text{for the direction toward the source } (\alpha=0), \\ \dot{N}_s(\alpha) & \text{for } \alpha > 0, \end{cases} \quad (1.1)$$

where

$$\dot{N}_0 = I_0\, F(0)\, (S_D/R^2)\, e^{-\varepsilon R}, \quad (1.2)$$

$$\dot{N}_s(\alpha) = I_0 \frac{d\Omega}{\sin(\alpha)} \frac{S_D}{R_2} (\sigma R) \times \times \int_0^{\pi-\alpha} \chi(\alpha+\beta)\, F(\beta)\, e^{-\varepsilon R(\alpha,\beta)}\, d\beta. \quad (1.3)$$

Here, I_0 is the source intensity, $F(\beta)$ is the directional pattern (for example, $F(\beta) = 1/4\pi$ for an isotropic source and $F(\beta) = \begin{cases} \cos(\beta)/\pi & \text{for } \beta \le \pi/2 \\ 0 & \text{for } \beta > \pi/2 \end{cases}$ for a Lambert source), S_D is the area of the detector, R is the distance between the source and the detector, and $R(\alpha, \beta)$ is the photon path length from the source to the detector

$$R(\alpha, \beta) = R \frac{\sin(\alpha) + \sin(\beta)}{\sin(\alpha+\beta)}. \quad (1.4)$$

In Eq. (1.1), $\dot{N}_0$ describes the contribution of the directly transmitted light and $\dot{N}_s$ - of the scattered light.[3–5] The result does not depend on the azimuth angle ϕ due to the axial symmetry. The scattering phase function χ in Eq. (1.3) is normalized by the condition

$$\int_{4\pi} \chi(\Omega)\, d\Omega = 2\pi \int_0^{\pi} \chi(\gamma) \sin(\gamma)\, d\gamma = 1. \quad (1.5)$$

To measure the brightness field, we used a device described in detail in Ref. 6. The variable baseline of the device allowed us to measure at distances R varying from 1.2 to 15 m. Scanning in the angle α could be performed in the range up to 180° with a minimum step of 2′. The aperture angle of the device in the scanning mode did not exceed 1.5°. The device was equipped with a quasi-isotropic source of light with a wide spectrum of radiation in the visible wavelength range. A set of changeable narrow-band (with a bandwidth of ~5 nm at half maximum) light filters was used for selection of the wanted wavelength. The device was

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controlled by a microprocessor that made it possible to use it for *in situ* measurements. Measurements were carried out in Lake Baykal (in its southern part) at a distance of 3.5 km from the coast at a depth of 1000–1100 m (the position of a neutrino telescope).

It is seen from Eqs. (1.1)–(1.3) that the full basis set of the aforementioned parameters ε, σ and χ are included into the magnitude of the brightness field. In general case, there are some difficulties in solving Eqs. (1.1)–(1.3). Below we consider a solution to these equations with additional assumption about the strong elongation of the scattering phase function in the forward direction - the property characteristic of the scattering phase functions of natural waters.

2. ABSORPTION COEFFICIENT

Equation (1.1) was derived for the number of photons coming to the detector from the elementary solid angle $d\Omega$. After summing up the contributions from all possible directions of photon arrival, we derive for the corresponding integral parameter

$$\dot{N}_t = \dot{N}_0 + \int_0^{2\pi} d\varphi \int_0^{\pi} \dot{N}_s(\alpha) \sin(\alpha)\, d\alpha. \quad (2.1)$$

Let us consider the case of an isotropic source of radiation for simplicity. From Eqs. (1.2) and (1.3) we derive

$$\dot{N}_t = I_0 \frac{S_D}{4\pi R^2} e^{-\varepsilon R} \times$$

$$\times \left(1 + \sigma R\, 2\pi \int_0^{\pi} d\alpha \int_0^{\pi-\alpha} d\beta\, \chi(\alpha+\beta)\, e^{\varepsilon(R - R(\alpha,\beta))} \right). \quad (2.2)$$

After replacing the variable $\gamma = \alpha + \beta$ in the second integral in the right side of Eq. (2.2) and changing the order of integration, we obtain

$$\int_0^{\pi} d\alpha \int_0^{\pi-\alpha} d\beta\, \chi(\alpha+\beta)\, e^{\varepsilon(R - R(\alpha,\beta))} =$$

$$= \int_0^{\pi} d\gamma\, \chi(\gamma) \int_0^{\gamma} d\alpha\, e^{\varepsilon(R - R(\alpha,\gamma-\alpha))}. \quad (2.3)$$

Integration in Eq. (2.3) is actually performed over small angles due to strong elongation of the scattering phase function in the forward direction. It follows from Eq. (1.4) that in this case $R(\alpha, \beta) \approx R$ and hence

$$\int_0^{\pi} d\gamma\, \chi(\gamma) \int_0^{\gamma} d\alpha\, e^{\varepsilon(R - R(\alpha,\gamma-\alpha))} \approx \int_0^{\pi} \chi(\gamma)\, \gamma\, d\gamma. \quad (2.4)$$

Taking into account normalization condition (1.5), we obtain

$$\int_0^{\pi} \chi(\gamma)\, \gamma\, d\gamma = \frac{1}{2\pi} (1 + K_\chi), \quad (2.5)$$

where

$$K_\chi = 2\pi \int_0^{\pi} \chi(\gamma)\, (\gamma - \sin(\gamma))\, d\gamma. \quad (2.6)$$

The condition

$$K_\chi \ll 1. \quad (2.7)$$

is satisfied for strongly forward-peaked scattering phase function.

Upon substituting Eqs. (2.3)–(2.7) in Eq. (2.2), we obtain

$$\dot{N}_t \approx I_0\, [S_D/(4\pi R^2)]\, e^{-\varepsilon R} (1 + \sigma R). \quad (2.8)$$

Initial Eq. (2.2) was derived in the single scattering approximation, when only the first power of σR was taken into account. The approximate equality

$$1 + \sigma R \approx e^{\sigma R} \quad (2.9)$$

is fulfilled to the given accuracy.

We finally obtain from Eqs. (2.8) and (2.9)

$$\dot{N}_t \approx I_0\, [S_D/(4\pi R^2)]\, e^{-\varkappa R}. \quad (2.10)$$

The accuracy of fulfilling Eq. (2.7) depends on the specific type of scattering phase function.

The condition $K_\chi \le 0.015$ was obtained for our case (see Sec. 3) and thus, relation (2.7) is actually fulfilled.

Although we have taken into account the light scattering for the first order of the constant σR, it is easy to show that taking into account higher scattering orders will lead to appearance of additional terms $\frac{(\sigma R)^2}{2!} + \frac{(\sigma R)^3}{3!} + \ldots$ in the right side of Eq. (2.8) and hence Eq. (2.9) will be replaced by the exact equality in this case. Equation (2.10) will be also fulfilled when one cannot ignore the multiple scattering, if the coefficient K_χ (correspondingly modified) remains small.

Equation (2.10) was tested for the case of multiple scattering in Ref. 7, where the experimental data were compared with the results of Monte Carlo simulation. It was pointed out that the deviations from Eq. (2.10) remain small even for very large distances R ($R \sim 3/\sigma$).

Then it is easy to obtain the absorption coefficient χ. Writing Eq. (2.10) for two different baselines R_1 and R_2, we obtain

$$\varkappa \approx \ln\left(\frac{\dot{N}_t(R_1)\, R_1^2}{\dot{N}_t(R_2)\, R_2^2} \right) / (R_2 - R_1). \quad (2.11)$$

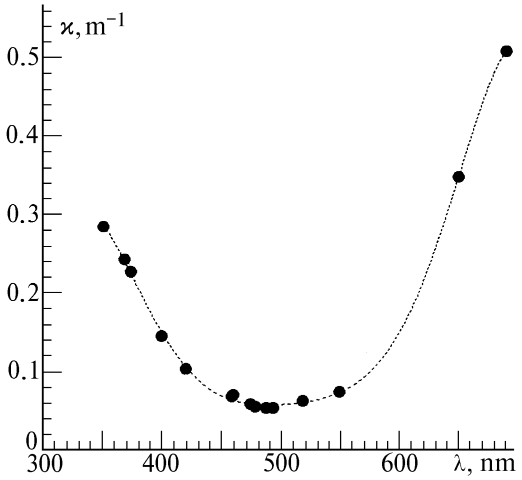

FIG. 1. Absorption coefficient (Lake Baykal, depth H = 1100 m, *August of 1993), data of* in situ *experiment.*

The results of calculation of the absorption coefficient from the data of measurement of the integral brightness field at different wavelengths are shown in Fig. 1. It is seen from Fig. 1 that there is a characteristic transmittance window with the maximum transmittance at the wavelength $\lambda \approx 490$ nm, where the absorption length $1/\varkappa$ reaches 18–20 m.

3. SCATTERING PHASE FUNCTION

According to Eq. (1.3), the number of photons of the scattered light coming to a collimated detector tilted at the angle α to the source–detector axis is described by the following expression:

$$\dot{N}_s(\alpha) = I_0 \frac{S_D}{R^2} (\sigma R) \int\limits_{\Omega_D} \frac{d\Omega}{\sin(\alpha)} \times$$

$$\times \int\limits_0^{\pi-\alpha} \chi(\alpha + \beta)\, F(\beta)\, e^{-\varepsilon R(\alpha,\beta)}\, d\beta. \qquad (3.1)$$

For the angles α such that $\sin\alpha \gg \Delta\alpha$ ($2\Delta\alpha$ is the aperture angle of the detector), we can consider the simple estimate of the first integral in Eq. (3.1) assuming the integrand function to be constant on the small interval Ω_D ($\Omega_D/4\pi \ll 1$). Introducing the coefficients, independent of the scattering angles, into the constant factor $C = I_0(S_D/R^2)(\sigma R)\Omega_D$, we obtain:

$$\dot{N}_s(\alpha) = \frac{C}{\sin(\alpha)} \int\limits_0^{\pi-\alpha} \chi(\alpha + \beta)\, F(\beta)\, e^{-\varepsilon R(\alpha,\beta)}\, d\beta. \qquad (3.2)$$

One can consider Eq. (3.2) as an equation for the unknown function χ. It is difficult to determine the constant C directly from the experiment; however, it is not incorporated into the resultant solution due to the normalization condition for scattering phase function (1.5).

To solve Eq. (3.2), we used the method described in our previous paper.[5] The comparison of the results of calculation of the scattering phase function from the brightness field by Eq.(3.2) with the results of measuring $\chi(\alpha)$ by the standard technique was also made there. In particular, it was obtained that the differences between the scattering phase function reconstructed from the brightness field and the measured one did not exceed the errors of the experiment.

The stability of the method, the influence of the measurement errors on the accuracy of reconstruction of the scattering phase function, as well as the problems related to ill-posed integral equation (3.2) were also discussed in Ref. 5 in detail.

The scattering phase function of water of Lake Baykal is shown in Fig. 2. It is seen that the scattering at the small angles is prevalent due to the strong elongation of the scattering phase function. The full range of variations of $\chi(\alpha)$ at scattering angles shown in the figure is approximately five orders of magnitude. The characteristic angles, at which the scattering phase function becomes more gently sloping, are $\alpha \sim 15$–20°.

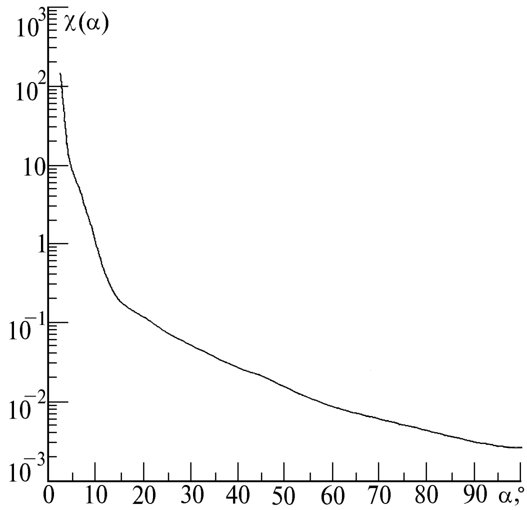

FIG. 2. Scattering phase function (Lake Baykal, depth H = 1000 m, *wavelength* λ = 497 nm, *March of 1988), data of* in situ *experiment.*

Let us obtain now the numerical estimate of the parameter K_χ from Eq. (2.6).

By definition,

$$K_\chi = \frac{\int\limits_0^{\pi} \chi(\alpha)\, (\alpha - \sin(\alpha))\, d\alpha}{\int\limits_0^{\pi} \chi(\alpha)\, \sin(\alpha)\, d\alpha} \qquad (3.3)$$

(we have obviously taken into account the normalization condition for scattering phase function (1.5) in this formula). To calculate K_χ by Eq. (3.3), it is necessary to know the scattering phase function for the entire angular range. However, usually the experimental data allow one to determining the scattering phase function only beginning with a certain minimum angle $\alpha_0 > 0$ (for example, $\alpha_0 = 2°$ for Fig. 2). Let us show how we can construct the estimate of K_χ in this case.

Let us divide the interval of integration in the nominator of Eq. (3.3) into two subintervals: from 0 to α_0 and from α_0 to π. Then we obtain

$$K_\chi = K_1 + K_2. \tag{3.4}$$

For K_1 we derive

$$K_1 = \frac{\int\limits_0^{\alpha_0} \chi(\alpha)\ (\alpha - \sin(\alpha))\ d\alpha}{\int\limits_0^{\pi} \chi(\alpha)\ \sin(\alpha)\ d\alpha} = $$

$$= \frac{\int\limits_0^{\alpha_0} \chi(\alpha) \left(\frac{\alpha - \sin(\alpha)}{\sin(\alpha)}\right) \sin(\alpha)\ d\alpha}{\int\limits_0^{\pi} \chi(\alpha)\ \sin(\alpha)\ d\alpha} \le$$

$$\le \max_{\alpha \in (0,\alpha_0)} \left(\frac{\alpha - \sin(\alpha)}{\sin(\alpha)}\right) \frac{\int\limits_0^{\alpha_0} \chi(\alpha)\ \sin(\alpha)\ d\alpha}{\int\limits_0^{\pi} \chi(\alpha)\ \sin(\alpha)\ d\alpha} \le$$

$$\le \frac{\alpha_0 - \sin(\alpha_0)}{\sin(\alpha_0)}\ . \tag{3.5}$$

Then we have for K_2

$$K_2 = \frac{\int\limits_{\alpha_0}^{\pi} \chi(\alpha)(\alpha - \sin(\alpha))\ d\alpha}{\int\limits_0^{\pi} \chi(\alpha)\ \sin(\alpha)\ d\alpha} =$$

$$= \frac{\int\limits_{\alpha_0}^{\pi} \chi(\alpha)\ (\alpha - \sin(\alpha))d\alpha}{\int\limits_0^{\alpha_0} \chi(\alpha)\ \sin(\alpha)\ d\alpha + \int\limits_{\alpha_0}^{\pi} \chi(\alpha)\ \sin(\alpha)\ d\alpha} \le$$

$$\le \frac{\int\limits_{\alpha_0}^{\pi} \chi(\alpha)\ (\alpha - \sin(\alpha))\ d\alpha}{\int\limits_{\alpha_0}^{\pi} \chi(\alpha)\ \sin(\alpha)\ d\alpha}\ . \tag{3.6}$$

When calculating by Eq. (3.6), we can ignore the contribution at very large angles. It is seen from Fig. 2 that this range cannot significantly affect the result of calculation.

TABLE I.

K_χ, see Eq. (2.6)	$\langle\alpha\rangle$	$\langle\alpha^2\rangle^{1/2}$	$\langle\alpha^3\rangle^{1/3}$	$\langle\cos\alpha\rangle$
≤ 0.015	$\le 8°$	$\le 16°$	$\le 24°$	≥ 0.96

Using Eqs. (3.4)–(3.6) and the data on the scattering phase function (Fig. 2), we obtain the upper estimate of the coefficient K_χ considered in the previous section. In such a way, we can obtain the estimates of different parameters, for example, of the average scattering angles. The results of such calculations are given in Table I.

4. SCATTERING AND EXTINCTION COEFFICIENTS

Simple technique for determining the extinction coefficient ε can be obtained from Eq. (1.1) at $\alpha = 0$. However, to separate the contribution of only directly transmitted light, one should apply the narrow-aperture collimator, that makes the adjustment of the device when finding the direction $\alpha = 0$ much more difficult. Furthermore, even small errors in determining the direction toward the source in this case can result in noticeable errors in measuring the parameter ε. The problem becomes dramatically difficult for the device operating *in situ*, when automated adjustment is needed. The use of a wide collimator leads to signal averaging over the collimator width and we have the situation already considered in Section 2.

We can avoid these difficulties with the use of signal $\dot{N}_s(\alpha)$ of the scattered light alone for reconstructing ε. For strongly forward-peaked scattering phase function and two different baselines R_1 and R_2, we obtain

$$\frac{\dot{N}_s(\alpha;\, R_1)\, R_1}{\dot{N}_s(\alpha;\, R_2)\, R_2} = \frac{\int\limits_0^{\pi-\alpha} \chi(\alpha + \beta)\ F(\beta)\ e^{-\varepsilon R_1(\alpha,\beta)}\ d\beta}{\int\limits_0^{\pi-\alpha} \chi(\alpha + \beta)\ F(\beta)\ e^{-\varepsilon R_2(\alpha,\beta)}\ d\beta} \approx$$

$$\approx \frac{e^{-\varepsilon R_1(\alpha,0)} \int_0^{\pi-\alpha} \chi(\alpha+\beta)\, F(\beta)\, d\beta}{e^{-\varepsilon R_2(\alpha,0)} \int_0^{\pi-\alpha} \chi(\alpha+\beta)\, F(\beta)\, d\beta} =$$

$$= e^{\varepsilon(R_2(\alpha,0)-R_1(\alpha,0))} = e^{\varepsilon(R_2-R_1)} \quad (4.1)$$

(we have taken into account that, according to Eq. (1.4), $R(\alpha, 0) = R$).

It follows from Eq. (4.1) that the extinction coefficient can be estimated as

$$\tilde{\varepsilon}(\alpha) = \ln\left(\frac{\dot{N}_s(\alpha, R_1)\, R_1}{\dot{N}_s(\alpha; R_2)\, R_2}\right) \Big/ (R_2 - R_1), \quad (4.2)$$

where, as follows from the derivation,

$$\tilde{\varepsilon}(\alpha = +0) = \varepsilon, \quad (4.3)$$

$$\tilde{\varepsilon}(\alpha) \approx \varepsilon \quad (4.4)$$

for $\alpha > 0$, if the scattering at small angles is prevalent.

To examine the accuracy of Eq. (4.4) for different angles α, let us do as follows. Let us select a certain initial value of the extinction coefficient and calculate $\dot{N}_s(\alpha, R)$ for two different baselines R_1 and R_2 by Eq. (1.3) using the data on the scattering phase function from Section 3. The specific value of the extinction coefficient has a little significance here, because it is seen from Eqs. (1.3) and (1.4) that ε is included only in combination with the factor R, where R is the baseline of the device, i.e., the parameter that can be selected by our choice. Then let us calculate the parameter $\tilde{\varepsilon}(\alpha)$ by Eq. (4.2) and compare it with the initial parameter ε.

The results of calculation for $\varepsilon = 0.1$ m^{-1}, $R_1 = 1.5$ m, and $R_2 = 3$ m are shown in Fig. 3. As seen, there is a plateau (framed) at angles varying from 0 to 10–12°, where $\tilde{\varepsilon}(\alpha)$ is practically no different from ε. It is also seen that the difference between the estimate of the extinction coefficient by Eq. (4.2) and the true extinction coefficient does not exceed 10% at large angles α ~ 10–30°.

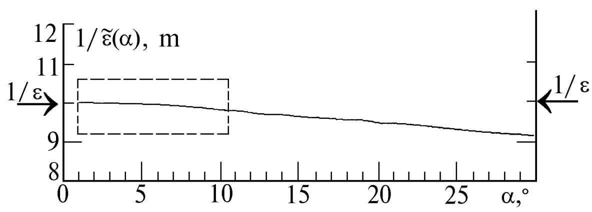

FIG. 3. Comparison of ε *with its estimate* $\tilde{\varepsilon}(\alpha)$ *by Eq. (4.2). The following values of the input parameters were taken for calculation:* $\varepsilon = 0.1$ m^{-1}, $R_1 = 1.5$ m, *and* $R_2 = 3$ m.

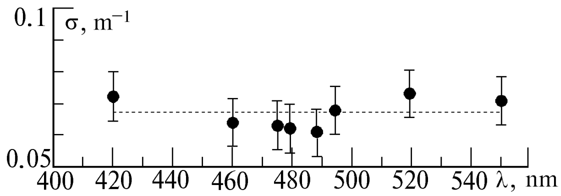

FIG. 4. Scattering coefficient (Lake Baykal, depth $H = 1100$ m, *October–November of 1993), data of* in situ *experiment.*

We have calculated the extinction coefficient ε at several wavelengths λ from the data on the brightness field at $\alpha = 4°$ for different baselines, using Eq. (4.2). Using the results obtained in Section 2, we can obtain the scattering coefficient σ_λ. The results of calculations are shown in Fig. 4. It is seen that the scattering length $1/\sigma$ is approximately 15 m for the maximum transmission.

5. CONCLUSION

In this paper, we have considered the approach that has allowed us to determine the primary hydrooptical characteristics in a unified way, from the light field produced by a point source of radiation.

The following characteristics were obtained using Eq. (1.1) and the data on the spatial-angular distribution of brightness in the single scattering approximation when the scattering at small angles was prevalent: the absorption coefficient $\varkappa$ (Fig. 1), scattering phase function $\chi(\alpha)$ (Fig. 2), and the scattering coefficient σ (Fig. 4). To solve Eq. (1.1) for three unknown parameters $\varkappa$, χ, and σ, it was written in the form of the system of equations for two different baselines R_1 and R_2. Normalization condition (1.5) was used as the third equation.

Determination of each parameter was accompanied with discussion of the employed assumptions and accuracy of the calculation technique.

This method for determining the primary hydrooptical characteristics is used for the study of the conditions of deep-water recording of elementary particles in Lake Baykal.

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