METHOD OF THE GREEN'S FUNCTIONS AND LINEAR–SYSTEMS APPROACH IN THE THEORY OF TRANSFER AND RECORDING OF OPTICAL RADIATION

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The correspondence between the linear–systems approach and method of the Green's functions as well as a procedure for constructing the Green's functions providing an account of peculiarities of image formation by optical systems are considered.

The solution of the problem of optical signal propagation through scattering and absorbing media is of great importance for fundamental and applied problems. The comprehensive analysis of the effect of molecular and dispersed components of the media forming the propagation channels of optical radiation on the signal characteristics expands the range of applicability of conventional optical and opto–electronic systems (for example, in aerospace photography, underwater photography, astronomical observations, etc.), and allows one to develop new methods and means for the study of the propagation channels themselves by optical methods. One of the most well–known lines of studying is, for example, laser sensing of the atmosphere as a scattering and absorbing medium. The theory of laser sounding and the commercially available lidars are based on the previously ascertained regularities of scattering and absorption of optical signals in dispersed and molecular–gaseous media. The efficiency of these (and others) opto–electronic systems is obviously determined by the degree of understanding of the physical processes chosen as the most informative for achieving the purposes being of concern to a developer of opto–electronic systems.

Let us consider in the most general form the problem of propagation of optical signals disregarding the concrete subject areas and formulate the most general approaches to its solution. Let us arrange to distinguish between the input and output optical signals. The signal specified at the point (in the region) of its emission or incidence on the boundary of the scattering and absorbing medium will be called the input signal P_{in}. The signal specified at the point (in the region) of its recording will be called the output signal P_{out}.

Thus, we will consider the signals P_{in} and $P_{\text{out}}(t)$ to be one–dimensional signals if they are specified at a fixed point (or in the region) of the space as functions of time t, two–dimensional signals P_{in}, $P_{\text{out}}(x, y)$ in the stationary case, and three–dimensional signals P_{in}, $P_{\text{out}}(x, y, t)$ in the case of their spatial–temporal dependence. The first case obviously pertains to the theory of optical detection and ranging, sounding, and communication, the second case is treated in the vision theory and in the theory of passive sounding of the underlying surface temperature. The third case is realized, for example, in observations of dynamic phenomena. We will assume that the radiation source and receiver are screened by the scattering medium whose optical properties are fixed at each point, unchanged with time, and specified by the values of the coefficients of scattering $\beta_{\text{sc}}(\mathbf{r})$, extinction $\beta_{\text{ext}}(\mathbf{r})$, and absorption $\beta_{\text{ab}}(\mathbf{r})$ and the scattering phase function $g(\mathbf{r}, \boldsymbol{\omega})$.

The studies of the optical signal propagation in the scattering media are aimed at finding the regularities and relations between the spatial, temporal, energetic, and other characteristics of the input and output signals depending on the optical properties and geometric parameters of the propagation channels of the short–wave radiation. In the most general form these relations in terms of the radiant intensity are given by the stationary

$$(\boldsymbol{\omega}, \operatorname{grad} I(\mathbf{r}, \boldsymbol{\omega})) = -\beta_{\text{ext}}(\lambda, \mathbf{r})\, I(\mathbf{r}, \boldsymbol{\omega}) + \\ + \beta_{\text{sc}}(\lambda, \mathbf{r}) \int\limits_{\Omega} I(\mathbf{r}, \boldsymbol{\omega}')\, g(\mathbf{r}, \boldsymbol{\omega}; \boldsymbol{\omega}')\, d\boldsymbol{\omega}' + \Phi_0(\mathbf{r}, \boldsymbol{\omega}) \qquad (1)$$

or nonstationary

$$\frac{1}{c}\frac{\partial I(\mathbf{r}, \boldsymbol{\omega})}{\partial t} + (\boldsymbol{\omega}, \operatorname{grad} I(\mathbf{r}, \boldsymbol{\omega}, t)) = -\beta_{\text{ext}}(\lambda, \mathbf{r})\, I(\mathbf{r}, \boldsymbol{\omega}) + \\ + b_{\text{sc}}(\lambda, \mathbf{r}) \int\limits_{\Omega} I(\mathbf{r}, \boldsymbol{\omega}')\, g(\mathbf{r}, \boldsymbol{\omega}', \boldsymbol{\omega})\, d\boldsymbol{\omega}' + \Phi_0(\mathbf{r}, \boldsymbol{\omega}) \qquad (2)$$

integro–differential equations of radiative transfer (here $I(\mathbf{r}, \boldsymbol{\omega}, t)$ is the intensity at the point $\mathbf{r}$ in the direction $\boldsymbol{\omega}$ at time t). Any problem of the theory of optical signal transfer in scattering media can be reduced to the solution of Eq. (1) or Eq. (2) with corresponding boundary and initial (in the nonstationary case) conditions.[1]

The problem having been formulated in such a way is analogous to the problem of the theory of analysis of radio–engineering systems. Let us disregard the peculiarities of the physical nature of propagation channels of signals in optics and radio engineering and the physical processes accompanying the energy transfer from a source to a receiver. Then to study formally the channels with distributed scatterers the well–known method of the theory of linear–system analysis can be applied (the linearity of the atmospheric optical channels follows from the linearity of Eqs. (1) and (2) in the intensity). When the linear systems are invariant under spatial and (or) temporal shift of the sources, the fundamental principles of this approach to the solution of the problems of the theory of the optical signal transfer in the scattering media (as applied to the problems of laser detection and ranging, sounding, and communication) can be reduced to the following relations:

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$$P_{\text{out}}(t) = \int_0^{\infty} P_{\text{in}}(t')\, h(t - t')\, \mathrm{d}t' , \qquad (3)$$

$$\dot{K}_{\text{out}}(\gamma) = \dot{K}_{\text{in}}(\gamma)\, \dot{H}(\gamma) . \qquad (4)$$

Here $h(t)$ is the unit–pulse response of the channel of sounding, detection and ranging, and communication to the $\delta(t)$ pulse; $\dot{K}_{\text{out}}(\gamma)$, $\dot{K}_{\text{in}}(\gamma)$, and $\dot{H}(\gamma)$ are the complex spectral representations of the signals $P_{\text{out}}(t)$, $P_{\text{in}}(t)$, and $h(t)$, respectively,

$$h(t) = F^{-1}[\dot{H}(\gamma)] , \quad \dot{H}(\gamma) = F[h(t)] , \qquad (5)$$

F and F^{-1} are the direct and inverse one–dimensional Fourier transforms, $\dot{H}(\gamma)$ is usually called the transfer function of the system.

As applied to the vision theory

$$P_{\text{out}}(x, y) = \int_{-\infty}^{\infty}\!\!\int P_{\text{in}}(x - x', y - y')\, h(x', y')\, \mathrm{d}x'\mathrm{d}y' ; \qquad (6)$$

$$h(x, y) = F^{-2}[\dot{H}(\gamma, \omega)] , \qquad \dot{H}(\gamma, \omega) = F^{-2}[h(x, y)] \qquad (7)$$

here $h(x, y)$ is the unit–pulse response of the vision channel at the point (x, y) of the image plane to the source $\delta(x^*, y^*)$ in the object plane and F^2 and F^{-2} are the direct and inverse two–dimensional Fourier transforms. The fundamental principles of the linear–system approach (LSA) to the solution of the atmospheric optics problems in the case of dynamic phenomena observed through scattering media can be written down analogously.

However, before using the obvious advantages of the LSA for the study of regularities of the optical signal transfer in dispersed media it is necessary to take into account a number of factors of principal importance.

Let us refine the notion of a signal in optics of dispersed media as applied to the problems of: laser (optical) sounding and detection and ranging (a) and vision theory (b).

a) We will consider two possible definitions of optical signals and unit–pulse responses of channels with scattering.

The first definition.

$$P_{\text{in}} = I(\mathbf{r}^*, t, \boldsymbol{\omega}^*, \lambda) = I(\mathbf{r}^*, t, \boldsymbol{\omega}^*) , \qquad (8)$$

i.e., here the intensity emitted by the source located at the point $\mathbf{r}^*$ at the wavelength λ (below for brevity λ is omitted if there is no special need in it) in the direction $\boldsymbol{\omega}^*$ at time t is considered to be the input signal. Similarly,

$$P_{\text{out}} = I(\mathbf{r}^{**}, t, \boldsymbol{\omega}^{**}) . \qquad (9)$$

Taking into account the unambiguous relation between the transfer functions and the unit–pulse responses, we restrict ourselves to the examination of the latter in most cases. The unit–pulse response $h(t)$ has the form

$$h(t) = I(\mathbf{r}^{**}, \boldsymbol{\omega}^{**}, t \,;\, \mathbf{r}^*, \boldsymbol{\omega}^*, \delta(t)) , \qquad (10)$$

hence, it is the intensity at the point $\mathbf{r}^{**}$ in the direction $\boldsymbol{\omega}^{**}$ at the time t given that $P_{\text{in}} = I(\mathbf{r}^*, \boldsymbol{\omega}^*, \delta(t))$: $[h] = [I]/\text{s}$.

The second definition.

$$P_{\text{in}} = \int_{\Omega^*} I(\mathbf{r}^*, \boldsymbol{\omega}^*, t)\, \mathrm{d}\boldsymbol{\omega}^* \qquad (11)$$

is the power of the optical signal emitted by the source located at the point $\mathbf{r}$ (or in the region centered at the point $\mathbf{r}^*$).

$$P_{\text{out}} = \int_{\Omega^{**}} I(\mathbf{r}^{**}, \boldsymbol{\omega}^{**}, t)\, \mathrm{d}\boldsymbol{\omega}^{**} , \qquad (12)$$

i.e., the intensity of radiation incident on the aperture of the optical receiving system is considered to be the output signal. Then

$$h(t) = P_{\text{out}}(\mathbf{r}^{**}, t \,;\, \delta(t)) . \qquad (13)$$

The unit–pulse response in this case is the output signal power given that the input signal $P_{\text{in}}(t)$ of the form given by Eq. (11) is radiated as the δ–pulse: $[h] = [\text{power}]/\text{s}$.

b) By analogy with the one–dimensional case let us consider two definitions of the optical signals and unit–pulse responses of the vision systems (we have this notion earlier introduced for the systems formed by the optical system, the object plane, and the scattering medium separating them). Hereafter it is assumed that the object plane is the XOY plane of a Cartesian coordinate system.

The first definition.

$$P_{\text{in}} = I(x^*, y^*, \boldsymbol{\omega}^*) . \qquad (14)$$

Thus, the intensity emitted by the object plane at the point (x^*, y^*) in the direction $\boldsymbol{\omega}^*$ is considered to be the input signal. The radiation intensity transmitted through the scattering medium and arriving at the point $\mathbf{r}^{**}$ in the direction $\boldsymbol{\omega}^{**}$ is considered to be the corresponding output signal P_{out}

$$P_{\text{out}} = I(\mathbf{r}^{**}, \boldsymbol{\omega}^{**}) . \qquad (15)$$

The unit–pulse response can be now written down as

$$h(\mathbf{r}^{**}, \boldsymbol{\omega}^{**}) = I(\mathbf{r}^{**}, \boldsymbol{\omega}^{**}; \quad \delta(x - x^*), \delta(y - y^*), \boldsymbol{\omega}^*) \qquad (16)$$

and means the intensity at the point $\mathbf{r}^{**}$ in the direction $\boldsymbol{\omega}^{**}$ created by the single–point source (x^*, y^*) located in the object plane and radiating in the direction $\boldsymbol{\omega}^*$: $[h] = [I]/\text{m}^2$.

The second definition.

$$P_{\text{in}} = \int_{\Omega^*} I(\mathbf{r}^*, \boldsymbol{\omega}^*)\, G(\boldsymbol{\omega}^*, x^*, y^*)\, \mathrm{d}\boldsymbol{\omega}^* \qquad (17)$$

is considered to be the radiation intensity at the point $\mathbf{r}^*$ of the object plane. Here $G(\cdot)$ is the directional pattern of radiation emitted from each point of the surface. The unit–pulse response

$$h(\mathbf{r}^{**},\boldsymbol{\omega}^{**}) = \int_{\Omega^*} I(\mathbf{r}^{**},\boldsymbol{\omega}^{**};\delta(x-x^*),\delta(y-y^*))G(\boldsymbol{\omega}^*,x^*,y^*)\mathrm{d}\boldsymbol{\omega}^* \qquad (18)$$

keeps the meaning and dimensionality of Eq. (16) given that the input signal is the point of the object plane having coordinates (x^*, y^*) and radiating in accordance with the directional pattern $G(\omega^*, x^*, y^*)$.

First of all we note that the unit–pulse responses in the form of Eqs. (10) and (16) are identical to the Green's functions known in the linear transfer theory as fundamental solutions of the radiative transfer equation. For them relations (3)–(7) are obviously satisfied. Moreover, it also follows from the linearity of Eqs. (1) and (2) that the responses $h(\cdot)$ in the form of Eqs. (13) and (18) can be easily obtained if the Green's functions (10) and (16) are known. Now the prospects for the development of optical systems intended for various purposes are associated with the creation of opto–electronic complexes. When they operate under real conditions an external channel of propagation of optical signals is added to their optical and electronic channels. It is natural to provide a basis, if it is possible, for describing all these channels by the same notions, terms, and parameters that simplifies the procedure of adjusting different units and their optimal choice for achieving the formulated purposes. Taking into account that in radio engineering and theory of optical systems such an adjustment is achieved by means of the power system characteristics, it looks natural to describe the external channels of the opto–electronic systems at the same conceptual level, i.e., with the help of or on the basis of the characteristics written down in the form of Eqs. (13) and (18). In this case, for example, the signal spectrum at the output of the opto–electronic complex can be represented in the form

$$\dot{K}_{\text{out}} = \dot{K}_{\text{in}}\,\dot{H}_{\text{ec}}\,\dot{H}_{\text{oc}}\,\dot{H}_{\text{eu}}\,, \tag{19}$$

where $\dot{H}_{\text{ec}}$ is the optical transfer function of the external channel of the complex, $\dot{H}_{\text{oc}}$ is the transfer function of its optical channel, and $\dot{H}_{\text{eu}}$ is the transfer function of its electronic unit. We note that Eqs. (10) and (16) are written down for radiation intensities at the input of the optical system. On the assumption of the linearity of the optical system its effect on the signal characteristics is accounted for by the factor $\dot{H}_{\text{oc}}$ in Eq. (19). In other words, it follows from Eq. (19) that we succeeded in separating the problem of an account of the effects of the optical system and of the scattering medium on $\dot{K}_{\text{out}}$ into two independent problems which can be solved separately. That is, to take into account the effect of the scattering medium on $\dot{K}_{\text{out}}$, we must only find the solution of the transfer equation (the Green's function) at the point $\mathbf{r}^{**}$ in the direction ω^{**}, and take into account the effect of the real optical system via its unit–pulse response or optical transfer function. It appears that even on the basis of the above assumptions the procedure of constructing the solution of Eq. (19) accounting for Eqs. (10), (13), and (18) is not always applicable. This is due to the fact that in the process of propagation of the information–bearing optical signal through the scattering medium the latter becomes a three–dimensional source of a background signal of secondary emission. Since the optical system is usually adjusted to the source of a valid signal and is characterized by the finite depth of sharpness in the image space, even the ideal optical system can introduce the distortions which were ignored in Eq. (19). It is impossible to take this effect into account in terms of the Green's functions in the form of Eqs. (10) and (16). It will be done, however, if we determine the Green's functions as functions of the coordinate l along the direction ω^{**}, i.e.,

$$h(\mathbf{r}^{**}, \omega^{**}, l, t) = I(\mathbf{r}^{**}, \omega^{**}, l, t, \mathbf{r}^*, \omega^*, \delta(t))\,; \tag{20}$$

$$h(\mathbf{r}^{**}, \omega^{**}, l, x^*, y^*) = I(\mathbf{r}^{**}, \omega^{**}, l; \delta(x-x^*), \delta(y-y^*), \omega^*). \tag{21}$$

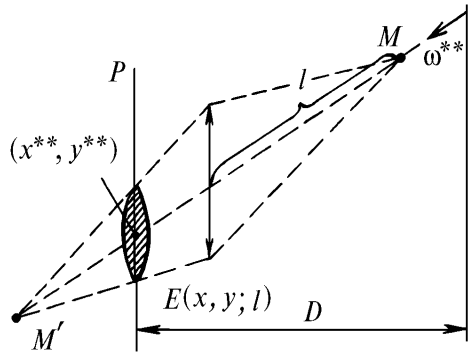

FIG. 1. Geometric diagram. P is the plane of ideal (unblurried) image of the point M; (x^*, y^*) are the coordinates of this image; M′ is the point of maximum illumination.

Let the ideal optical system have a finite depth of sharpness in the image space and the boundary of this region nearest to the system be at the distance D from it. Let us place the single–point source (center of scattering) on a ray in the direction ω^{**}. Then its radiation intensity incident on the input pupil of the system is transformed by it into the illumination of the object plane according to the rule:

$$\beta(l; x^{**}, y^{**}) = \begin{cases} 1, & l > D\,; \\ E(l; x^{**}, y^{**}) \Big/ \int\int E(\cdot)\,\mathrm{d}x^{**}\mathrm{d}y^{**}\,, & l < D, \end{cases}$$

where $E(l, x, y)$ is the distribution of the illumination over the blur circle.

If the values of $I(\mathbf{r}^{**}, \omega^{**}; l)$ at each point of the ray in the direction ω^* are known and $\beta(l)$ is specified then by summing over the intensities $I(\mathbf{r}^{**}, \omega^{**}; l)$ coming from all points of the ray with the weight $\beta(l)$ we will determine the unit–pulse response (to the point perturbation in the object plane at the point (x^{**}, y^{**}) in the direction ω^{**}) of the ideal optical system taking into account the finite depth of sharpness in the image space. Taking this into account Eq. (19) can be written down as

$$K_{\text{out}} = K_{\text{in}}\,H_{\text{vs}}\,H_{\text{oc}}\,H_{\text{eu}}\,,$$

where H_{vs} is the optical transfer function of the vision system formed by the medium, the object plane, and the ideal optical system.

Thus, the regularities of the effect of the scattering medium on the transfer of optical signals can be studied by the methods of the Green's function in the form of Eqs. (20) and (21) or of the linear–systems characteristics determined by Eqs. (13) and (18). We stress that the above–given definitions of signals, unit–pulse characteristics, and all the relations are valid only for the single point in the object plane of the optical system (or for the intensity $I(\mathbf{r}^{**}, \omega^{**})$ or $I(\mathbf{r}^{**}, \omega^{**}; l)$). However, making use of the notion of isoplanarity of images we can generalize all the results obtained at this point to isoplanar area comprising it. Then the evaluation of the dimensions of isozones turns out to be a problem of particular importance for the theory of transfer and recording of optical signals through the scattering media. These problems were studied in Ref. 2.

REFERENCES

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