DISTRIBUTION LAWS OF ATMOSPHERIC TRANSMISSION IN THE IR ON HORIZONTAL PATHS

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Analyzing the experimental data on the atmospheric attenuation of laser radiation at $\lambda = 10.6$ μm *we obtained the distribution functions of atmospheric transmission. These distribution functions are compared with the same dependences at* $\lambda = 0.55$ *and* 1.06 μm.

The atmospheric laser systems (ALS) intended for different purposes are being increasingly employed in the national economy.[1] To a large extent the efficiency of the ALS performance is determined by the state of the atmosphere. That circumstance stimulates active development of techniques for forecasting the optical weather[2] whose important characteristics is the atmospheric transmission T, which serves as an input parameter in assessing the reliability of the ALS performance.[3] Atmospheric transmission is directly related to the coefficient of attenuation of laser radiation.

As well known,[4] that relation at λ = 0.55 μm is given by the formula:

$$T(0.55) = [-\alpha(0.55)L] = \exp\left(-\frac{3.91}{S_m}L\right), \qquad (1)$$

where S_m is the meteorological visibility range, L is the path length, and α(0.55) is the coefficient of attenuation of radiation.

As for near–IR within the atmospheric transparency windows, the same dependence, associated primarily with scattering of radiation, may be represented in the form[4]:

$$T(\lambda) = \exp\left[-L\frac{3.91}{S_m}\left(\frac{0.55}{\lambda}\right)^q\right], \qquad (2)$$

where $q = 0.585 S_m^{1/3}$ for $S_m < 6$ km and $q = 1.3$ for $S_m > 6$ km and λ is the wavelength.

It follows from Eqs. (1) and (2) that studying both the temporal variations and the statistical characteristics of S_M is quite important. Such studies were performed in different geographic regions on the basis of long term observations of S_m (see Refs. 5–7). Using the dependences obtained for S_m in the Leningrad district the distribution laws of the atmospheric transmission at λ = 0.55 μm and λ = 1.06 μm were directly retrieved. They were approximated well by the Weibull modified distribution on the paths of the lengths > 5 km in length.[8]

However applying Eq (2) in middle–IR results in significant errors in the estimated attenuation of radiation.[9] Therefore, to obtain the statistical characteristics of atmospheric transmission at λ = 10.6 μm, we used the (experimental data on the attenuation of a CO_2 laser radiation. Measurements were performance in the Leningrad district.[10] An exponential dependence of the attenuation was observed in all the measurement range, so the corresponding laws of transmission distributions were compared with those in the visible and IR ranges for different path lengths.

To measure the attenuation of the CO_2 laser radiation, the IKAU–I IR atmospheric device was employed.[11] It is built around a multipaas optical system, which permits to obtain a number of the paths 400–4000 m in length by means of multiple reflections of radiation propagated through a homogeneous 100 m horizontal base.

The technique of such measurements was outlined in Ref. 11; the experiments were performed for three years and in different seasons; 19062 readings were accumulated during the operating time of 170 hours. Continuous measurement runs stretched from 30 minutes to 6 hours, depending on the given situation along the beampath and the operating conditions of the instruments. Practically all data arrays referred to the three lines in the spectrum of lasing of a CO_2 laser: P_{20}, P_{22}, and P_{24}.

The coefficient of attenuation of the CO_2–laser radiation is known to be determined[12] by the sum of contribution of the molecular absorption due to atmospheric carbon dioxide, α_{CO_2} and by the H_2O lines and due to the water vapor continuum α_{H_2O} as well as of the aerosol attenuation α_a. We found that estimated contribution of the selective absorption by the H_2O lines remains within 5% of the contribution of CO_2, therefore we neglected this effect in our subsequent estimates. The contribution of the molecular absorption due to atmospheric CO_2 was estimated based on the CO_2 concentrations measured with an analytic gas analyzer. Its value was (330 ± 5) ppm during our experiments. The value of α_{CO_2} for the principal lasing line of the CO_2 laser P_{20} was determined approximately from the formula[13]

$$\alpha_{CO_2} \simeq \frac{6.36\cdot 10^5}{\theta^{3/2}}\exp\left(-\frac{2230}{\theta}\right), \qquad (3)$$

where θ is the air temperature in Kelvins.

TABLE I. Statistical characteristics of the parameters of the data run.

Serial number of data run		Interval of sampling of the given parameter	Mean values and rms deviations of the meteorological parameters								Mean values and rms deviations of the attenuation coefficients			
			$\bar{\theta}$°C	σ_θ	$\bar{P}$, hPa	σ_p	$\bar{f}$, %	σ_f	S_m, km	S_{s_m}	$\bar{\alpha}_\Sigma$, km^{-1}	σ_{α_Σ}	$\bar{\alpha}_a$, km^{-1}	σ_{α_a}
		Sampling of p (hPa)												
I		< 5	0.6	2.2	4.5	0.3	71.5	8.1	17	12	0.09	0.05	0.07	0.05
II		5 – 10	6.0	5.7	6.7	1.3	74.5	8.6	15	6	0.12	0.06	0.08	0.05
III		10 – 15	17.9	3.8	12.9	1.3	64.5	6.3	31	7	0.18	0.09	0.06	0.08
IV		> 15	23.1	2.3	17.0	0.9	60.4	7.1	25	11	0.20	0.03	0.02	0.02
		Sampling of f (%)												
V		50 – 70	11.5	8.8	9.6	5.2	61.1	3.2	26	9	0.12	0.07	0.05	0.04
VI		70 – 90	1.5	10.5	6.6	4.6	78.3	4.6	17	5	0.13	0.05	0.07	0.04
VII		90 – 100	–2.3	8.6	5.7	3.0	94.4	2.2	8	6	0.15	0.06	0.11	0.04
		Sampling of S_m (km)												
VIII		10 – 20	12.8	6.9	12.5	2.8	84.7	8.3	18	2	0.22	0.03	0.10	0.05
IX	Summer	20 – 100	14.7	6.4	11.1	3.8	63.4	12.1	35	11	0.13	0.06	0.04	0.03
X		1 – 10	–0.6	9.6	5.5	3.8	83.0	4.6	7	3	0.14	0.06	0.10	0.04
XI	Winter	10 – 20	–1.9	3.7	4.2	0.9	74.7	8.2	13	2	0.09	0.02	0.07	0.02

TABLE II. Pairwise of the correlation coefficients.

Serial number of run	Pairwise of the correlation coefficients							
	$R_{f\theta}$	R_{fp}	$R_{p\alpha_\Sigma}$	$R_{\theta\alpha_\Sigma}$	$R_{p\alpha_a}$	$R_{f\alpha_a}$	$R_{s_m\alpha_\Sigma}$	$R_{s_m\alpha_a}$
I	–0.88	–0.004	0.33	–0.26	0.30	0.46	–0.69	–0.69
II	–0.86	–0.14	0.46	–0.27	0.02	0.50	–0.59	–0.68
III	–0.93	0.45	0.48	–0.42	–0.10	0.32	–0.53	–0.61
IV	–0.89	0.07	0.60	0.28	0.21	0.52	–0.40	–0.23
V	0.09	0.32	0.79	0.66	–0.40	–0.009	–0.28	–0.75
VI	–0.53	–0.31	0.71	0.70	–0.52	–0.07	–0.47	–0.70
VII	0.65	0.63	0.60	0.53	0.13	0.50	–0.30	–0.09
VIII	–0.94	–0.82	0.28	–0.22	–0.77	0.82	–0.43	–0.50
IX	–0.29	0.34	0.83	0.51	0.04	0.25	–0.39	–0.73
X	–0.50	–0.23	0.70	–0.55	–0.17	0.17	–0.36	–0.34
XI	–0.17	0.41	0.27	–0.39	–0.52	–0.05	–0.28	–0.24

In that case the coefficient of the continuous attenuation is given by the difference between the experimentally measured total attenuation and the calculated value of α_{CO_2}:

$$\alpha_\Sigma = \alpha_{exp} - \alpha_{CO_2} = \alpha_{H_2O} + \alpha_a\,. \qquad (4)$$

To identify purely aerosol attenuation in α_Σ, the coefficient of absorption by the water vapor continuum was calculated using the well–known Burch formula[14]:

$$\alpha_{H_2O}(10.6) = 0.177\frac{p^2}{\theta}\exp\left(\frac{1745}{\theta} - \frac{1745}{296}\right), \qquad (5)$$

where p is the partial pressure of water vapor, in hPa, and α_{H_2O} is taken in km^{-1}.

The majority of experimental points (18387 readings) was obtained in haze of various density. The total array of data was *a priori* divided[15] into 11 runs corresponding to the given intervals of the partial pressure of the water vapor p, of the relative humidity f, and of the meteorological visibility range S_m, and for these runs the average values of the coefficient of attenuation of laser radiation were calculated in the process of the of preliminary statistical processing of the data together with the meteorological parameters, their rms deviations (see Table I), and their pairwise of correlation coefficients (Table II).

It can be seen from Table II that the highest pairwise of the coefficient correlation coefficient is $P_{p\alpha_\Sigma}$. This fact testifies to a significant effect of water vapor continuum an the attenuation of the CO_2 laser radiation. Meanwhile the aerosol attenuation coefficient is only weakly related to water vapor pressure.

The physically explainable tendency for $\bar{\alpha}_a$ to increase at lower S_m as well as the decrease of the pairwise of the correlation coefficients $R_{S_m\alpha_\Sigma}$ and $R_{S_m\alpha_a}$ at higher partial pressure of the water vapor may be noted too. In other words the contribution of the aerosol attenuation to the total one became more pronounced at lower partial pressures of the water vapor.

The data in Table II show that after substraction of α_{H_2O} the correlations between α_a and S_m become quite significant, so that S_m may be used to estimate α_a in the longwave atmospheric transparency window. To this end it is expedient to introduce a coefficient A relating α_a to S_m:

$$\alpha_a = \frac{A}{S_m} . \qquad (6)$$

The following values of A were obtained in the experiment: $A = 0.28$ for the summer hazes and $A = 0.98$ for the winter hazes.

Although in principle the coefficient A may vary with S_m (e.g. following a non–linear dependence[16]), a large spread of α_a makes it possible to use the average attenuation coefficient for the quantitative estimation of the characteristics of attenuation of the IR laser radiation.

The results shown illustrate individual statistical characteristics of α_Σ and α_a. However to forecast the level of reliability of the ALS performance in a continuous regime one needs certain generalized statistical parameters which should characterize the state of the medium through which the signal propagates. To this end it is expedient to relate α_Σ and α_a to the atmospheric transmission T, so as to obtain certain distribution laws of this variable. Thus the arrays of data on the atmospheric transmission, related to the attenuation coefficient α via the Bouguer law

$$T = \exp(-\alpha L)$$

were subjected to further statistical processing on a computer. Such computer processing followed the technique developed in Refs. 5, 7, and 8, and the following possible distributions were tested: truncated Weibull, modified arcsine, truncated exponential, truncated Rayleigh, truncated Maxwell, truncated normal, beta, and truncated log–normal. That choice accounted for the fact that the random value T could vary from 0 to 1.

The statistical characteristics of the atmospheric were on a computer calculated transmission for the various hypothetical path lengths chosen analogously to Ref. 8 (L = 1,5, 10, 20, and 50 km). They incorporated the average value $\overline{m}_T$, the variance, $\overline{D}_T$, the unbiased and consistent estimates of the third $\overline{\mu}_3$ and the fourth $\overline{\mu}_4$ central sampling moments of T. Since the family of the Pearson distributions was used to find the best approximation of the actual distributions of the atmospheric transmission, the parameters of that family were calculated $\overline{\beta}_1 = \overline{\gamma}_1^2$ and $\overline{\beta}_2 = \overline{\gamma}_2 + 3$, where $\overline{\gamma}_1 = \overline{\mu}_3 \overline{D}_T^{-3/2}$ is the asymmetry coefficient, and $\overline{\gamma}_2 = \overline{\mu}_4 \overline{D}_T^{-2} - 3$ is the coefficient of excess.

TABLE III. Statistical characteristics of the atmospheric transmission at l = 10.6 μm *with an account of the total attenuation.*

L, km	$\overline{m}$	$\overline{D}$	$\overline{\mu}_3$	$\overline{\mu}_4$	$\overline{\beta}_1$	$\overline{\beta}_2$	$\overline{m}_T/\sqrt{\overline{D}}$
1	$8.54\cdot10^{-1}$	$1.84\cdot10^{-2}$	$-1.08\cdot10^{-1}$	$8.32\cdot10$	18.65	24.44	6.29
5	$5.20\cdot10^{-1}$	$3.39\cdot10^{-2}$	$-1.08\cdot10^{-3}$	$3.96\cdot10$	0.103	3.44	2.82
10	$2.64\cdot10^{-1}$	$2.83\cdot10^{-2}$	$-1.08\cdot10^{-3}$	$2.19\cdot10$	0.741	2.73	1.57
20	$9.86\cdot10^{-2}$	$1.37\cdot10^{-2}$	$-1.08\cdot10^{-3}$	$8.85\cdot10$	2.52	4.75	0.844
50	$1.33\cdot10^{-2}$	$9.91\cdot10^{-4}$	$-1.08\cdot10^{-4}$	$2.11\cdot10$	14.18	21.48	0.424

Table III lists the statistical characteristics of the atmospheric transmission, calculated with an account of the total attenuation of laser radiation by the water vapor continuum and by the aerosol (i.e. of α_Σ); both the experimental and theoretical distribution functions $F(T)$ of the atmospheric transmission, T for that case are plotted in Fig. 1.

The analysis showed that for L = 10 km the truncated gamma–distribution fitted best to describe the empirical distributions of the atmospheric transmission (see curves 1′, 2′, 3′, 4′, and 5′ in Fig. 1).

When L = 1 and L = 5 km the approximation by the gamma–distribution may be used for the estimating calculations when $T \geq 0.6$ and $T \geq 0.3$, respectively. It should be noted that when L = 5 km the truncated gamma–distribution is close in values of the parameters to the normal one (see Table III and curve 7 in Fig. 1), and may also be applied to approximate the empirical distributions.

The corresponding data array on the atmospheric transmission associated with the purely aerosol component of attenuation (α_a) was also subjected to the same statistical analysis. The corresponding statistical characteristics of atmospheric transmission are presented in Table IV and in Fig. 2. The results of comparison of the data in Tables III and IV demonstrates a significant discrepancy in all the statistical characteristics expecially when the path length increases. In particular, the behaviour of the average transmission, $\overline{m}_T$, testified to a significant contribution of water vapor to the attenuation of the CO_2–laser radiation.

TABLE IV. Statistical characteristics of atmospheric transmission at λ = 10.6 μm *taking into account aerosol attenuation only.*

L, km	$\overline{m}$	$\overline{D}$	$\overline{\mu}_3$	$\overline{\mu}_4$	$\overline{\beta}_1$	$\overline{\beta}_2$	$\overline{m}_T/\sqrt{\overline{D}}$
5	$7.04\cdot10^{-1}$	$3.73\cdot10^{-2}$	$-1.83\cdot10^{-3}$	$2.57\cdot10^{-3}$	$6.44\cdot10^{-2}$	1.84	3.64
10	$5.35\cdot10^{-1}$	$7.04\cdot10^{-2}$	$9.61\cdot10^{-4}$	$7.73\cdot10^{-3}$	$2.63\cdot10^{-3}$	1.55	2.01
20	$3.59\cdot10^{-1}$	$8.72\cdot10^{-2}$	$1.07\cdot10^{-2}$	$1.18\cdot10^{-2}$	$1.75\cdot10^{-1}$	1.56	1.21
50	$1.91\cdot10^{-1}$	$6.12\cdot10^{-2}$	$1.38\cdot10^{-2}$	$7.73\cdot10^{-2}$	$8.32\cdot10^{-1}$	2.06	7.72

It follows from the analysis that the truncated Weibull distribution fits best to describe the empirical distributions of the atmospheric transmission in the case of the aerosol attenuation on the paths of different length, starting from

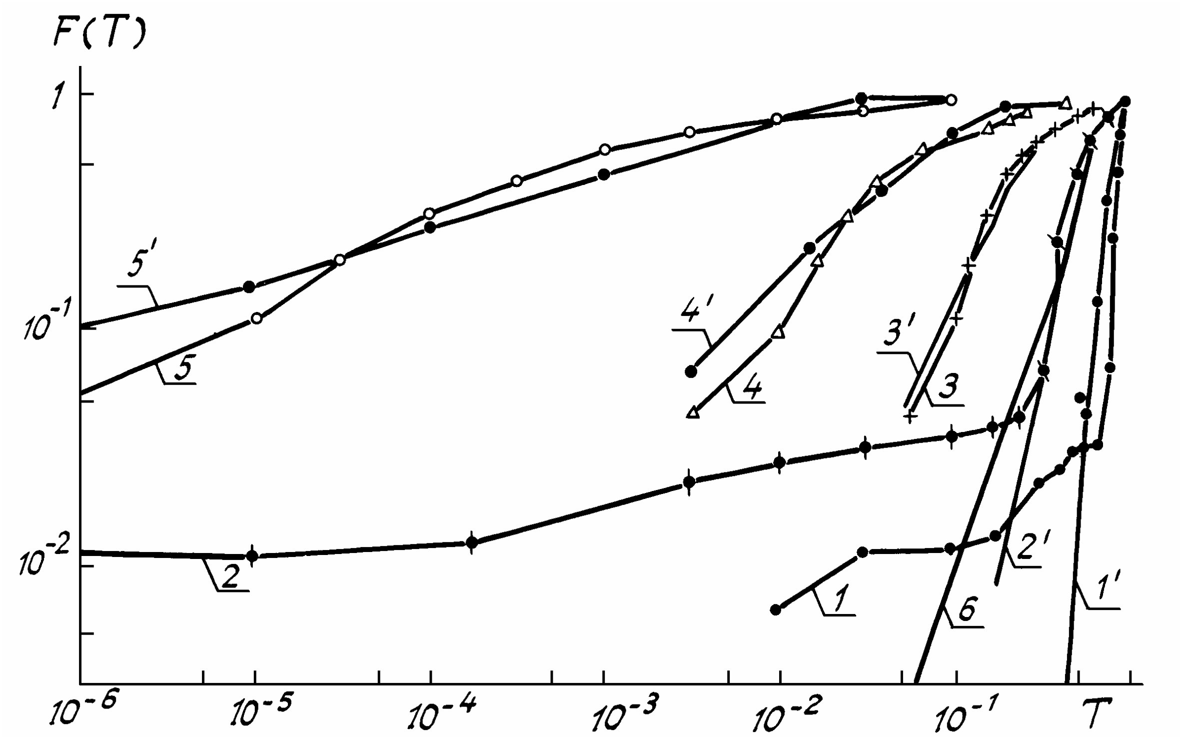

FIG. 1. Experimental and theoretical distribution functions of the atmospheric transmission λ = 10.6 μm *with an account of the total attenuation of laser radiation. Calculations from the experimental data: 1) L* = 1; *2) L* = 5; *3) L* = 10; *4) L* = 20; *5) L* = 50 km. *Theory: 1′–5′) gamma–distribution; 6) truncated normal distribution.*

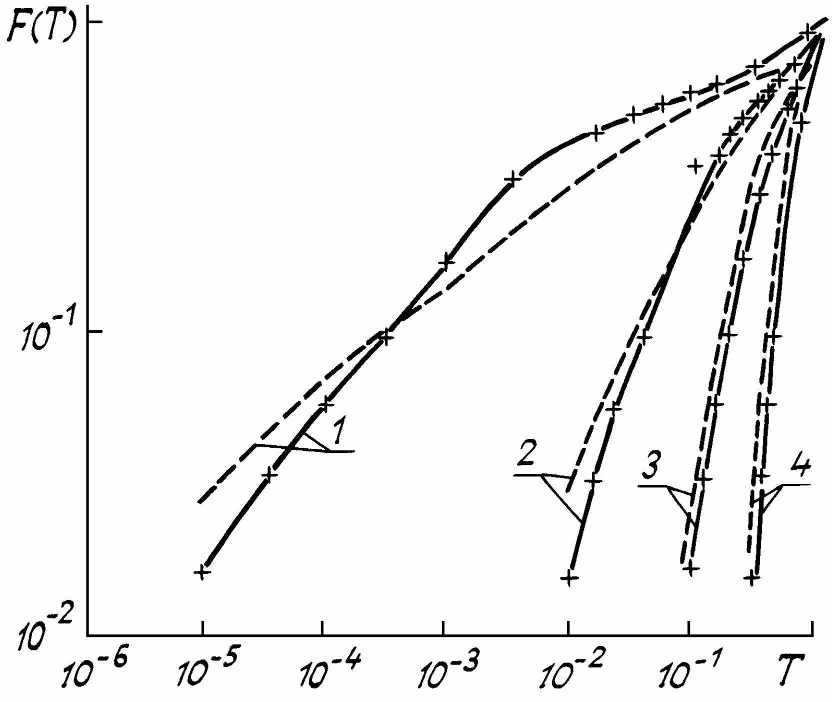

FIG. 2. Experimental and theoretical distribution functions of atmospheric transmission at λ = 10.6 μm *taking into account aerosol attenuation only: 1) L* = 50; *2) L* = 20; *3) L* = 10; *4) L* = 5. *Solid lines show experimental distributions and dashed lines theoretical ones*

$L \geq 5$ km (see Fig. 2). The same distribution describes well the empirically observed variations of the atmospheric transmission at $\lambda = 0.55$ μm and $\lambda = 1.06$ μm,[8] on the paths of length $L > 5$ km. However the adjustable parameter of the Weibull distribution in the middle–IR range $r \simeq 2\frac{\lambda}{Z}$ differs from the corresponding values in the visible and near–IR ranges. That difference is physically explained by different ratios of the size of the aerosol particles and the radiation wavelength in these spectral ranges.

Thus the following conclusions may be formulated:

1. Stable correlations have been found between the parameters determining the state of the atmosphere and the radiation attenuation at $\lambda = 10.6$ μm.

2. The distribution laws have been obtained for atmospheric transmission at $\lambda = 10.6$ μm for the North–Western region of the European part of the USSR.

3. The truncated Weibull distribution which approximates quite well the empirical distributions of the atmospheric transmission in the visible and near–IR spectral ranges is also applicable for describing the distribution of the atmospheric transmission in the middle–IR range, associated with the aerosol attenuation of radiation.

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