APPROXIMATE SOLUTION OF THE TRANSFER EQUATION: THREE-FLUX APPROXIMATION

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The solution of the transfer equation on the basis of the three-flux approximation is studied. The solution is constructed in two stages. At the first stage the fluxes are determined and at the second stage the radiation intensity is determined. A general approximate analytical expression is presented for the intensity of the atmospheric haze. The results of calculations performed with different initial angular distributions of the intensity, given at the first stage, are analyzed. It is shown that the solution with the initial approximation proposed in this paper satisfies the transfer equation more accurately than do the solutions obtained on the basis of the well-known approaches of R. Turner and Y. Kaufman.

Data from remote sounding of the earth's surface in the optical region of the spectrum are distorted by multiple scattering and absorption of solar radiation in the atmosphere. For this reason, to improve the accuracy of the identification of aerospace images an atmospheric correction, based on the solution of the radiation transfer equation in the system "atmosphere underlying surface", must be introduced in the images.[1,2]

Approximate methods for calculating the radiation transfer are of great importance for practical implementation of the procedure of introducing atmospheric corrections in the fluxes of aerospace video information, since highly accurate methods require significant amounts of computer time and large immediate-access computer memories. In addition, the use of highly accurate methods is justified when the optical parameters of the atmosphere are known with high accuracy at the time the aerospace images are recorded. However synchronous measurements of these parameters is a quite difficult technical problem, and usually such data are not available.

An approximate analytical expression for the intensity of the scattered radiation can be obtained by solving the transfer equation with an approximate analytical expression which can be obtained for the source function under different simplifying assumptions.[3–5] In this paper we examine a solution of the transfer equation on the basis of the three-flux approximation. The solution is constructed in two stages: at the first stage the upward and downward fluxes are determined and at the second stage the intensity is calculated.

The radiation transfer equation for a plane-parallel uniform atmosphere with optical thickness τ_0 has the form

$$\mu \frac{dI}{d\tau}(\tau, \mu, \varphi) = I(\tau, \mu, \varphi) - \frac{\omega_0}{4\pi} \times \int_0^{2\pi} d\varphi' \int_{-1}^{1} d\mu' P(\mu, \varphi, \mu', \varphi') I(\tau, \mu', \varphi') - \frac{\omega_0 S}{4} e^{-\tau/\mu_0} \cdot P(\mu, \varphi, -\mu_0, \varphi_0 + \pi), \tag{1}$$

where $I(\tau, \mu, \varphi)$ is the intensity of the scattered radiation; $\mu = \cos\theta$, $\mu_0 = \cos\theta_0$; θ, φ are the vertical and azimuthal angles of propagation of the radiation; θ_0 and φ_0 are the vertical and azimuthal angles for the direction toward the sun; ω_0 is the single scattering albedo; $P(\mu, \varphi, \mu', \varphi')$ is the scattering phase function for scattering from the direction μ', φ' into the direction μ, φ; τ is the optical depth; and, πS is the illumination at the top boundary of the atmosphere for an area perpendicular to the direction of propagation of the direct solar radiation. The boundary conditions for the intensity of atmospheric haze (there is no underlying surface) had the following form:

for downward radiation:

$$I(0, \mu, \varphi) = 0;$$

for upward radiation: (2)

$$I(\tau_0, \mu, \varphi) = 0.$$

At the first stage of the solution of the problem the intensity of the upward-scattered radiation $\hat{I}_1(\tau, \mu, \varphi)$ and the intensity of the downward-scattered radiation $\hat{I}_2(\tau, \mu, \varphi)$ are represented in the form

$$\hat{I}_{1,2}(\tau, \mu, \varphi) = E_{1,2}(\tau) i_{1,2}(\mu, \varphi), \tag{3}$$

where the upward-scattered flux $E_1(\tau)$ and the downward-scattered flux $E_2(\tau)$ reflect the vertical dependence of the intensity, while the functions $i_1(\mu, \varphi)$ and $i_2(\mu, \varphi)$ reflect the relative angular dependence. It is assumed that these functions do not depend on τ and are normalized as follows:

$$\int_0^{2\pi} d\varphi \int_0^1 d\mu\mu \hat{I}_{1,2}(\tau, \mu, \varphi) = E_{1,2}(\tau), \text{ i.e.,}$$

$$\int_0^{2\pi} d\varphi \int_0^1 d\mu\mu i_{1,2}(\mu, \varphi) = 1. \tag{4}$$

Here and below, for directions toward the lower hemisphere, μ is replaced by $-\mu$ and for this reason $\mu > 0$.

Assume that the initial angular distribution of the intensity $i_{1,2}(\mu, \varphi)$ is known. Substituting the approximate expressions for the intensity (3) into the transfer equation (1) and carrying out the integration over angles within the upper and lower hemispheres we obtain a system of differential equations for the upward and the downward diffuse fluxes:

$$\begin{cases} \dfrac{dE_1(\tau)}{d\tau} = \alpha_1 E_1(\tau) - \gamma_2 E_2(\tau) - \kappa_1 E_0(\tau); \\ \dfrac{dE_2(\tau)}{d\tau} = -\alpha_2 E_2(\tau) + \gamma_1 E_1(\tau) + \kappa_2 E_0(\tau) \end{cases} \tag{5}$$

with the boundary conditions $E_1(\tau_0) = E_2(0) = 0$, where $E_0(\tau) = \pi S\mu_0 \exp(-\tau/\mu_0)$ is the flux of the direct solar radiation;

$$\alpha_{1,2} = \beta_{1,2} + \gamma_{1,2},$$

$$\beta_{1,2} = (1 - \omega_0) \int_0^{2\pi} d\varphi \int_0^1 d\mu \, i_{1,2}(\mu, \varphi);$$

$$\gamma_{1,2} = \frac{\omega_0}{4\pi} \int_0^{2\pi} d\varphi \int_0^1 d\mu \int_0^{2\pi} d\varphi' \int_0^1 d\mu' P(-\mu, \varphi; \mu', \varphi') \times$$

$$\times \, i_{1,2}(\mu', \varphi');$$

$$\kappa_1 = \frac{\omega_0}{4\pi\mu_0} \int_0^{2\pi} d\varphi \int_0^1 d\mu \, P(\mu, \varphi, -\mu_0, \varphi_0 + \pi),$$

$$\kappa_2 = \omega_0/\mu_0 - \kappa_1.$$

The coefficients $\gamma_{1,2}$ and $\beta_{1,2}$ are the scattering and absorption coefficients for the upward and downward diffuse fluxes; and, κ_1 and κ_2 are the scattering coefficients for the direct solar flux in the upward and downward diffuse fluxes, respectively. It should be noted that the system of differential equations (5) is a particular case of the general system of different equations obtained on the basis of the four-flux theory.[6] In this case there is no upward-directed flux, since there is no reflecting surface.

If the two conditions $\omega_0 = 1$ and $\gamma_1 = \gamma_2$ are not satisfied at the same time, then the solution of the system (5) has the form[7]

$$E_1(\tau) = A\, e^{k_1\tau} + B\, e^{-k_2\tau} + C\, e^{-\tau/\mu_0};$$

$$E_2(\tau) = (1/\gamma_2)\left(AFe^{k_1\tau} + BGe^{-k_2\tau} + H\, e^{-\tau/\mu_0}\right),$$

where

$$C = \frac{4\pi S\omega_0\mu_0^2}{(a\mu_0 - 2)^2 - \lambda^2\mu_0^2} \left(\frac{\kappa_1}{\pi S\omega_0} \left[\frac{1}{\mu_0} - \alpha_2\right] - \gamma_2 \right);$$

$$A = \left(-H\, e^{-k_2\tau_0} + C\, G\, e^{-\tau_0/\mu_0} \right)/D;$$

$$B = \left(H\, e^{k_1\tau_0} - C\, F\, e^{-\tau_0/\mu_0} \right)/D;$$

$$D = F\, e^{-k_2\tau_0} - G\, e^{k_1\tau_0}, \qquad a = \alpha_2 - \alpha_1,$$

$$\lambda^2 = (\alpha_1 + \alpha_2)^2 - 4\gamma_1\gamma_2;$$

$$F = \alpha_1 - k_1, \; G = \alpha_1 + k_2, \quad H = (\alpha_1 + 1/\mu_0)C - \kappa_1;$$

$$k_1 = (\lambda - a)/2, \qquad k_2 = (\lambda + a)/2.$$

In the case when the conditions $\omega_0 = 1$ and $\gamma_1 = \gamma_2$ are satisfied at the same time the solution has the simpler form

$$E_1(\tau) = C_1\tau + C_2 + \mu_0^2 Z e^{-\tau/\mu_0};$$

$$E_2(\tau) = C_1(\tau - 1/\alpha) + C_2 + (\mu_0^2 Z - \pi S\mu_0) e^{-\tau/\mu_0}$$

where

$$C_1 = \alpha\left(\mu_0^2 Z\left(1 - e^{-\tau_0/\mu_0}\right) - \pi S\mu_0\right)/(1 + \alpha\tau_0);$$

$$C_2 = \left(\alpha\pi S\mu_0 - \mu_0^2 Z\left(\alpha\tau_0 + e^{-\tau_0/\mu_0}\right)\right)/(1 + \alpha\tau_0);$$

$$Z = \pi S(\kappa_1 - \alpha), \quad \alpha = \alpha_1 = \alpha_2 = \gamma_1 = \gamma_2.$$

Using the analytical expressions derived above for upward and downward fluxes, we substitute the approximate expressions for the intensity(3) into the multiple scattering integral on the right side of the transfer equation (1). Thus at the second stage of the solution we obtain two separate differential equations for the intensities of the upward and downward radiation that can also be solved analytically:

$$\mu \frac{dI_1}{d\tau}(\tau, \mu, \varphi) = I_1(\tau, \mu, \varphi) - U_1(\mu, \varphi)e^{k_1\tau} -$$

$$- W_1(\mu, \varphi)e^{-k_2\tau} - V_1(\mu, \varphi)e^{-\tau/\mu_0}; \tag{6}$$

$$-\mu \frac{dI_2}{d\tau}(\tau, \mu, \varphi) = I_2(\tau, \mu, \varphi) - U_2(\mu, \varphi)e^{k_1\tau} - W_2(\mu, \varphi)e^{-k_2\tau} - V_2(\mu, \varphi)e^{-\tau/\mu_0}, \quad (7)$$

where

$$U_{1,2}(\mu, \varphi) = A\left(P_{1,2}(\mu, \varphi) + Q_{1,2}(\mu, \varphi)F/\gamma_2\right);$$

$$W_{1,2}(\mu, \varphi) = B\left(P_{1,2}(\mu, \varphi) + Q_{1,2}(\mu, \varphi)G/\gamma_2\right);$$

$$V_{1,2}(\mu, \varphi) = CP_{1,2}(\mu, \varphi) + Q_{1,2}(\mu, \varphi)H/\gamma_2 + \frac{\omega_0 S}{4} P\left(\pm\mu, \varphi, -\mu_0, \varphi_0 + \pi\right);$$

$$P_{1,2}(\mu, \varphi) = \frac{\omega_0}{4\pi}\int_0^{2\pi} d\varphi' \int_0^1 d\mu' \, P\left[\mu, \varphi, \pm\mu', \varphi'\right) \times i_1(\mu', \varphi');$$

$$Q_{1,2}(\mu, \varphi) = \frac{\omega_0}{4\pi}\int_0^{2\pi} d\varphi' \cdot \int_0^1 d\mu' \, P\left(\mu, \varphi, \pm\mu', \varphi'\right) \times i_2(\mu', \varphi'),$$

here the plus sign in front of μ corresponds to the index 1 and the minus sign corresponds to the index 2, and the boundary conditions for Eqs. (6) and (7) are determined by the equalities (2).

The solutions of the differential equations have the form

$$I_1(\tau, \mu, \varphi) = \frac{U_1(\mu, \varphi)}{1 - k_1\mu}\left[e^{k_1\tau} - e^{k_1\tau_0}\cdot e^{-(\tau_0-\tau)/\mu}\right] + \frac{W_1(\mu, \varphi)}{1 + k_2\mu}\left[e^{-k_2\tau} - e^{-k_2\tau_0}\cdot e^{-(\tau_0-\tau)/\mu}\right] + \mu_0\frac{V_1(\mu, \varphi)}{\mu + \mu_0}\left[e^{-\tau/\mu_0} - e^{-\tau_0/\mu_0}\cdot e^{-(\tau_0-\tau)/\mu}\right]; \quad (8)$$

$$I_2(\tau, \mu, \varphi) = \frac{U_2(\mu, \varphi)}{1 + k_1\mu}\left[e^{k_1\tau} - e^{-\tau/\mu}\right] + \frac{W_2(\mu, \varphi)}{1 - k_2\mu}\left[e^{-k_2\tau} - e^{-\tau/\mu}\right] + \mu_0\frac{V_2(\mu, \varphi)}{\mu - \mu_0}\left[e^{-\tau/\mu_0} - e^{-\tau/\mu}\right]. \quad (9)$$

In the case when the conditions $\omega_0 = 1$ and $\gamma_1 = \gamma_2$ are satisfied simultaneously the expressions for the intensities have the simpler form

$$I_1(\tau, \mu, \varphi) = \mu X_1(\mu, \varphi)\left[(\tau/\mu + 1) - \left(\tau_0/\mu + 1\right)e^{-(\tau_0-\tau)/\mu}\right] + Y_1(\mu, \varphi)\left(1 - e^{-(\tau_0-\tau)/\mu}\right) + \mu_0\frac{Z_1(\mu, \varphi)}{\mu + \mu_0}\left[e^{-\tau/\mu_0} - e^{-(\tau_0-\tau)/\mu}\cdot e^{-\tau_0/\mu_0}\right]; \quad (10)$$

$$I_2(\tau, \mu, \varphi) = \mu X_2(\mu, \varphi)\left[(\tau/\mu - 1) + e^{-\tau/\mu}\right] + Y_2(\mu, \varphi)\left(1 - e^{-\tau/\mu}\right) + \mu_0\frac{Z_2(\mu, \varphi)}{\mu + \mu_0}\left[e^{-\tau/\mu_0} - e^{-\tau/\mu}\right], \quad (11)$$

where

$$X_{1,2}(\mu, \varphi) = C_1\left(P_{1,2}(\mu, \varphi) + Q_{1,2}(\mu, \varphi)\right);$$

$$Y_{1,2}(\mu, \varphi) = C_2\left(P_{1,2}(\mu, \varphi) + \left(C_2 - C_1/\alpha\right)Q_{1,2}(\mu, \varphi)\right);$$

$$Z_{1,2}(\mu, \varphi) = Z\mu_0^2 P_{1,2}(\mu, \varphi) + \left(\mu_0^2 Z - \pi S\mu_0\right) \times Q_{1,2}(\mu, \varphi) + \frac{S}{4}P\left(\pm\mu, \varphi, -\mu_0, \varphi + \pi\right).$$

The approximate analytical expressions, derived above, for the intensity of the scattered radiation (8)–(11) can be interpreted as follows. The last interaction of the radiation with the atmosphere is taken into account exactly by substituting Eq. (3) into the multiple scattering integral in the transfer equation (1). In so doing it is assumed that the result of all preceding acts of interaction of radiation with the atmosphere is described by the expression (3), in which the angular variables p and <p and the vertical variable τ are separated. In contrast to multiple scattering processes, the contribution of single scattering is taken into account exactly (the last term on the right side of the transfer equation (1)).

An important aspect in the foregoing approach to the calculation of the intensities, which is based on the three-flux approximation (E_0, E_1, and E_2) is that the initial angular distribution of the intensity $i_{1,2}(\mu, \varphi)$ is given. In Ref. 4, where questions concerning the atmospheric correction to satellite images are examined, it is suggested that the aerosol scattering phase function is strongly elongated, and as a result the initial angular distribution of the intensity of atmospheric haze is described in terms of δ-functions:

$$\begin{cases} i_1(\mu, \varphi) = \delta(\mu - \mu_0)\delta(\varphi - \varphi_0)/\mu_0, \\ i_2(\mu, \varphi) = \delta(\mu - \mu_0)\delta(\varphi - \varphi_0 + \pi)/\mu_0. \end{cases} \quad (12)$$

In Ref. 5, on the other hand, it is suggested that the initial angular distribution within the lower and upper hemispheres is uniform:

$$i_{1,2}(\mu, \varphi) = 1/\pi. \quad (13)$$

We note that Eqs. (12) and (13) satisfy the condition (4).

These two different limiting cases of the initial angular distribution of the intensity do not adequately reflect the real situation. For this reason, it is more natural to use for the initial angular distribution the expressions for the upward and downward single-scattering intensities averaged over the optical thickness[8];

$$i_1(\mu,\ \varphi) = \frac{\omega_0\mu_0 S}{4(\mu + \mu_0)\tau_0}\left[\mu_0\left(1 - e^{-\tau_0/\mu_0}\right) - \right.$$
$$\left. - \mu e^{-\tau_0/\mu_0}\left(1 - e^{-\tau_0/\mu}\right)\right]\frac{P\left(\mu,\ \varphi,\ -\mu_0,\ \varphi_0 + \pi\right)}{S_1}; \quad (14)$$
$$i_2(\mu,\ \varphi) = \frac{\omega_0\mu_0 S}{4(\mu - \mu_0)\tau_0}\left[\mu\left(1 - e^{-\tau_0/\mu_0}\right) - \right.$$
$$\left. - \mu_0\left(1 - e^{-\tau_0/\mu_0}\right)\right]\frac{P\left(-\mu,\ \varphi,\ -\mu_0,\ \varphi_0 + \pi\right)}{S_2},$$

where S_1 and S_2 are constants which are obtained by substituting Eq. (14) into the condition (4).

To compare the intensities of the upward and downward radiation, which are obtained on the basis of the approach examined above with different initial approximations (11)–(14), we performed a numerical experiment. The scattering phase function was given in the form

$$P\left(\mu,\ \varphi,\ \mu',\ \varphi'\right) = \left(\tau_R/\tau_0\right)P_R\left(\mu,\ \varphi,\ \mu',\ \varphi'\right) +$$
$$+ \left(\tau_a/\tau_0\right)P_a\left(\mu,\ \varphi,\ \mu',\ \varphi'\right),$$

where P_R is the Rayleigh scattering phase function; P_a is the aerosol scattering phase function, corresponding to haze of the continental type Haze L[9]; and, τ_R and τ_a are the Rayleigh and aerosol optical thicknesses, and $\tau_0 = \tau_R + \tau_a$.

Since the obtained intensities (8)–(11) have a simple analytical dependence on the optical depth τ, it is easy to obtain their derivatives with respect to τ. For this reason, the approximate solution obtained can be checked by substituting it directly into the starting transfer equation (1). For this, we shall examine the relative error in the approximate solution of the equation

$$R(\tau,\ \mu,\ \varphi) = \frac{\Delta I(\tau,\ \mu,\ \varphi)}{I(\tau,\ \mu,\ \varphi)}\ 100\%, \quad (15)$$

where

$$- \frac{\omega_0}{4\pi}\int_0^{2\pi} d\varphi' \int_0^1 d\mu'\ P\left(\mu,\ \varphi,\ \mu',\ \varphi'\right)I(\tau,\ \mu',\ \varphi) -$$
$$- \frac{\omega_0 S}{4}\ e^{-\tau/\mu_0}\ P\left(\mu,\ \varphi,\ -\mu_0,\ \varphi_0 + \pi\right),$$

and the approximate solution was taken for $I(\tau,\ \mu,\ \varphi)$. It should be noted that the relative error (15) shows how closely the approximate solution satisfies the initial transfer equation (1).

The calculations were performed for the top boundary of the atmosphere ($\tau = 0$) with $\omega_0 = 1$ for three variants of the values of the optical thickness of the atmosphere: 1) $\tau_0 = 0.3$, $\tau_R = 0.1$, $\tau_a = 0.2$, 2) $\tau_0 = 0.3$, $\tau_R = 0$, $\tau_a = 0.3$, and 3) $\tau_0 = 0.1$, $\tau_R = 0.1$, $\tau_a = 0$. These values were chosen in connection with the fact that the first variant corresponds to the continental atmosphere with average optical thickness $\tau_0 = 0.3$ for the wavelength 0.55 μm. The second variant corresponds to a purely aerosol, maximally elongated scattering phase function. The third variant corresponds to a minimally elongated Rayleigh scattering phase function. The same optical thickness τ_0 was chosen for the variants 1 and 2, in order to study the effect of only the elongation of the scattering phase function.

The errors (15) for different vertical angles θ and azimuthal angle $\varphi = 0°$ with different vertical angles of the sun θ_0 and $\varphi_0 = 0°$ are given in Table I. The columns 1–3 contain the data obtained for the initial approximations (12)–(14), respectively. It follows from Table I that the intensity of the scattered radiation, obtained with the initial approximation (14), satisfies significantly better the starting transfer equation than does the solution with the initial approximations (12) and (13) for all three variants of the calculations and all values of the vertical angles θ and θ_0. (This is also true for other azimuthal angles φ.)

The large errors R for the intensities with the initial approximations (12) and (13) indicate that on the basis of these approaches less accurate assumptions are made about the angular distribution of the intensity in the entire range of the angles θ and φ. However these initial approximations were employed for the calculations intended for the atmospheric correction of scanner satellite images, obtained by viewing the underlying surface at angles close to the nadir.[4,5] The values of I_1/S at $\theta = 0°$ for different values of θ_0 are given in Table II. One can see from the table that the values of the intensities of the upward radiation are quite close at $\theta_0 = 30°$. For $\theta_0 = 0°$, the intensity with the initial approximation (12) differs by approximately a factor of two from the intensity with the initial approximations (13)–(14) for the variants 1 and 2. This is because the direction of strong anisotropy of the initial approximation (12) is also the direction of propagation of the radiation $\theta = \theta_0$. There is no such difference for the Rayleigh scattering phase function (variant 3). In this case the optical thickness τ_0 is smaller, and for this reason the contribution of multiple scattering, which is determined by the initial approximations (12)–(14), is smaller and the contribution of single scattering, which is taken into account exactly, is larger. For the Rayleigh scattering phase function all three initial approximations give close results. For $\theta_0 = 30°$ the intensities obtained for different initial approximations (12)–(14) exhibit small differences, but at $\theta_0 = 60°$ these differences increase.

TABLE I. The values of the relative error R of the approximate expressions for the intensity of atmospheric haze.

θ	variant 1			variant 2			variant 3		
	1	2	3	1	2	3	1	2	3
$\theta_0 = 0°$									
0	401	17	6	431	45	12	-27	-15	6
30	-4	13	5	-4	40	10	-36	-18	7
60	-68	-5	2	-97	-1	-1	-46	-19	7
90	-30	-11	3	-31	-20	4	-12	-5	2
$\theta_0 = 30°$									
0	13	24	7	14	70	15	-34	-20	8
30	371	6	7	396	32	14	-25	-20	8
60	-15	-14	5	-20	-9	8	-26	-18	6
90	-28	-13	4	-38	-23	5	-9	-5	2
$\theta_0 = 60°$									
0	-71	51	7	-129	181	6	-43	-36	15
30	39	16	8	38	93	15	-10	-30	11
60	271	-17	7	281	7	10	-5	-22	8
90	1	-15	6	-5	-17	8	-3	-5	2

TABLE II. The values of the intensity of the upward haze of the atmosphere I_1/S for observation in the direction of the nadir.

θ	variant 1			variant 2			variant 3		
	1	2	3	1	2	3	1	2	3
$\theta_0 = 0°$									
0	0.101	0.049	0.047	0.037	0.017	0.015	0.036	0.036	0.037
30	0.040	0.042	0.041	0.012	0.014	0.012	0.031	0.032	0.033
60	0.025	0.030	0.030	0.006	0.010	0.008	0.022	0.023	0.024

In conclusion we note that the characteristics of the atmospheric correction to the data obtained by an airborne multichannel scanning system (AMSS) were studied in Refs. 10–12; the angle of the total field of view of the AMSS was equal to 51.2°. It was shown that to increase the accuracy of the classification processing of the AMSS data the angular dependence of the intensity of the atmospheric haze on the scanning angle must be taken into account correctly.[11] As shown in this paper, the approximation (12) gives the least satisfactory results. The errors increase rapidly as the direction of viewing opposite to the direction of propagation of the primary solar radiation is approached: in this approximation Eq. (12) results in the appearance of a spurious "peak" in the angular distribution of the intensity of atmospheric haze; this was shown previously in Ref. 12 on the basis of a somewhat simplified approach. The approximation (13) gives more accurate results, which, however, are not as accurate as the approximation (14); this undoubtedly will result in the appearance of additional errors in classification when this approximation is employed. We underscore another drawback of the approximation (13) — the intensity of the background haze is calculated ; using the angular distribution of the intensity of the downward-cattered radiation $I_2(\tau, \mu, \varphi)$, which has a strong maximum in the direction of propagation of the direct solar radiation (in view of the forward "elongation" of the atmospheric scattering phase function) and agrees poorly with the approximation (13).

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