Inverse problem of determining variance of the power of an atmospheric source of aerosol pollutants

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The "inverse" problem of determining variance of the power of a point stationary source of atmospheric pollutions is formulated in this paper. Based on our earlier investigations on determining coordinates and mathematical expectation of a source's power from data on a pollutant concentration and its variance at several observation points, the algorithm for determining variance of a source power is developed. The obtained theoretical results are illustrated by calculated spread of a pollutant in the atmosphere over Novosibirsk.

In Ref. 1 we considered the problem of determining coordinates x_0, y_0, z_0, and mathematical expectation $\overline{q}$ from the values of pollutant concentration obtained at a limited number of test points. The problem was solved using an equation conjugate to the equation of turbulent diffusion.[2] In this paper, we describe the procedure of determining one more characteristic of a source, namely, its power variance.

The "direct" problem on the spread of a pollutant from a stationary point source in the half-space $z \geq 0$ can be stated in the following form:

$$\overline{U}_i \frac{\partial \overline{C}}{\partial x_i} - \frac{\partial}{\partial x_i} K_{ij} \frac{\partial \overline{C}}{\partial x_j} = \overline{q}\, \delta(x - x_0)\, \delta(y - y_0)\, \delta(z - z_0)\ ;$$

$$\overline{C}(\pm\ \infty, y, z) = \overline{C}(x, \pm\ \infty, z) = \overline{C}(x, y, \infty) = 0\ ;$$

$$- V_s\ \overline{C} - K_{zj} \frac{\partial \overline{C}}{\partial x_j} + V_g\ \overline{C} = 0 \quad \text{at } z = 0\ , \qquad (1)$$

where $\overline{U}_i$ is the mathematical expectation of the ith component of the medium velocity; $\overline{C}$ is mathematical expectation of the pollutant concentration; K_{ij} are the coefficients of turbulent diffusion; V_s is the sedimentation rate of aerosol particles; V_g is the rate of particles' fallout onto the underlying surface; $\delta(...)$ is the Dirac delta function. In the general case, one should add sedimentation rate of aerosol particles to the z-component of wind velocity in Eq. (1). The bar over symbols means averaging over the statistical ensemble. The repeated indices denote summation. Let us put the problem of determining the variance of pollutant concentration, σ^2:

$$\overline{U}_i \frac{\partial \sigma^2}{\partial x_i} - \frac{\partial}{\partial x_i} K_{ij} \frac{\partial \sigma^2}{\partial x_j} = 2K_{ij} \frac{\partial \overline{C}}{\partial x_i} \frac{\partial \overline{C}}{\partial x_j} - \frac{\varepsilon}{c_0\, b}\, \sigma^2 +$$

$$+ 2\overline{(\hat{C}\hat{q})}\ \delta(x - x_0)\ \delta(y - y_0)\ \delta(z - z_0)\ ;$$

$$\sigma^2(\pm\ \infty, y, z) = \sigma^2(x, \pm\ \infty, z) = \sigma^2(x, y, \infty) = 0\ ; \quad (2)$$

$$- 2V_s\, \sigma^2 - K_{zj} \frac{\partial \sigma^2}{\partial x_j} + V_g\, \sigma^2 = 0 \quad \text{for } z = 0\ ,$$

where $\hat{C}$ is pulsation of the pollutant concentration; $\hat{q}$ is pulsation of the source power; b is the kinetic energy of turbulence; ε is its dissipation rate; c_0 is the empirical constant.[3] The boundary condition for the variance in Eq. (2) is set in accordance with Ref. 4.

Mathematical expectation of the product of the pulsations in the right-hand side of Eq. (2) can be written as $\overline{(\hat{C}\hat{q})} = r_{cq}\, \sigma\sigma_q$, where r_{cq} is the coefficient of correlation between concentration pulsation and the source power; σ_q is the variance of the source power. If the instantaneous source power is changed by several times, the pollutant concentration at a source's point evidently changes similarly, so $r_{cq} = 1$. Therefore, the third term in the right-hand side of Eq. (2) can be written in the form $2\sigma\sigma_q\, \delta(x - x_0)\, \delta(y - y_0)\, \delta(z - z_0)$. Now let us put the problem conjugate to Eq. (2). To do this, let us multiply Eq. (2) by a function σ_*^2 and integrate it over the half-space in which the pollutant is spread. Performing some calculations with the account of the boundary conditions for σ^2, and assuming the following boundary conditions for σ_*^2:

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$$\sigma_*^2(\pm\,\infty, y, z) = \sigma_*^2(x, \pm\,\infty, z) = \sigma_*^2(x, y, \infty) = 0\ ; \quad (3)$$

$$-\,V_s\,\sigma_*^2 - K_{zj}\,\frac{\partial \sigma_*^2}{\partial x_j} + 2V_g\,\sigma_*^2 = 0 \quad \text{at } z = 0\ ,$$

we obtain

$$\int \sigma^2 R \mathrm{d}V = \int \sigma_*^2\, 2K_{ij}\,\frac{\partial \overline{C}}{\partial x_i}\,\frac{\partial \overline{C}}{\partial x_j}\,\mathrm{d}V + 2\sigma_q(\sigma_*^2\,\sigma)\ ,$$

$$-\,\overline{U}_i\,\frac{\partial \sigma_*^2}{\partial x_i} - \frac{\partial}{\partial x_j}\,K_{ij}\,\frac{\partial \sigma_*^2}{\partial x_j} + \frac{\varepsilon}{c_0\,b}\,\sigma_*^2 = R\ . \qquad (4)$$

The term in the round brackets of Eq.(4) is taken at the point $x = x_0$, $y = y_0$, $z = z_0$.

Let the values of mathematical expectation of the pollutant concentration be known at some test points (as it was supposed in Ref. 1). Let the variance of the pollutant concentration σ_1^2 be also known at least at one point with the coordinates x_1, y_1, z_1.

Now one can write the following algorithm for solving the problem on determining power variance of a source.

1. Find the source's coordinates and mathematical expectation of power by use of known pollutant concentration.[1]

2. Solve the direct problem of spread (1) and find the field of mathematical expectation of the pollutant concentration.

3. Assume that $R = \delta(x - x_1)\,\delta(y - y_1)\,\delta(z - z_1)$, and solve the problem (3) and (4), and find the field σ_*^2.

4. Taking into account the form of R, known value σ_1^2, and fields $\overline{C}$ and σ_*^2 obtained above, find the product $\sigma\sigma_q$ at the source's point by use of Eq. (4).

5. After this, it is possible to solve the problem (2) for concentration variance and determine σ at the source's point. Now the sought value σ_q can be easily obtained by the product $\sigma\sigma_q$ and σ at the source's point.

Before considering a concrete example of calculation of the variance of a source's power, let us discuss σ_q as a qualitative function of the source's power and pulsation intensity of the pollutant concentration $I = \sigma/\overline{C}$ at an observation point. According to Eq. (1), we have $\overline{C} \sim \overline{q}$. It follows from Eq. (2) that $\sigma^2 \sim (\overline{q})^2 + \sigma\sigma_q$. Therefore,

$$\sigma_q \sim q(I - \text{const}/I)\ . \qquad (5)$$

It follows from Eq. (5) that the variance of the source's power must monotonically increase with the increase of intensity of concentration pulsation. Since σ_q is non-negative, by definition, intensity of concentration pulsation cannot be less than a certain critical value regardless of the source's power.

Calculations were performed for daytime meteorological conditions, typical for Novosibirsk on June 25. The city was blown by southwest wind of 3 m/s speed at the height of the wind vane at the suburban weather station. Using the method from Ref. 5, the field of wind velocity over the city was determined. Then the calculations were performed in accordance with the algorithm discussed above. The Figure 1 presents σ_q as a function of I for different values of the source's power. The values presented in the figure are given in conventional units. As seen from the Figure, the calculated results agree well with the estimate (5). The critical value of I is of the order of 0.29 in this case.

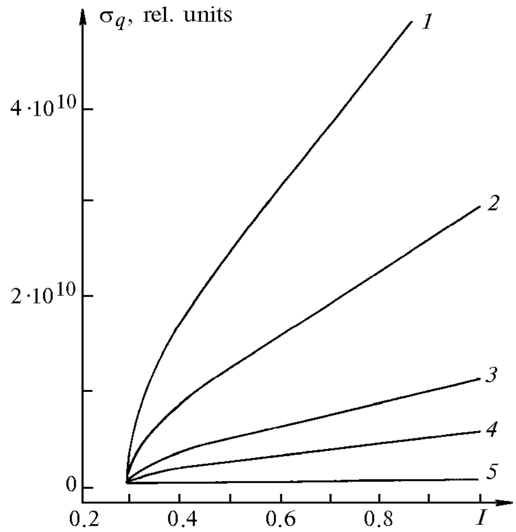

Fig. 1. The parameter σ_q as a function of I calculated for different values of power of the pollutant source. The figures *1–5* at curves correspond to the values q equal to 10^{10}; $0.5{\cdot}10^{10}$; $0.2{\cdot}10^{10}$; 10^9; 10^8 of arbitrary units.

In connection with the calculated values of the variance, the following remark must be done. Variance of the measured concentration values is defined by three main causes: the variance of the source's power, statistical nature of the process of pollutant spread in the atmosphere, and the instrumental errors in measurements of concentration. It is evident that the variance of the source's power is equal to that of concentration in the ideal case when the boundary layer of the atmosphere is laminar and measurement of concentration is absolutely accurate. In an actual case, the values of the variance of the source's power calculated by the algorithm proposed above must be necessarily interpreted as an estimate of the error of the calculated value of the source power. This error is always different from zero, even in the case when the real value of the source's variance is zero.

References

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