Identification of the atmospheric pollution sources using a network of mobile monitoring means

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The inverse problem concerning determination of the emission rate and coordinates of a stationary point source of atmospheric pollution from data on pollutant concentration obtained with a mobile monitoring network is considered. In the sample calculations considered, the results of direct problems solution are used as input data. It is shown that the approach proposed can be effectively used in solving the problems of determination of the parameters of unknown atmospheric pollution source from information about the pollutant concentrations obtained from the data collected at a mobile monitoring network.

The dispersal of the aerosol and gaseous atmospheric pollution is usually presented within the two classes of problems. The first one is the solution of direct problems, when the task is to find the pollution field from the known characteristics of pollution sources. The second class is solution of inverse problems, when the information about the pollutant concentration measured at some control points is used for determination of the type, coordinates, and emission rate of the pollution sources. Under the Euler approach to description of turbulent diffusion process, the most fruitful is the use of a semiempirical equation.[1] For stationary pollution sources, it can be written as

$$\frac{\partial \overline{U}_i\,\overline{C}}{\partial x_i}-\frac{\partial}{\partial x_i}K_{ij}\frac{\partial \overline{C}}{\partial x_j}=Q\quad (i,\ j=\overline{1,3}),\tag{1}$$

where $\overline{C}$ and $\overline{U}_i$ are the mathematical expectations of the pollutant concentration and wind velocity components; K_{ij} are components of the tensor of turbulent diffusion coefficients (we believe that $K_{ij}=0$ at $i\neq j$); Q is the term describing pollution sources; $x=x_1$ and $y=x_2$ correspond to the horizontal coordinates and $z=x_3$ is for the vertical one. The overbar denotes averaging over the statistical ensemble. Repeating indices mean summation. The direct problem is solved in a rectangular area G with the surface S consisting of the lateral surface Σ, bottom Σ_0 (at $z=0$) and top Σ_H (at $z=H$). The boundary conditions for Eq. (1) are specified as follows:

$$\overline{C}=0 \text{ at } \Sigma,\ \Sigma_H;\quad K_{zz}\frac{\partial \overline{C}}{\partial z}+V_s\,\overline{C}=\beta\overline{C}\ \text{ at } \Sigma_0,\tag{2}$$

where V_s is the particle sedimentation rate; β is the parameter of pollutant interaction with the underlying surface.

An effective method for solution of inverse problems was developed by Marchuk.[2] It is based on application of the equation conjugate with the semiempirical turbulent diffusion equation. The equation conjugate with Eq. (1) has the form

$$-\frac{\partial \overline{U}_i\,\overline{C}_*}{\partial x_i}-\frac{\partial}{\partial x_i}K_{ij}\frac{\partial \overline{C}_*}{\partial x_j}=P\tag{3}$$

with the system of the boundary conditions

$$\overline{C}_*=0 \text{ at } \Sigma,\ \Sigma_H;\quad K_{zz}\frac{\partial \overline{C}_*}{\partial z}+V_s\,\overline{C}_*=\beta\overline{C}_*\ \text{ at } \Sigma_0.\tag{4}$$

According to the Marchuk's method, the dual presentation of the functional[2]:

$$\int_G P\,\overline{C}\,\mathrm{d}G=\int_G Q\overline{C}_*\,\mathrm{d}G,\tag{5}$$

allows solution of a wide spectrum of inverse problems. Particular examples are considered, for example, in Refs. 3 and 4.

Besides the stationary sites for monitoring of atmospheric pollution, it is also possible to use a mobile monitoring network. In the latter case, instrumentation for measuring the pollutant concentration is installed on a mobile platform. One of the advantages of the mobile network over the stationary one is the possibility of collecting information on the atmospheric pollutant concentration over extended territories. In this paper, we consider the inverse problem, namely, determination of the emission rate and coordinates of a stationary point source of atmospheric pollution from the monitoring data obtained on a mobile network.

Let a mobile network be represented by K routes, along which the mobile laboratory measures pollutant concentrations. Assume that the kth route ($k=\overline{1,K}$) is the straight line segment with the initial coordinates

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$x = x_k$, $y = y_k$, $z = z_k$. The mobile laboratory moves at the kth route along the y-axis at a constant speed V_k, the measurement cycle starts at t_k and continues until T_k. The length of the kth route is $L_k = V_k T_k$.

Further reasoning will be aimed at solution of the problem formulated by the finite-difference method. Let a stationary point pollution source be located at the mth node ($m = \overline{1, M}$) of the difference grid covering the area G and have the coordinates x_m, y_m, z_m and the emission rate Q_m. This means that in Eq. (1)

$$Q = Q_m \, \delta(x - x_m)\, \delta(y - y_m)\, \delta(z - z_m). \qquad (6)$$

In the general case, the concentration measured along the kth route is an ensemble of instantaneous concentration values averaged over the periods Δt

$$C_{ik} = \frac{1}{\Delta t} \int\limits_{t_k + i\Delta t}^{t_k + (i+1)\Delta t} C(x_k, y_k + V_k t, z_k, t)\, \mathrm{d}t, \qquad (7a)$$

where $C(x, y, z, t)$ are the instantaneous random pollutant concentrations at the kth route; i is the number of a current node of the difference grid at the kth route, $i = \overline{0, I_k}$, $I_k = L_k/(V_k \Delta t) = L_k/\Delta y$, Δy is the grid step along the axis y. The change of the variables $y = y_k + V_k t$ in Eq. (7a) leads to

$$C_{ik} = \frac{1}{\Delta y} \int\limits_{y_k + i\Delta y}^{y_k + (i+1)\Delta y} C\left(x_k, y, z, \frac{y - y_k}{V_k}\right) \mathrm{d}y. \qquad (7b)$$

By definition, C_{ik} is random value, but the semiempirical equation deals with the concentrations averaged over the statistical ensemble $\overline{C}$, which are time independent in the considered case. The result of averaging of Eq. (7b) over the statistical ensemble is

$$\overline{C}_{ik} = \frac{1}{\Delta y} \int\limits_{y_k + i\Delta y}^{y_k + (i+1)\Delta y} \overline{C}(x_k, y, z_k)\, \mathrm{d}y. \qquad (8)$$

In the general case $C_{ik} \neq \overline{C}_{ik}$. The rigorous analysis of conditions, under which the equality $C_{ik} = \overline{C}_{ik}$ is true, is beyond the scope of this work. However, we can present rather simple restrictions ensuring approximate fulfillment of this equality. If a random process $C(t)$ is ergodic during the movement from one node to another, then time averaging is equivalent to averaging over the statistical ensemble. The integral in Eq. (7a) is an example of sliding time average. If the characteristic Euler time scale of pollutant concentration pulsations τ is much smaller then the time step Δt, then the ergodicity condition is approximately fulfilled.[5] In the general case, τ is the estimated duration of the autocorrelation function of concentration pulsations on the time axis.[1,6]

Thus, under this restriction on τ and Δt, we can believe that $C_{ik} \approx \overline{C}_{ik}$. Note that these restrictions also correspond to the conditions of applicability of the semiempirical and conjugate equations.[6]

If $\Delta t = \Delta y / V_k$, then when solving the inverse problem by numerical methods the measured values of the pollutant concentration at the ith point of the kth route correspond to the values $C_{ik} \approx \overline{C}_{ik}$ at the grid nodes. Assume, for the kth route, that

$$P_{ik}(x, y, z) = Q_* \delta(x - x_k)\, \delta(y - y_k - i\Delta y)\, \delta(z - z_k), \qquad (9)$$

where Q_* is arbitrary nonzero constant measured in the same units as Q_m. In view of Eqs. (5), (6) and (9), we have

$$Q_* \overline{C}_{ik} = Q_{ikm} \overline{C}_{*ik}(x_m, y_m, z_m), \qquad (10)$$

where $\overline{C}_{*ik}$ is the solution of the conjugate problem for the ith point of the kth route. Unlike Eqs. (1) and (3), no summation over repetitive indices is assumed in Eq. (10). The parameters Q_{ikm} describe the emission rate of a stationary source located at the mth node of the computational grid and producing the measured value of the concentration $\overline{C}_{ik}$ at the ith point of the kth route. Thus, Eq. (10) determines the set of Q_{ikm} at each of the M nodes of the computational grid for each of the K routes. Determine also

$$\overline{Q}_{km} = \frac{1}{I_k + 1} \sum_{i=0}^{I_k} Q_{ikm};$$

$$\sigma^2_{km} = \frac{1}{I_k} \sum_{i=0}^{I_k} \left(Q_{ikm} - \overline{Q}_{km}\right)^2. \qquad (11)$$

Because of the condition of a unique solution to the inverse problem, the source producing the integral concentration values measured at the preset routes can be at only one node of the computational grid. It is obvious that the node of the computational grid with the minimal variance σ^2_{km} just determines the sought coordinates of the source, and the value of $\overline{Q}_{km}$ estimates the sought emission rate of the source. Similar, in many aspects, procedure for determination of the emission rate and coordinates of an instantaneous point source from the data of a stationary monitoring network is presented in Ref. 3. Note also that the coordinates and emission rate of a source can be determined by combining the information on the pollutant concentration obtained at different routes.

As an example illustrating the approaches described above, consider dispersal of a pollutant from a stationary point source located in the western suburbs of Novosibirsk (Fig. 1). The city of Novosibirsk is situated on both banks of Ob River as

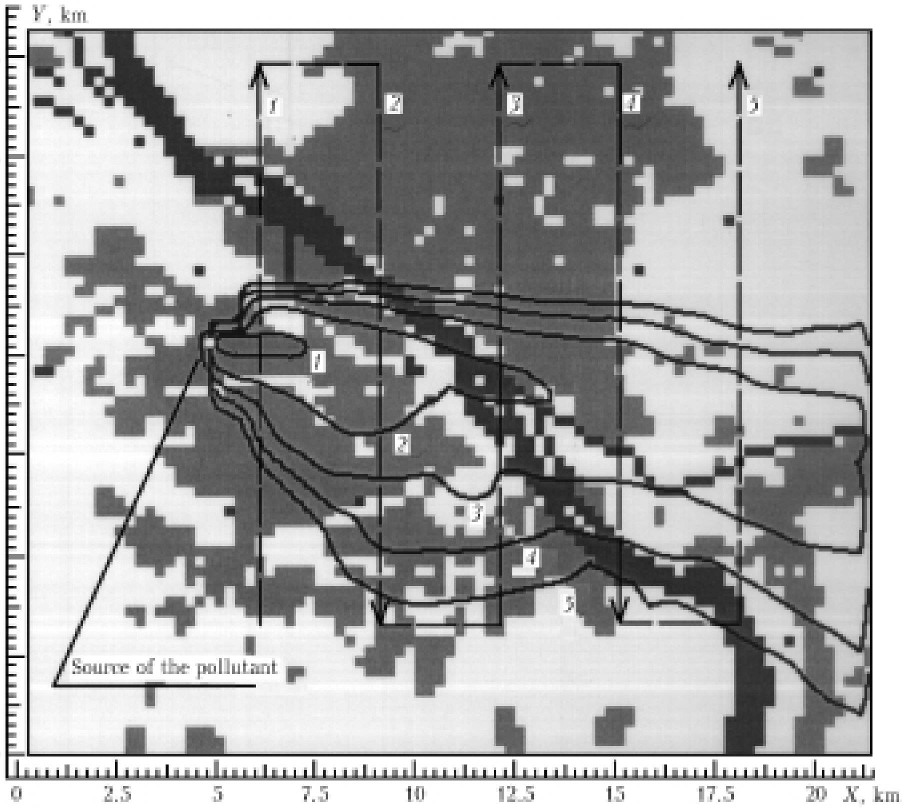

Fig. 1. Illustration to the problem formulated. Isolines *1–5* correspond to the pollutant concentrations of 10^4, 10^3, 10^2, 10^1, 10^0 rel. units at the height z = 100 m. The routes of the mobile monitoring network are shown by dashed lines.

shown by light gray in Fig. 1. Ob River shown by dark gray divides the city into the right and left bank parts. In our calculations we specified the weather conditions typical of this territory at 15:00 LT in July at the westerly winds with the speed of 3 m/s at the vane height. The pollution source was located at the point with the coordinates $x_m = 4.75$ km, $y_m = 10.25$ km, $z_m =$ 50 m and had the emission rate $Q_m = 10^{10}$ rel. units. The wind velocity field over the city was determined using a numerical-analytical model.[7] Then the direct problem (1), (2) was solved. The turbulent diffusion coefficients were specified by the algebraic model similar to that from Ref. 8. The solution of the direct problem $\overline{C}_{ik}$ was taken as the concentrations measured along the routes. The isolines shown in Fig. 1 give the idea of the pollutant concentration field in the cross section z = 100 m. The five routes considered (K = 5) are also depicted in Fig. 1. Their characteristics such as the initial coordinates, length, speed and time of movement of the mobile laboratory along them are given in Table 1.

The characteristic Euler time scale of pollutant concentration pulsations in the atmospheric surface layer is known to be about tens of seconds. With the speed of the mobile laboratory chosen and the time step Δt = 100 s, the pollutant concentrations were set with the interval of 2000 m for each route in accordance with the above restrictions. Thus, at each route we considered eight measurement points.

Table 1. Characteristics of routes of the mobile monitoring network

Characteristic	Route number k				
	1	2	3	4	5
Initial coordinates					
x_k, km	6	9	12	15	18
y_k, km	3.25	17.25	3.25	17.25	3.25
z_k, m	100	100	100	100	100
Length L_k, km	14				
Measurement duration T_k, s	700				
Speed V_k, m/s	20				

For each of the K routes, we solved the conjugate problem and determined the set of Q_{km} for all the M nodes of the computational grid by Eq. (10). Then, according to Eq. (11), we calculated $\overline{Q}_m$ and σ_m^2 and then determined the nodes of the computational grid with the minimal values of σ_m^2.

The calculated results given in Table 2 illustrate how the calculated source characteristics approach the true values as information on the concentration values measured at the routes is complemented.

We can see that in the considered case to reconstruct the pollution source parameters, it is sufficient to invoke the information about the pollutant concentration measured at four to five points along each route. Additional calculations show that if the points selected for measurements lie rather far from the boundaries of the computational domain and have sufficiently high, significant absolute concentration values, then to reconstruct the source characteristics, we need the information about the pollutant concentration measured at only two to three points of a route.

Table 2. Examples of determination of source characteristics in the process of accumulating information about pollutant concentration during movement along preset routes

Calculations involved the pollutant concentrations at points of the kth route with the numbers		Determined source characteristics			
		x_m, km	y_m, km	z_m, m	Q_m, rel. units
$k = 1$	$i = \overline{1, 2}$	5.25	7.50	110	$1.41 \cdot 10^5$
	$i = \overline{1, 3}$	5.00	8.25	120	$8.71 \cdot 10^5$
	$i = \overline{1, 4}$	5.00	10.0	130	$5.28 \cdot 10^9$
	$i = \overline{1, 5}$	4.75	10.25	50	10^{10}
	$i = \overline{1, 8}$	4.75	10.25	50	10^{10}
$k = 2$	$i = \overline{1, 2}$	7.50	6.75	160	$3.42 \cdot 10^6$
	$i = \overline{1, 3}$	4.75	10.25	20	$9.74 \cdot 10^9$
	$i = \overline{1, 4}$	4.75	10.25	50	$9.99 \cdot 10^9$
	$i = \overline{1, 5}$	4.75	10.25	50	$9.99 \cdot 10^9$
	$i = \overline{1, 8}$	4.75	10.25	50	$9.99 \cdot 10^9$
		Preset source characteristics			
		4.75	10.25	50	10^{10}

Table 3 presents the generalized results concerning determination of the pollution source characteristics from the complete information about the concentration values measured at each of the considered routes. One can see a close agreement between the reconstructed and preset values. The observed deviations of the determined z coordinate of the source correspond to one step of the difference grid of the computational grid, which is 10 m in our case, and, also, can be considered as good enough.

Thus, this paper formulates and considers the main principles of solution of the inverse problem aimed at determination of the characteristics of a stationary point source of atmospheric pollution from the data on the pollutant concentration measured along the routes of a mobile network of atmospheric monitoring. In a particular example of solution of such a problem, we used the information about the concentrations of atmospheric pollutants obtained by solving the direct problem of atmospheric pollutant dispersal. The approach proposed in this paper can be used rather efficiently for determination of unknown characteristics of atmospheric pollution sources from the information about the pollutant concentrations obtained in the mobile monitoring network.

Table 3. Results of determination of source parameters

Route number*	Determined source characteristics			
	x_m, km	y_m, km	z_m, m	Q_m, rel. units
1	4.75	10.25	50	10^{10}
2	4.75	10.25	50	$9.99 \cdot 10^9$
3	4.75	10.25	60	$1.01 \cdot 10^{10}$
4	4.75	10.25	40	$9.90 \cdot 10^9$
5	4.75	10.25	40	$9.90 \cdot 10^9$
	Preset source characteristics			
	4.75	10.25	50	10^{10}

* The pollutant concentrations measured at these routes were used in the calculations.

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