LIDAR DETECTION OF ANOMALOUS STATES OF MESOSPHERIC SODIUM

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This paper presents optimal schemes for processing post-detector signals from a resonance fluorescence lidar intended for detecting anomalous states of mesospheric sodium. The schemes are best suited for three cases: when no a priori *information on the* Na *content is available at different states of the mesosphere; when information on the* Na *profile is available; and, when information on the* Na *profile and its statistical properties is available. An algorithm for estimating the detection efficiency and some results of numerical simulations estimating the lidar detector capabilities are presented.*

INTRODUCTION

As is well known,[1] at altitudes R from 80 to 110 km there .continuously exists a regular sodium layer with concentration maximum M at height R equal to 88–95 km, and sometimes one can observe a weaker layer with maximum at R equal to 100–105 km. Formation of the latter is connected with meteoric fluxes and is accompanied by a substantial (up to fourfold[2]) increase of M within the regular layer.

We will develop schemes of optimal processing of the photocounts[3] in a lidar detector[4] which make use of the effect of resonance fluorescence (RF) of Na to detect anomalous concentrations of Na in the regular layer and the sporadic layer. We will then analyze the efficiency of these schemes.

RF signals from Na received by a ground lidar[4] are so weak that for a given M the conditional distribution of the signal photocounts n_s during the time gate ΔT obeys the Poisson law[5]: $P_{Pois}(n_s; \bar{n}_s) = \bar{n}_s^{n_s} \exp(-\bar{n}_s) / n_s!$, where the conditional mean signal photocount is given by the lidar equation $\bar{n}_s = KM / R^2$, where K is the instrumentation atmospheric coefficient. The conditional distribution of the background-dark counts n_p during ΔT is equal to $P_{Pois}(n_p, \bar{n}_p)$, where $\bar{n}_p$ is the mean value, which can be taken to be known since it is estimated within a sufficiently large region between echo-signals.

DETECTION OF THE CONCENTRATION IN THE REGULAR LAYER

We shall considered two possible situations:

1) anomalous(M_a) and normal (M_n) concentrations of Na within the altitude $(R - \Delta R)(R + \Delta R)$, $\Delta R = c\Delta T/2$, is known *a priori*.

2) M_{a1} and M_{n1} in each of $l = \overline{1, L}$ altitude range regions $(R_1 - \Delta R_1, R_1 + \Delta R_1)$ are given *a priori*, $\Delta R_1 = c\Delta T_1/2$.

In the first case, the scheme of optimal processing of the photocounts n_μ detected during N ($\mu = \overline{1, N}$) sounding acts, or a decision rule based on the theory of testing the statistical hypotheses and the Neuman–Pearson criterion,[3] consists in acceptance of the hypothesis H_0: $M = M_n$ if $n < c_{th}$ and the hypothesis H_1: $M = M_a$ if $n \geq n_n$. Here

$$n = \sum_{\mu=1}^{N} n_\mu$$

and the threshold c_{th} is found from the inequality

$$\alpha \geq \sum_{n=c_{th}}^{\infty} P_{Pois}(n; \bar{n}_0) , \tag{1}$$

where

$$\bar{n}_0 = N(\bar{n}_0^c + \bar{n}_p), \quad \bar{n}_0^c = KM_n/R^2.$$

The efficiency of this scheme is characterized by a prescribed probability of false alarm a and a calculated probability of detection

$$P_d = 1 - \beta, \quad \beta = \sum_{n=0}^{c_{th}-1} P_{Pois}(n, \bar{n}_1), \tag{2}$$

where β is the probability of not detecting an anomalous concentration, and $\bar{n}_1 = N(\bar{n}_1^c + n_p)$, where $\bar{n}_1^c = KM_a / R^2$. For $\bar{n}_0$ □ 1, using the Gaussian approximation for $P_{Pois}(n; \bar{n}_1)$, i = 0, 1, we obtain

$$c_{th} = \Phi^{-1}(1 - \alpha)\bar{n}_0^{-1/2} + \bar{n}_0,$$

$$P_d = 1 - \Phi\left[(c_{th} - \bar{n}_1)/\bar{n}_1^{1/2}\right], \quad (3)$$

where Φ and Φ^{-1} are the standard distribution function and its inverse function.[6]

In the second case, the decision rule is H_0: $M_1 = M_{n1}$, if $\Lambda < c_{th}$ and H_1: $M_1 = M_{a1}$, if $\Lambda \geq c_{th}$. Here, the photocounts $n_{\mu l}$ detected in N ($\mu = \overline{1, N}$) acts of sounding and in L ($l = \overline{1, L}$) altitude gates enter into a functional of the likelihood ratio:

$$\Lambda = \sum_{l=1}^{L}\left[\sum_{\mu=1}^{N} n_{\mu l} \ln \frac{\bar{n}_{1l}}{n_{10}} + \bar{n}_{10} - \bar{n}_{11}\right],$$

and the threshold c_{th} and P_d are given approximately by

$$c = \Phi^{-1}(1 - \alpha)\sigma_0 + \Lambda_0, \quad P_d = 1 - \Phi\left[(c - \bar{\Lambda}_1)/\sigma_1\right], \quad (4)$$

where

$$\bar{\Lambda}_i = \sum_{l=1}^{L}\left[\bar{n}_{1i} \ln \frac{n_{11}}{n_{10}} + \bar{n}_{10} - \bar{n}_{11}\right];$$

$$\sigma_i^2 = \sum_{l=1}^{L} \bar{n}_{1i}\left(\ln \frac{\bar{n}_{11}}{n_{10}}\right)^2, \quad i = 0,\ 1;$$

$$\bar{n}_{10} = N\left(K_1 M_{n1}/R_1^2 + \bar{n}_p\right), \quad n_{11} = N\left(K_1 M_{a1}/R_1^2 + \bar{n}_p\right),$$

K_1 is the instrumentation atmospheric coefficient corresponding to ΔR_1.

DETECTION OF THE SPORADIC LAYER

Accounting for the general decrease of M with increase of R within the height interval $R = 95$–110 km, we use[3] the photocounts $n_{\mu 1}$ and $n_{\mu 2}$ in N ($\mu = \overline{1, N}$) sounding acts and two ($L = 2$) adjacent gates 2 ΔR located at heights R_1 and R_2 of the bottom (local minimum M) and the top maximum M) of the sporadic layer. If the values of R_1 and R_2 are *a priori* unknown, then we record the photocounts in all the gates in the height interval $R = 95$–100 km and carry out the detection procedure for every pair of adjacent gates.

Let us consider two possible situations:

1) the quantities M_{nl} and M_{al} ($l = 1,\ 2$) are *a priori* unknown and are determined from maximum likelihood estimates (MLE) $\hat{M}_1$ over the sample $n_{\mu l}$.

2) Prescribed *a priori*: the probability density distributions of M at R_1 in H_0 $P_{Par}(M_1;\ M_2,\ \alpha_0 \mid H_0]$ and in H_1 $P_{gam}[M_1;\ \kappa_1,\ \beta_1 \mid H_1]$ and at R_2 in H_0 $P_{gam}(M_2;\ \kappa_0,\ \beta_0 | H_0)$ and in H_1 $P_{Par}(M_2;\ M_1,\ \alpha_1 \mid H_1)$ as Pareto distributions[7] $P_{Par}(x;\ x_0,\ \alpha) = \alpha x_0^{\alpha} x^{-\alpha-1}$ for $x > x_0$ and gamma-distributions[7] $P_{gam} = (x;\ \alpha,\ \beta) = \beta^{\alpha} x^{\alpha-1} \exp(-\beta x)/\Gamma(\alpha)$ for $x \geq 0$; the probabilities of the states are $P(H_0) = q$ and $P(H_1) = 1 - q = p$; the loss (cost) matrix is $\{\Pi_{ij}\}$, i, $j = 0,\ 1$, for acceptance of the solutions.[7]

In the first case Mnl and Mal are assigned the values $M_{n1} = \hat{M}_1$, $M_{n2} = \hat{M}_2$, $M_{a1} = \hat{M}_1$, $M_{a2} = \hat{M}_2$ in the first variant when $\hat{M}_1 > \hat{M}_2$ and $M_{n1} = \hat{M}_1$, $M_{n2} = \hat{M}_2$, $M_{a1} = \hat{M}_1$, $M_{a2} = \hat{M}_2$ in the second variant for $\hat{M}_1 < \hat{M}_2$ where the MLE's

$$\hat{M}_1 = \left(n_1/N - \bar{n}_p\right)/KP_1^{-2}, \quad n_1 = \sum_{\mu=1}^{N} n_{\mu 1}$$

have zero shift and relative rms deviations (errors)[5]

$$\delta_{M1} = \frac{1}{\sqrt{N}} \cdot \frac{\sqrt{KM_1/R_1^2 + \bar{n}_{\mu 1}}}{KM_1/R_1^2}. \quad (5)$$

The decision rule is as follows: H_0 (the layer does not exist), if n_1, $n_2 \in \Omega_0$, and H_1 (the layer exists), if n_1, $n_2 \neq \Omega_0$, where the region Ω_0 in the space n_1, n_2 lies within the ellipse

$$(n_1 - \bar{n}_{10})^2/\bar{n}_{10} + (n_2 - \bar{n}_{20})^2 / \bar{n}_{20} = -2\ln\alpha.$$

Bounds for β may be found in the approximations of its over- and underestimated values in the first variant as

$$P_{\chi n}\left(-\ln\alpha,\ \frac{(\bar{n}_{21} - \bar{n}_{20})^2}{\bar{n}_{11}}\right) > \beta >$$

$$> P_{\chi n}\left(-\frac{\bar{n}_{20}}{\bar{n}_{21}}\ln\alpha,\ \frac{(\bar{n}_{21} - \bar{n}_{20})^2}{\bar{n}_{21}}\right),$$

and in the second variant as

$$P_{\chi n}\left(-\frac{\bar{n}_{10}}{\bar{n}_{21}}\ln\alpha,\ \frac{(\bar{n}_{21} - \bar{n}_{20})^2}{\bar{n}_{21}}\right) > \beta >$$

$$> P_{\chi n}\left(-\frac{\bar{n}_{20}}{\bar{n}_{11}}\ln\alpha,\ \frac{(\bar{n}_{21} - \bar{n}_{20})^2}{\bar{n}_{11}}\right), \quad (6)$$

where

$$P_{\chi n}(\chi^2,\ \lambda) = \sum_{j=0}^{\infty} \exp(-\lambda/2)\frac{(\lambda/2)^j}{j!} P_{\chi}\left(\chi^2 | 2 + 2j\right),$$

$$P_{\chi}(\chi^2|\nu) = \left[2^{\nu/2}\Gamma(\nu/2)\right]^{-1} \int_0^{\chi^2} t^{\nu/2-1} \cdot \exp(-t/2)dt$$

are the noncentral χ^2-distribution function with 2 degrees of freedom and the χ^2-distribution, respectively.[6] The value

$$\beta = P_{\chi n}\left(-\ln\alpha\left(1 - \frac{\left(\bar{n}_{21}^{-1/2} - \bar{n}_{20}^{-1/2}\right)^2}{\bar{n}_{11}^{-1/2} + \bar{n}_{21}^{-1/2}}\right)^2\right),$$

$$4\left[\left(\frac{\bar{n}_{21}-\bar{n}_{20}}{n_{11}^{-1/2}+n_{21}^{-1/2}}\right)^{2}\right] \qquad (7)$$

may serve as an estimate for β.

In the second case the decision rule, based on the Bayes criterion, is: H_0 if $y < c^*$, and H_1 if $y \geq c^*$. Here the threshold $c^* = q(\Pi_{01} - \Pi_{00})//p(\Pi_{10} - \Pi_{11})$,, the ratio of the unconditional likelihood functions is equal to $y = A_1/A_0$ when $\bar{n}_{20}^{c} \ll \bar{n}_p$, where

$$A_i = \frac{\beta_i^{\kappa_i}\alpha_i\Gamma(n_1+n_2+\kappa_i)}{\Gamma(\kappa_i)(n_{2-i}+\kappa_i+\alpha_i)} \times$$

$$\times\left[\frac{NK}{R^2}+\frac{NK}{R^2}+\beta_i\right]^{-n_1-n_2-\kappa_i}\cdot {}_2F_1\left[1,\ n_1+n_2+\kappa_i;\right.$$

$$\left. n_{2-i}+\kappa_i+\alpha_i+1;\ \frac{NK/R_{2-i}^2+\beta_i}{NK/R_1^2+NK/R_2^2+\beta_i}\right],$$

Γ is the gamma-function, and $_2F_1$ is the hypergeometric Gaussian function.[6]

The efficiency of detection according to this processing scheme is determined by the values of the average losses

$$R_p = q\Pi_{00}+p\Pi_{10}+q(\Pi_{01}-\Pi_{00})\alpha-p(\Pi_{10}-\Pi_{11})(1-\beta),$$

where $\alpha = A_{RO}$ and $\beta = 1 - A_{R1}$ if the region n_1, $n_2 \in y \geq c$ coincides with the region Ω_1 lying in the origin of the coordinate plane n_1, n_2 and bounded by the line $y = c^*$, and $\alpha = 1 - A_{R0}$ and $\beta = A_{R1}$ if the region n_1, $n_2 \in y < c^*$ coincides with Ω_1

$$A_{Ri} = \sum_{n_1}\sum_{n_2\in\Omega}\frac{1}{n_1!}\cdot\frac{1}{n_2!}\left[NK/R_1^2\right]^{n_1}\left[NK/R_2^2\right]^{n_2}A_i,$$

$$i = 0,\ 1.$$

PROBABILITY OF DETECTION OF ANOMALOUS STATES

For model calculations of the probability of detection, let us take the optical properties of the atmosphere[6] for a meteorological visibility range of 13 km, the profile of the normal concentration of Na in the form of the average monthly distribution function of the Na concentration for January 1970 (Ref. 1), night sounding conditions with spectral sky radiance 2.4 W m^{-2} sr^{-1}, the cross section of the resonance fluorescence of Na equal to $2 \cdot 10^{-17} m^2\ sr^{-1}$ (Ref. 4) and the following lidar parameters: laser wavelength 589.0 nm; radiation band width 8 pm and receiving band width 2 nm; effective area of the receiving aperture 1.1 m^2; solid angle of the field of view $0.25 \cdot 10^{-6}$ sr; transmission coefficient of the transmitting optics 0.1 and the receiving optics and filters 0.2; quantum efficiency of the photomultiplier 0.1; intensity of the dark photoelectrons of the photomultiplier 15 s^{-1} height resolution 1 km, pulse duration 3.5 ms. The radiated power and pulse repetition rate were taken to be equal to 5 W and 0.2 Hz, which is readily obtained using a rhodamine-6G laser with intracavity Fabry-Perot interferometers with flashlamp pumping[4], for the case of detecting and measuring the regular layer concentration, and 50 W and 250 Hz, e.g. , an exciter-laser pumped laser, for the case of detecting the sporadic layer. The potential efficiency of detection was calculated using formulas (1)–(4), (6), (7) and that of measurement using formula (5).

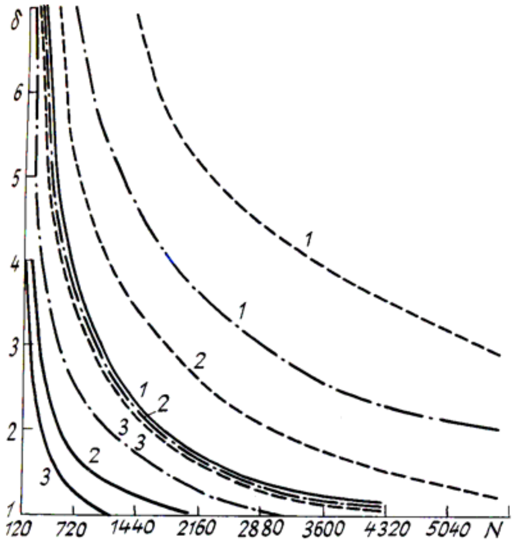

FIG. 1. Minimum relative deviation $\delta = (M_a - M_n)/M_n$ of the anomalous Na *concentration as a function of the number of soundings N, detected with prescribed efficiency*

$\alpha = 10^{-1}$, $P_d = 0.9$ (———);
$\alpha = 10^{-2}$, $P_d = 0.99$ (—·—·—);
$\alpha = 10^{-3}$, $P_d = 0.999$ (– – –);
$L = 1$ (*1*), 3 (*2*), 9 (*3*).

The calculated detection probabilities are shown in Figs. 1–4. In addition, the probabilities of measuring the Na concentration are shown for comparison in Fig. 2. The curves for single-gate detection in Fig. 1 ($L = 1$, $R = 88$ km) and in Fig. 2 were obtained using formulas (1)–(3) and show good probabilities of detection usually revealing the anomalous Na concentrations connected with meteoric fluxes. Gating and reception of the RF signals is optimal from the height of the maximum Na concentration, in this case, from a height of 88 km. At this height there can be weak ($\alpha = 10^{-1}$, $P_d = 0.9$), intermediate (10^{-2}, 0.99), and highly effective (10^{-3}, 0.999) detection of a meteoric flux with $\delta = 3$ (a fourfold increase of the Na concentration) at $t = 1$ h 28 m, 4 h 33 m, 7 h 30 m or at $t = 6$ h of fluxes with $\delta = 1.1$; 2.25; 3.45, respectively. However, reception of signals from a set of height gates located below and above as well as at $R = 88$ km is more optimal, as is illustrated for $L = 3$, 9 in Fig. 1

(relations (4) are used here). As the calculations show, the efficiency of detection increases sharply with L and then saturates. Thus, for $\alpha = 10^{-1}$, $N = 780$, $\delta = 2$, and $L = 1, 3, 5, 7, 9$, we obtain $P_d = 0.625$ (using Eqs. (1)–(3)) and 0.745 (using relations (4)), 0.945, 0.982, 0.988, and 0.992, respectively. Hence, proceeding from a compromise between accuracy of detection and simplicity of instrumentation, $L = 3$–5 is the most acceptable range of values. It should be noted that for $\delta_l \neq \delta\ (l = \overline{1, L})$ the efficiency of multigate detection is insensitive to models of variation of δ_l with respect to l which have the same mean $\bar{\delta} = \sum_{l=1}^{N} \delta_l / L$.

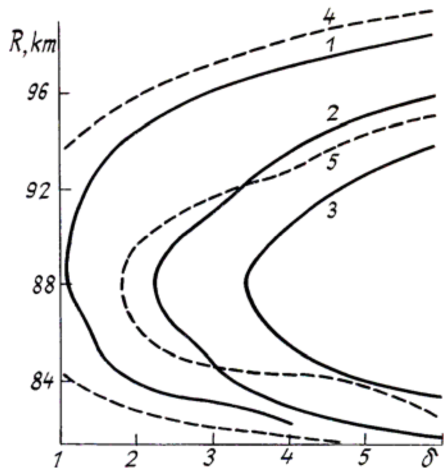

FIG. 2. The required sounding *height R as a function of* δ *for detecting* (———) *with* $\alpha = 10^{-1}$ $P_d = 0.9$ *(1);* $\alpha = 10^{-2}$ $P_d = 0.99$ *(2);* $\alpha = 10^{-3}$ $P_d = 0.999$ *(3), and measuring the anomalous concentration* M_a *(— — —)with* $\delta_\mu = 0.4$ *(4),* 0.2 *(5) for* $N = 4320$.

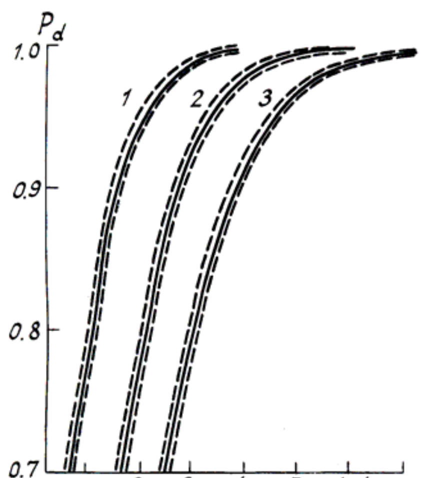

FIG. 3. Attainable probability *of detection* P_d *of the sporadic layer* (——) *and its boundaries* (— — —) *as functions of the sounding time t for* $\alpha = 10^{-1}$ *(1),* 10^{-2} *(2),* 10^{-3} *(3).*

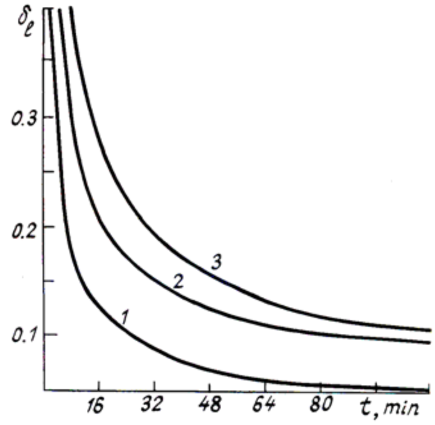

FIG. 4. Necessary $\delta_{sl} = (M_2 - M_1)/M_1$ *of the* Na *concentration at the maximum of the sporadic layer compared to its concentration at its beginning as a function of t during the detection of the sporadic layer with* $\alpha = 10^{-1}$, $P_d = 0.9$ *(1);* $\alpha = 10^{-2}$, $P_d = 0.99$ *(2);* $\alpha = 10^{-3}$, $P_d = 0.999$ *(3).*

The curves in Fig. 3 were obtained using formulas (6)–(7); those in Fig. 4 were obtained using formula (7) and illustrate the probabilities of detection of the sporadic layer by means of signals from $R_1 = 100$ km, $R_2 = 103$ km. It turns out that t equal to a few minutes is required for effective detection ($\alpha \leq 10^{-1}$, $P_d \geq 0.9$) of the absence of this layer when it does not exist and $\hat{M}_1 > \hat{M}_2$, and a few hours for the effective detection of the layer when it does exist and $\hat{M}_1 < \hat{M}_2$. This is connected with the substantial difference between $|\delta_{la}|$ in the first case (~ 100%) and in the second case (~ 5%). Taking into account the rarity of occurrence of the layer, it can be assumed that the proposed signal processing scheme from R_1, R_3 is entirely suitable and acceptable. Figure 3 shows the probabilities of detection of the sporadic layer with $\delta_l \sim 5\%$, taken from Ref. 1, and Fig. 4 shows the probabilities of detection of other more intense layers with $\delta_l \geq 5\%$. It can be seen that the detection of anomalous states of mesospheric Na by means of detecting the sporadic layer is entirely possible and quite effective. For example, weak, intermediate, and highly effective detection of a meteoric flux is possible with $t \leq 30$ min, when $\delta_l \geq 9$, 15, 19.5%.

CONCLUSIONS

The obtained technical schemes for lidar signal processing for the purpose of detecting anomalous states of mesospheric Na are easily realized and may serve both in available RF-lidars and in those which are presently under development. This being the case, they can complement existing schemes for measuring the Na concentration. The efficiency of the detection schemes is highest when prescribed profiles of the Na concentration in the normal and anomalous states are used. In the first stage, when this information is absent, the scheme of detecting the sporadic layer can be applied. After accumulating statistical data on the Na concentration one can use a more effective scheme for detecting the anomalous concentration in the regular layer.

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