Account for the detector's "dead time" using Monte Carlo method

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Numerical experiment is described to study the effect of "dead time" of a counting system on the statistics of readouts of laser radiation passed through the turbulent atmosphere at sampling time varying in a wide range. The numerical results agree well with the experimental data.

To record optical signals in the photon counting mode, a receiver usually consists of a photomultiplier tube, discriminator, and a shaper of standard pulses. Each of the above units has a finite response time, therefore some single-electron pulses are not recorded, and characteristics of the photoelectron flux (primary flux) and the flux of readouts (secondary flux) differ. Many authors have earlier considered the transformations of statistical characteristics of the primary flux by counters due to various types of the "dead time" (DT). It proves to be a rather complicated problem to take into account the effect of DT on the probability distribution of readouts, and this problem can be solved only approximately and only in some asymptotic cases.

The detailed analysis of the effect of DT on the statistics of readouts of laser radiation passed through the turbulent atmosphere and having Gaussian distribution of the field is given in Refs. 1–3.

In our earlier paper,[4] we have presented some results on the effect of DT on the statistics of readouts at $T \ll \tau$, where T is the sampling time and τ is the correlation time of the intensity fluctuations in the atmosphere, assuming also the lognormal intensity distribution. According to Ref. 4, the probability distribution of photocurrent P_2 with the allowance made for the DT effects and the turbulent atmosphere can be obtained from the distribution of photo readouts P_1 for the amplitude-stabilized radiation that accounts for DT through averaging over an ensemble of intensity fluctuations I

$$P_2(n, N, \varepsilon) = \langle P_1(n, N, \varepsilon)\rangle_I, \qquad (1)$$

where n is the number of readouts in a sample; N is the mean number of readouts; $\varepsilon = \Delta t/T$, Δt is the counter's dead time.

Using the lognormal distribution of the radiation intensity,[5] after averaging we have the following equation:

$$P_2(n; N', \varepsilon) = \begin{cases} \left[F_n(N'(1 - n\varepsilon); \varepsilon)\right] - F_{n-1}\{N'[1 - (n - 1)\varepsilon]; \varepsilon\}, & n < \dfrac{1}{\varepsilon}, \\ 1 - F_{n-1}\{N[1 - (n - 1)\varepsilon]; \varepsilon\}, & \dfrac{1}{\varepsilon} \le n < \dfrac{1}{\varepsilon} + 1, \\ 0, & n \ge \dfrac{1}{\varepsilon} + 1, \end{cases} \qquad (2)$$

where

$$F_n\left[N'(1 - n\varepsilon); \varepsilon\right] = \frac{1}{\sqrt{2\pi\sigma}} \sum_{k=0}^{n} \frac{[N'(1 - n\varepsilon)]^k}{k!} \times \int_{-\infty}^{\infty} \exp\left\{ky - N'(1 - n\varepsilon)e^y - \frac{1}{\sqrt{2\sigma^2}}\left(y + \frac{\sigma^2}{2}\right)^2\right\} dy$$

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and $\sigma^2 = \langle y^2\rangle - \langle y\rangle^2$ are fluctuations of the logarithm of relative intensity $y = \ln(I/I_0)$; I and I_0 are the intensity of the radiation propagated through the atmosphere and the intensity of incident radiation. Scattering is ignored.

At the sampling time $T \ll \tau$, Eq. (2) describes the experimental results from Ref. 4 sufficiently accurate. But, at the sampling time, comparable with the correlation time of intensity fluctuations of laser radiation in the atmosphere, the distributions of photo readouts and the effect of DT on the distributions of photo readouts are poorly studied. In Ref. 6, we presented the results of experimental studies of the probability distribution of readouts at the sampling time comparable with the correlation time of intensity fluctuations. These studies were performed under laboratory conditions, and this enabled providing for stationarity and control of the main parameters of a path to a degree inaccessible in field atmospheric experiments.

The studies were conducted with various states of the induced turbulence, which corresponded to weak and moderate turbulence in actual atmosphere. As the experiments showed, the probability distribution of readouts of a non-Gaussian field at $T \approx \tau$ differs strongly from the known approximate distributions. As the distributions known are badly suit the description of experimental probability distributions at $T \approx \tau$, it was needed to find new distributions describing the experimental data. The first step was to find the dependence of the relative variance of photo readouts β_n^2 on the sampling time from the experimental results. It is known that in the case of lognormal intensity distribution at $T \ll \tau$, this parameter is connected with the variance of the logarithm of relative intensity σ^2 by the equation[7]:

$$\sigma^2 = \ln\,(1 + \beta_n^2). \qquad (3)$$

As is shown below, Eq. (3) keeps true at $T \approx \tau$, however in this case σ^2 has the meaning of the variance of logarithm of the relative integral intensity σ_U^2 and β_n depends on the exposure time T. At $T \to 0$ $\sigma_U^2 \to \sigma^2$.

Then, to describe experimental probability distributions at $T \approx \tau$, we can use Mandel equation[8]:

$$P(n, T) = \int\limits_0^\infty \frac{(\eta U)^n}{n!} \exp(-\eta U)\ \omega(U)\ \mathrm{d}U, \qquad (4)$$

where $U = \int\limits_0^\infty I(t)\mathrm{d}t$ is the energy measured with the detector during the sampling time; ω is the energy probability density; η is the quantum efficiency of the detector.

To solve Eq. (4), we should specify the intensity distribution. It is known that at $T \ll \tau$ the intensity distribution is unambiguously determined by the distribution of photo readouts through inversion of Eq. (4), and at the sampling time $T \approx \tau$ the distribution of photo readouts must determine the distribution of the integral intensity. Then, according to Eq. (4), the probability that during time T no readouts happen (at a given mean number of photo readouts N) is equal to

$$P(0, N) = \int \exp(-Nx)\ \omega(x)\mathrm{d}x, \qquad (5)$$

where $\omega(x)$ is the probability distribution of the normalized energy measured by the receiver, $x = U/\langle U\rangle$. Thus, $P(0, N)$ can be considered as Laplace transform of the function $\omega(x)$, and if the dependence $P(0, N)$ is known (for example, from experiment), then the corresponding $\omega(x)$ can be found by applying the inverse Laplace transform to the function $P(0, N)$:

$$\omega(x) = \frac{1}{2\pi i} \int\limits_{b+i\infty}^{b-i\infty} P(0, N)\ \exp(Nx)\mathrm{d}N. \qquad (6)$$

Expanding $\omega(x)$ into a series over Laguerre functions, after some transformations according to Ref. 8, we obtain a solution of the inverse problem in the form

$$\omega(x) = \sum a_n(x) P(n, N), \qquad (7)$$

where

$$a_n(x) = 2N(-2)^n \sum_{m=0}^{\infty} \binom{m}{n} l_n(2Nx) =$$

$$= 2N(-2)^2 \sum_{k=0}^{\infty} \binom{n+k}{n} l_{n+k}(2Nx)$$

and $l_n(y)$ are the Laguerre functions.

Dots in Fig. 1 denote the distributions $\omega(x)$ obtained by solving the inverse problem. The results are presented for one state of turbulence at different sampling time. The curves correspond to the lognormal distribution, for which the parameter σ^2 is assumed dependent on the sampling time T. In this case, the parameter of the lognormal distribution was the variance of the integral intensity σ_U^2 instead of the intensity variance σ^2 used earlier. The variance of the integral intensity can be found from the experimental results, as well as by the use of Eq. (3). As is seen from Fig. 1, the lognormal distribution with the parameter σ_U^2 describes the distribution of the integral intensity accurate enough. The lognormal character of the distribution of the integral intensity is the basis for applying Eq. (3) at $T \approx \tau$, because it was derived just for the lognormal distribution.

Some results of numerous experiments are shown in Fig. 1, these results suggest that the general character of the distribution of integral intensity is independent on the sampling time. Only the distribution parameter σ_U^2 depends on the sampling time.

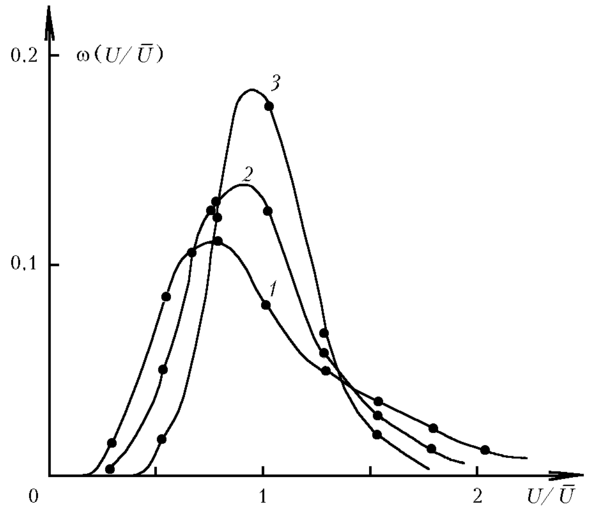

Fig. 1. Distribution of integral intensity. $\tau_c = 0.68$ ms. Dots are found from the distribution of photo readouts by solving the inverse problem, the curves are for the lognormal distribution: $\sigma_U^2 = 0.18$ and $T = 0.125$ ms (*1*), $\sigma_U^2 = 0.09$ and $T = 0.5$ ms (*2*), $\sigma_U^2 = 0.05$ and $T = 4$ ms (*3*).

One more conclusion that can be drawn from the above results, is the possibility of using the Diament–Teich distribution to describe the probability distribution of readouts in a wide range of sampling time, including $T \approx \tau$. To do this, it is sufficient to replace the variance of instantaneous intensity by the variance of integral intensity in the above distribution.

However, this conclusion does not allow Eq. (2) to be used to account for the DT effect on the probability distribution of readouts at $T \approx \tau$, because this equation was derived in the approximations based on the assumption that the sampling time is small as compared to the correlation time of intensity fluctuations. In this connection, we have developed an algorithm for numerical simulation of the experiment described above. Using the Monte Carlo method,[9] we have made a generator of the flux of readouts with a preset distribution. Below we give a detailed description of this algorithm.

It is known that the probability density of the length of an empty interval in the photoelectron flux is determined as[8]:

$$f(T) = \partial P(0, T)/(\partial T).$$

On the other hand, the probability distribution of photo readouts for a continuous-wave source of radiation is described by the Poison statistics; as a result, the probability of the empty readout is

$$P(0, T) = \exp(-I_0 t),$$

and, consequently, the distribution density of the length of the empty interval is

$$f_0(T) = I_0 \exp(-I_0 t).$$

To determine the length of the empty interval between two readouts in a sequence of random photoelectrons obeying the Poison statistics, it is necessary to solve the integral equation

$$x_0 = \int_0^t f_0(T)\mathrm{d}T, \qquad (8)$$

where x_0 is the random value uniformly distributed over the interval [0, 1].

Then

$$x_0 = \exp(-I_0 t)$$

and, consequently,

$$t = -\ln x_0/I_0. \qquad (9)$$

Then, drawing a random number x_0, from Eq. (9) we determine t_1 – the time of entry of the first readout during a sample of length T. If $t_1 > T$, then the sample is empty and the number of readouts in the zero channel of a histogram increases by one. If $t_1 < T$, then the counter contents increases by unity and a new random number x_0 is drawn, t_2 is determined, and the condition $t_1 + t_2 > T$ is checked. If the condition is fulfilled, then unity is added in the first channel of a histogram. Otherwise, the counter contents increase by unity, the next random number is drawn, and t_3 is determined. This process continues until the condition

$$t_1 + t_2 + \ldots + t_{n+1} > T \qquad (10)$$

is fulfilled.

As a result, we get the sample with n readouts and, consequently, increase the contents of the nth channel of the histogram by unity. To determine the number of readouts in the succeeding samples, the process is repeated. Repeating the process N_{tot} times, we obtain the histogram of distribution of the photo readouts. Then the obtained histogram is used for calculation of the probability of readouts, its mean value, and other parameters of the distribution.

To take into account the counter's DT in this algorithm, it is sufficient to introduce a gap time Δt after every readout with regard for the character and duration of the counter's DT. For the dead time of inextensible type, the algorithm is implemented especially easily. Thus, for example, if $t_i < \Delta t$, then under condition (10) the ith readout is missed (unity is not added to the counter contents), summation of t continues with the allowance for t_i, and the resulting histogram is thus distorted by the dead time.

In the case of fluctuating intensity $I \neq I_0 = \text{const}$, the modeling process becomes somewhat more complicated, because in this case it is needed to model random I with the given distribution and correlation of intensity fluctuations. In the absence of correlation, we can restrict ourselves to solution of the equation

$$x_0 = \int_0^I \omega(I)\mathrm{d}I,$$

where $\omega(I)$ is intensity distribution. In the case of lognormal intensity distribution, we have

$$x_0 = \int_0^{\ln I} \frac{1}{\sqrt{2\pi\sigma^2}} \exp[-(\ln I' - \langle \ln I' \rangle)^2/2\sigma^2] \, d\ln I'. \quad (11)$$

To take into account the effect of time correlation, the correlation function should be specified, in addition to the probability density.

The simplest solution of this problem consists in reduction of a non-Gaussian process to a Gaussian one through inertialess nonlinear transformation.[10] In the case of lognormal distribution, this transformation has the form

$$I_i = I_0 \exp(\sigma x_i - \sigma^2/2), \quad (12)$$

where x_i is a normally distributed random parameter with the variance equal to unity and the correlation function $\rho(t)$. In this paper, to describe the experimental correlation function of intensity fluctuations, we use the following empirical equation:

$$r(t) = 1/(1 + at), \quad (13)$$

where the parameter a was fitted to the experimental data.

The corresponding equation for $\rho(t)$ (see Ref. 10, p. 184) has the form

$$\rho(t) = \ln[r(t)(\exp(\sigma^2 - 1) + 1)]/\sigma^2, \quad (14)$$

and the values of x_i can be obtained using the recursion equation (see Ref. 10, p. 188)

$$x_i = \rho(t_i - t_{i-1})x_{i-1} + \sqrt{1 - \rho^2(t_i - t_{i-1})z_i}, \quad (15)$$

where z_i is the independent normally distributed random parameter; $x_1 = z_1$, and $t_i - t_{i-1}$ is the time interval, during which the intensity is thought constant (the sampling time T was divided into n intervals each $\Delta T = t_i - t_{i-1}$ long and it was assumed $\Delta T = \text{const} \le \tau$ for simplification).

To generate a flux with the preset distribution and correlation function, an independent normally distributed random parameter z_i is drawn and converted, using Eq. (15), into the normally distributed random parameter x_i with the preset correlation function (14). Substituting x_i in Eq. (12), we obtain the lognormally distributed intensity with the correlation function (13). Then, using Eq. (9) and the substitution $I_0 \to I_i$ according to the algorithm described above, the empty interval t is drawn and so on.

Using the described algorithm, we have conducted numerical experiments aimed at evaluation how the DT affects the statistics of photo readouts and, in particular, the central moments up to the fourth one, inclusive. Figure 2 shows the results of the experiments for one state of turbulence ($\sigma_U^2 = 0.678$) at different values of the ratio $\Delta t/T$. Here m_i is the normalized moment of the ith order, and m_{i0} is the normalized moment of the ith order of distribution not distorted by the DT effect. It is seen from the figure that the DT effect on the distribution moments up to $\Delta t/T < 10^{-5}$ is negligibly small. Starting from $\Delta t/T \approx 10^{-5}$, the DT effect increases gradually, and at $\Delta t/T \approx 10^{-4}$ the distortion of the fourth moment is 4–8%. Lower moments, as could be expected, experience smaller distortions, although at $\Delta t/T \approx 10^{-3}$ distortion of the first moment, i.e., the mean value, reaches 5–7%. At further increase of the ratio $\Delta t/T$, the effect of the dead time rapidly increases and experimental results become strongly distorted.

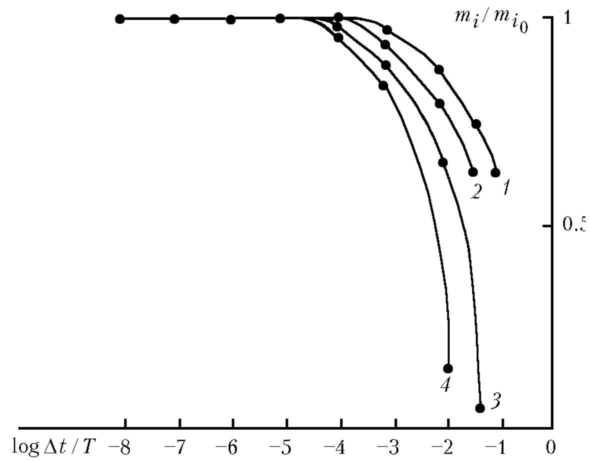

Fig. 2. Dependence of the normalized central moments on the ratio $\Delta t/T$: the mean (*1*), variance (*2*), asymmetry coefficient (*3*), and excess coefficient (*4*).

Thus, our studies showed that the proposed approach allows one to assess the effect of the dead time on the statistics of photo readouts and introduce the corresponding corrections for DT.

The results obtained could be useful in interpretation of experimental results obtained under field conditions in the atmosphere with photodetectors operating in the photon counting mode.

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