OPTIMIZATION OF THE HIGH-POWER HORN-TYPE ACOUSTIC EMITTERS

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The conditions are found that allow one to optimize the acoustic parameters of an exponential-type horn in high-power acoustic emitters designed to work in the atmosphere. It is shown that a consecutive numerical solution of the horn equation and the Khokhlov–Zabolotskaya equation allows an improvement in the accuracy of calculations in the problems on optimizing the acoustic parameters of high-power acoustic emitters.

Horn heads, having high efficiency, are widely used in high-power acoustic emitters (HPAE) designed to be operated in the atmosphere, for example, in acoustic sounding and sound broadcasting devices. The optimal horn design is often responsible for HPAE to realize its potentialities.

The range of range-finding systems using HPAE is one of the most important indicators of their efficiency. It was shown in Refs. 1 and 3 that numerical simulation with the use of the Khokhlov–Zabolotskaya–Kuznetsov (KhZK) equation provides a possibility of optimizing the parameters of powerful acoustic beam in order to achieve the maximum possible range. Sound propagation in that case was considered starting from the plane of the HPAE aperture ($z = 0$, z is the dimensionless longitudinal coordinate in the KhZK equation). However, before the sound wave, generated by a diaphragm of an electrodynamic converter, reaches the plane of HPAE aperture, it propagates in the HPAE horn and undergoes significant nonlinear distortions, that should be taken into account in the further consideration of the process of sound propagation in the atmosphere. Thus, the optimization of parameters of a separate horn as an element of the HPAE will be most efficient if done along with solving the problem of powerful acoustic beam propagation in the atmosphere.

In this paper we consider a numerical optimization of separate horn parameters simultaneously with the solution of KhZK equation,[1] i.e., with the regard for the sound beam diffraction, wave dissipation, and medium nonlinearity.

As known,[2] the exponential profile of a horn [$A(x) = A(0)\exp(\alpha_{\rm h}\, x)$, where $A(x)$ is the horn cross section, x is the longitudinal coordinate, and $\alpha_{\rm h}$ is the horn coefficient] is most efficient since it allows one to make the energy flux along a horn as much constant as possible. The one-dimensional wave equation for such a horn and solution to it are well-known. Written down for the sonic pressure, they have the form

$$\frac{\partial^2 P}{\partial t^2} = c_0^2\left(\frac{\partial^2 P}{\partial x^2} + \alpha_{\rm h}\frac{\partial P}{\partial x}\right); \qquad (1)$$

$$P = P_0 \exp[i\omega t - i(\omega\, c_0^{-2} - \alpha_{\rm h}^2/4)^{0.5}\, x - \alpha_{\rm h}\, x/2],$$

where $\omega = 2\pi f$, f is the sonic wave frequency, and c_0 is the speed of sound.

The view of solution to Eq. (1) does not change if the following condition is fulfilled:

$$\omega > \alpha_{\rm h}\, c_0/2\ . \qquad (2)$$

(for practical purposes it should be fulfilled with a large safety margin).

The limiting value $\omega = \alpha_{\rm h}\, c_0/2$ is the threshold frequency. For a horn, being a part of a multielement array, the threshold frequency is about a half of the lowest frequency in the operating frequency range of the whole array.

Nonlinear propagation of the wave along the horn with regard for distortions of the wave profile for a medium without dispersion can be followed with the help of the simple-wave equation allowing for the uniform shift of wave profile (the Riemann equation)

$$\frac{\partial V}{\partial T} + V\frac{\partial V}{\partial X} = 0, \qquad (3)$$

where

$$V = v/V_0(x);\;\; T = \int\limits_0^x V_0(x)\,{\rm d}x;\; X = \int\limits_0^x \frac{{\rm d}x}{c_0} - t;$$

$$v = 1/c_0 - 1/(u + c);\;\; c = c_0 + u(\gamma - 1)/2;$$

$$c_0^2 = \gamma P/\rho_0;\; \gamma = c_{\rm h}/c_V;$$

ρ_0 is the air density; $c_{\rm h}$ and c_V are heat capacity at constant pressure and volume, respectively; $V_0(x)$ is the scaling function of the form, e.g. $V_0(x) = [A(x)\rho_0]^{-0.5}\, c_0^{-5/2}$ for the ideal gas with the constant heat capacity limited by solid walls.

It can be shown that for the exponential horn the following relationship

$$x_h = -\frac{2}{\alpha_h}\ln\left(-\frac{\alpha_h\,\rho_0\,c_0^3}{(\gamma+1)\max(\partial P/\partial t)_{x=0}} + 1\right) \qquad (4)$$

holds true (x_h is distance to wave break in the horn), which follows from the solution of Eq. (3).

The relationships (2) and (4) yield the following system of conditions

$$-\frac{(\gamma+1)[\exp(-\alpha_h\,l/2)-1]}{\rho_0\,c_0^3}\max\left(\frac{\partial P}{\partial t}\right)_{x=0} < \alpha_h < \frac{2\omega}{c_0}, \quad (5)$$

$$\tan\varphi_h = \sqrt{\pi}\,r_0\,\frac{\alpha_h}{2}\exp(\alpha_h\,l/2),\ P < 0.2\ \text{MPa},$$

where l is the horn length; φ_h is the flare angle at the horn mouth; r_0 is the radius of horn throat. For the portion of sound energy reflected from horn mouth to throat to be negligible, the flare angle should be no less than 40° for a single horn and somewhat less for a horn in a multielement array. The system of conditions (5) allows one to promptly estimate the optimal parameters for a given horn.

To solve the problem of maximizing the range of HPAE operating in the atmosphere, the use of conditions (5) only is not sufficient. It is also necessary to perform numerical analysis of Eq. (3), that allows one to determine distortions of the wave profile and to estimate the upward transfer of the wave energy in spectrum. Then one should use the harmonic amplitudes, obtained when solving Eq. (3), at the plane of horn mouth as the initial condition for solving the KhZK equation. Such a numerical analysis was performed when preparing this paper. Let us note that Ref. 3 presents the description of the modified algorithm for numerical solution of the KhZK equation and the comparison of simulation results with the experimental data,[4] but the analysis was done without regard for nonlinear distortions at $z = 0$ (horn plane in a multielement HPAE).

In order to compare the simulation results with the experimental data, we, in this paper, used the data from Ref. 4 as well as the measurement results on the coefficient of nonlinear distortions (CND) at the mouth of a separate exponential horn, being a part of a multielement HPAE, for the first four harmonics of the sonic signal. CND varied from 2.5 to 21% when increasing the electric power delivered to HPAE from 25 to 3600 W. The horns comprising the HPAE described in Ref. 4 were close to the optimal criterion and had the following characteristics: r_0=0.009 m, l = 0.815 m, α_h=5.721.

By allowing for the system of conditions (5), Eq. (3) and the KhZK equation were numerically solved using the algorithm from Ref. 3. In so doing, the result of solution of Eq. (3) (harmonic amplitudes) was used in the determination of the initial conditions of the form

$$\rho_m(z = 0) = K_m\exp(-R^n)\cos(\omega_m\tau)$$

for the KhZK equation (n=16 corresponds to the hyper-Gauss beam, K_m are the harmonic amplitudes at the sound beam axis, m is the number of harmonic). Calculations were done using the following values of the initial parameters for solution of the KhZK equation (the designations are the same as in Refs. 1 and 3): $B = 0.5$, $N = 0.125$–1.5 (the peak sonic pressure P, generated by the electric power of 100 W, was 11.2 Pa), $z = 1.1$. The sonic energy dissipation was taken into account, when solving numerically the system of equations from Ref. 3, by replacing the product Mm^2, where M is the dissipation coefficient, in the KhZK equation by the current values M_m, calculated by formulas from Ref. 5. To obtain an accuracy of calculations not worse that 4%, it was sufficient to take into account ten harmonics and to make six steps in the range z. Comparison of the simulation results and experimental data has demonstrated their good agreement. The discrepancy is no more than 10% for 1–4 harmonics of the signal studied. Thus, the efficiency of the proposed scheme for numerical simulation of HPAE acoustic parameters is confirmed experimentally.

The result of optimization of HPAE design and acoustic parameters depends on some additional requirements imposed on the HPAE design: its size, electric power, etc. Therefore it is impossible to give some general recommendations for increasing the HPAE efficiency to satisfy a specific problem.

For example, the calculations showed that the maximum range of sound waves propagation in the atmosphere can be reached using the multielement HPAE, consisting of horns far from the optimal criterion (the system of conditions (5)) in their design. In this case the use of a shortened horn gives the best result.

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