RESTORATION OF PLANT CANOPY PARAMETERS FROM MEASUREMENTS OF THE SPECTRAL BRIGHTNESS COEFFICIENT

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An algorithm, for estimating the optical and geometrical parameters of plant canopies using bidirectional reflectance measurements of scattered solar radiation is described. The algorithm is based on the well-known Newton method. Formulas for calculating the derivatives of the spectral density coefficient with respect to the retrieval parameters by the Monte Carlo method and some results of model calculations for solving the inverse problem are given.

1. INTRODUCTION

In the last few years the rapid development of remote sensing of vegetation has made it necessary to develop algorithms for solving the inverse problems of determining the optical and geometrical parameters of plant canopies (PC) from data on the spectral brightness of the system "soil—vegetation". The development of plant canopy reflection models plays a key role in the solution of these problems. A complete review of existing models is contained in Refs. 1 and 2. Two fundamentally different approaches to the description of the PC radiation regime can be found:

1) modeling the vegetation by geometric figures with given dimensions and reflectance,

2) modeling the radiation transfer in a dense turbid medium filled with "leaves" of infinitessimally small dimensions with given distributions of their normal vectors and their optical properties.

While the first model is especially well-suited for the description of nonuniform thin crops, the second model, on the contrary, is used for a thick nonuniform PC in which the sizes of the phytoelements are smaller than the height of the PC.[3] Tere are also mixed models which account for the sizes of the lamella within the context of a turbid lamellar medium.[4,5] One of these models was used in Ref. 6 to solve the problem by the Monte Carlo method.

The first efforts at inverting reflection models for vegetation were performed by Goel et al. (see Ref. 1 and the references cited therein). They succeeded in solving the inverse problems for different models of uniform and nonuniform PC. The method for solving these problems consisted of minimizing a certain quadratic functional. In Ref. 7 the optical parameters of crops, a model of which is described in Ref. 10, were retrieved using a technique for solving inverse problems which makes use of the Monte Carlo method.[8]

The reflection model of Nilson-Kuusk was inverted by the same authors in Ref. 4. The retrieval technique was of such a kind that the retrieved parameters could not enter the "unphysical" region. The inversion procedure was used simultaneously for two sets of optical parameters in different spectral regions and one set of geometrical parameters.[4]

The aim of this paper is to invert the reflection model of Ref. 6 by using the Monte Carlo method. In this model the process of radiation transfer in the plant canopy is described by the integral transfer equation in a turbid lamellar medium whose elements have fixed dimensions. This equation is solved by the Monte Carlo method.

2. NOTATION AND STATEMENT OF THE PROBLEM

We will use the notation of Refs. 5 and 6. $x = (t, \Omega)$ is a point of the phase space x; $t = t(z)$ is the total leaf surface index at height z; Ω is the direction of motion of the photons immediately prior to the collision; $\Omega_0 = (\mu_0, \varphi_0)$, $\mu_0 < 0$, and $\Omega_0^* = (\mu_0^*, \varphi_0^*)$, $\mu_0^* > 0$ are the direction vectors of the solar radiation and observation, respectively; $\Omega_L = (\mu_L, \varphi_L)$ is a random leaf surface normal vector. The optical distance from the point z to the upper boundary of the layer in the direction Ω is $r(x) = t(z)G(\Omega)$, where

$$G(\Omega) = \frac{1}{2\pi} \int\limits_{\{\Omega^+\}} g_L(\Omega_L) |\Omega\Omega_L| d\Omega_L$$

is the average projection of the leaf normals onto the direction Ω. This quantity is equal to the extinction coefficient along the direction Ω not taking into account the sizes of the leaves. Here $g_L(\Omega_L)$ is the density of the leaf normal distribution

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$$g_L(\Omega_L) = \frac{1}{2\pi} g_\Theta(\Theta_L),$$

$g_\Theta(\Theta_L)$ is the density of the distribution of the polar angles Θ_L of the leaf normals[10]:

$$g_\Theta(\Theta_L)d\Omega_L = \left[2/\pi + b\cos 2\Theta_L + c\cos 4\Theta_L\right]d\Theta_L d\varphi_L.$$

Here the parameters b and c determine the distribution of the leaf tilt angles. The parameter[4–6]

$$\sigma_\kappa(t, \Omega', \Omega) = \begin{cases} G(\Omega), & \mu\mu' < 0, \\ G(\Omega)h_\kappa(t, \Omega', \Omega), & \mu\mu' > 0, \end{cases}$$

where

$$h_\kappa(t, \Omega', \Omega) = 1 - \left(\frac{G(\Omega')\mu}{G(\Omega)\mu'}\right)^{1/2} \exp\left(-\frac{\Delta(\Omega, \Omega')t}{\kappa H}\right),$$

$$\Delta(\Omega, \Omega') = \left(\mu^{-2} + \mu'^{-2} + 2(\Omega\Omega')/|\mu\mu'|\right)^{1/2},$$

and τ_κ is a new extinction coefficient, which accounts for the effect of glint and the relative size of the lamellas. Here $H = t(T)$ is the leaf surface index of a layer with height T. The conditional probability of scattering in the direction Ω from the direction Ω' is equal to

$$P(\Omega' - \Omega) = \frac{1}{2\pi}\int g_L(\Omega_L)|\Omega\Omega'|f(\Omega' - \Omega, \Omega_L)\, d\Omega_L/G(\Omega'),$$

where

$$f(\Omega' - \Omega, \Omega_L) = \begin{cases} r_L|\Omega\Omega_L|/\pi, & (\Omega'\Omega_L)(\Omega'\Omega_L) < 0, \\ t_L|\Omega\Omega_L|/\pi, & (\Omega'\Omega_L)(\Omega'\Omega_L) > 0, \end{cases}$$

is the reflection-transmission phase function of the Lambertian surface of a leaf with diffuse reflection coefficient r_L and transmission coefficient t_L. Denoting the albedo of the underlying surface by r_s, we obtain a set of seven parameters b, c, κ, H, r_L, t_L, and r_s to be retrieved.

Statement of the inverse problem: a monodirectional flux of solar radiation falls on the outer surface of a planar layer of plants. Detectors which measure the spectral brightness coefficient (SBC) of the vegetation from which the three optical parameters of the layer (r_L, t_L, and r_s) and the four geometrical parameters (b, c, κ, and H) are to be determined are located on the upper boundary of the layer. The geometrical parameters are more important because they are difficult to measure.

We assume that 1) the detector is located not high above the upper boundary of the layer and 2) the optical density of the atmosphere is negligibly small in comparison with the optical density of the vegetation layer. Hence, atmospheric scattering of light inside the vegetation layer and along the trajectory from the upper boundary of the layer to the radiation detector can be neglected. To account for the effect of the atmosphere on the downwelling solar radiation flux, the diffuse component should be added to the monodirectional flux. This has no effect on the solution algorithm of the problem, hence this effect was not taken into account in the first numerical experiments with the model problem.

3. SOLUTION ALGORITHM

The retrieval parameters are included in the transfer equation for the intensity in a complicated way, hence it is difficult (or even impossible) to form a closed analytical equation for them. In addition, these attempts should lead to different equations for different parameters. The use of a universal algorithm is preferable. We choose a modification of the Newton method.[12] Let $\alpha_1^*, \ldots, \alpha_n^*$ be the unknown values of the parameters $\alpha_1, \ldots, \alpha_n$, which are to be retrieved. Let $R_k \equiv R_k(\alpha_1, \ldots, \alpha_n)$, $k = 1, \ldots, N > n$, be the SBC's along the direction of sighting Ω^* for vegetation with the parameters $\alpha_1, \ldots, \alpha_n$, respectively. Let $R_k^* \equiv R_k(\alpha_1^*, \ldots, \alpha_n^*)$ be the measured SBC's. To determine $\alpha_1^*, \ldots, \alpha_n^*$ we consider the system of nonlinear equations

$$R_k(\alpha_1, \ldots, \alpha_n) = R_k^*, \quad k = 1, \ldots, N, \tag{1}$$

where R_k are quite complicated functions. Solution of this system by the Newton–Kantorovich method[11] leads to the iterations

$$\alpha_i^{l+1} = \alpha_i^l + \delta_i^l, \quad i = 1, \ldots, n, \quad l = 0, 1, \ldots, \tag{2}$$

where δ_1^l satisfy the system of linear equations

$$\sum_{i=1} \frac{\partial R_k}{\partial \alpha_i}\delta_1^l = R_k^* - R_k(\alpha_1^l, \ldots, \alpha_n^l),$$

$$k = 1, \ldots, n. \tag{3}$$

The prognostic values $\alpha_1^0, \ldots, \alpha_n^0$ are taken as the initial approximation of the retrieval parameters. Procedure (2) is repeated until the inequality

$$|R_k^* - R_k(\alpha_1^l, \ldots, \alpha_n^l)| < \varepsilon$$

is satisfied, where ε is some given small number. System (3), generally speaking, is overdetermined. It can be solved by the method of least squares. If the system is ill-defined, regularization may be used.[12] The values of the SBC s and their derivatives are computed by the Monte Carlo method(see Eqs. (4) and(5)).

Sometimes it is convenient to employ a modification of the above scheme. Let us represent the value of R as the sum $R = R_1 + (R - R_1)$, where R_1 is the

SBC of the singly scattered photons. Often in the important cases (e.g., in the spectral range of photosynthetically active radiation) the role of singly scattered radiation in the total flux is great. It can be readily verified that this assumption is also valid for the corresponding derivatives. Hence, it is natural to attempt to substitute the derivatives $dR_1/d\alpha$ for the derivatives $dR/d\alpha$ in system (3). The advantage of this substitution is a direct consequence of the algorithm used for calculating the derivatives (Section 5), namely, the derivatives $dR_1/d\alpha$ are calculated faster and more accurately than the derivatives $dR/d\alpha$. Of course, the theoretical rate of convergence in this case is slightly lower. However, in practice this may be compensated by the greater accuracy of the calculation derivatives $dR_1/d\alpha$. In addition, the relative difference of the derivatives $dR/d\alpha$ and $dR_1/d\alpha$ is not large for small values of the leaf surface index.

4. CALCULATION OF SBC BY THE MONTE CARLO METHOD

Let $J(t,\ \Omega)$ be the value of intensity of the scattered radiation at the point $(t,\ \Omega)$. The function J satisfies the integral equation[6]

$$J(t,\ \Omega) = \int_0^H \int_{\{\Omega\}} k[(t',\ \Omega') - (t,\ \Omega)] \times J(t',\ \Omega')\, dt' d\Omega' + F(t,\ \Omega), \tag{4}$$

where

$$F(t,\ \Omega) = I_0 \delta(\Omega - \Omega_0) G(\Omega) \exp[-G(\Omega)t]/|\mu|,\ \mu < 0, \tag{5}$$

I_0 is the radiation intensity incident along the direction Ω_0, and the kernel $k(x' - x)$ ($x = (t,\ \Omega)$ is a point in the phase space X) is equal to

$$k[(t',\ \Omega') - (t,\ \Omega)] = \begin{cases} \dfrac{1}{|\mu|}\exp\left[\dfrac{1}{\mu}\displaystyle\int \sigma_K(\tau,\ \Omega',\ \Omega)d\tau\right] P(\Omega' - \Omega)G(\Omega), & (t - t') < 0, \\ 0, & (t - t') > 0. \end{cases} \tag{6}$$

The SBC along the direction of sighting can be found by solving Eq. (4):

$$R(\Omega^*) = J(0,\ \Omega^*)/[G(\Omega^*) I_0 |\mu_0|]. \tag{7}$$

Equation (4) is solved by the Monte Carlo method.[6,8] Let us write it in the operator form $J = KJ + F$. It is known that when $\|K\| < 1$ the solution of Eq. (4) can be represented by a Neumann series[11]

$$J = F + KF + K^2F + \dots .$$

Let the scalar product in the space X be of the form

$$(\psi,\ \varphi) = \int_X \psi(x)\varphi(x)\, dx = \int_0^H \int_{\{\Omega\}} \psi(t,\ \Omega)\varphi(t,\ \Omega)\, dt\, d\Omega.$$

Then

$$J(0,\ \Omega^*) = J(x^*) = (J,\ \delta_{x^*}) =$$
$$= (F,\ \delta_{x^*}) + (KF,\ \delta_{x^*}) + (K^2F,\ \delta_{x^*}) + \dots$$
$$= (F,\ K^*\delta_{x^*}) + (KF,\ K^*\delta_{x^*}) + (K^2F,\ K^*\delta_{x^*}) + \dots \tag{8}$$

Here $x^* = (0,\ \Omega^*)$, δ_x^* is the delta-function, and K^* is the operator conjugate to K

$$[K\ \delta^*_{x^*}](x) = \psi(x)G(\Omega^*), \tag{9}$$

$$\psi(x) = \Psi(t,\ \Omega) = \frac{P(\Omega - \Omega^*)}{\mu^*} \times \exp\left[-\frac{1}{\mu^*}\int_0^t \sigma_K(t',\ \Omega,\ \Omega^*)\, dt'\right]. \tag{10}$$

From Eqs. (5), (7), (8), and (9) it follows that

$$R(\Omega^*) = (Q,\ \Psi) + (KQ,\ \Psi) + (K^2Q,\ \Psi) + \dots, \tag{11}$$

where Q is the initial collision density

$$Q(t,\ \Omega) = F(t,\ \Omega)\pi/I_0|\mu_0| = \frac{\pi G(\Omega)}{|\mu|}\exp\left[-\frac{G(\Omega)}{|\mu|}t\right]\delta(\Omega - \Omega_0). \tag{12}$$

Expression (11) corresponds to the following algorithm based on the Monte Carlo method. The Markov chain $x_0^n - x_1^n - \dots - x_m^n$ with transition density k is numerically simulated, where m is the random number of the last collision for the nth trajectory. After the collision at the point x_1^n the value $W_1^n\psi(x_1^n)$ is added to the estimate R, and correspondingly for the other collisions, so that finally we have

$$R(\Omega^*) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{i=0}^{m} W_i^n \psi(x_i^n), \tag{13}$$

where N is the number of trajectories and W_i^n is the weight of the nth photon after the ith collision.

For the reflection from the soil the contribution function is equal to

$$\psi_s(t,\ \Omega) = \frac{r_s|\mu|}{\pi G(\Omega)} \times$$

$$\times \exp\left[-\frac{1}{\mu^*}\int_0^t \sigma_K(t', \Omega, \Omega^*)\, dt' \right]\delta(t - H),$$

where r_s is the SBC of the soil, and $|\mu^*|/\pi$ is the angular density of scattering from the underlying Lambertian surface.

5. CALCULATION OF SBC DERIVATIVES BY THE MONTE CARLO METHOD

Let us write the Neumann' series (11) in compact form

$$R = \int Q_0\psi_0 + \int Q_0K_{01}\psi_1 + \int Q_0K_{01}K_{12}\psi_2 + \dots \quad (14)$$

Here $Q_0 = Q(x_0)$, $\psi_0 = \psi(x_0)$, $k_{12} = k(x_1 - x_2)$. To calculate the derivative $\partial R/\partial\alpha$ we differentiate the series (14), denoting $\partial f/\partial\alpha$ as f':

$$R = \int \left[Q_0'\psi_0 + Q_0\psi_0'\right] +$$

$$+ \int \left[Q_0'K_{01}\psi_1 + Q_0K_{01}'\psi_1 + Q_0K_{01}\psi_1'\right] + \dots$$

$$= \int Q_0\psi_0\left[\frac{Q_0'}{Q_0} + \frac{\psi_0'}{\psi_0}\right] + \int Q_0K_{01}\psi_0\left[\frac{Q_0'}{Q_0} + \frac{K_{01}'}{K_{01}} + \frac{\psi_0'}{\psi_0}\right] +$$

$$= \int Q_0\psi_0W_0 + \int Q_0K_{01}\psi_0W_{01} + \dots$$

The latter representation corresponds to the following calculation algorithm. We model the Markov chains as before, and after each collision at the point x the contribution $\psi(x)W$ is added to the estimate of $\partial R/\partial\alpha$. Thus, the derivatives are calculated from the same trajectories as the SBC's. A rigorous grounding of the calculation algorithm is given in Ref. 8. It is evident that the calculational requirements of the derivative calculation in comparison with those of the SBC's is determined by the computational requirements of the calculation of the factor $W_{0\dots i}$.

As has already been mentioned, the accurate calculation of all the values $W_{0\dots i}$ demands a considerable expense of computer time. Sometimes one can simplify the problem and calculate the derivative only from the first term in Eq. (11). Let $R_1 = \int Q_0\psi_0$. Then it is sufficient to calculate the factor $W_0 = Q_0'/Q_0 + \psi_0'/\psi_0$ to estimate the derivative $\partial R_1/\partial\alpha$.

Let us write an expression for R_1. There are two types of collisions — on the leaf surfaces and on the soil surface

$$R_1 = R_{1L} + R_{1s}, \quad (15)$$

where R_{1L} is the SBC of radiation singly scattered from the leaves and R_{1s} is the corresponding value for the soil. From Eqs. (10)–(12) we obtain

$$R_{1L}(\Omega^*) = \pi\frac{G(\Omega)}{|\mu_0|}\,\frac{P(\Omega - \Omega^*)}{\mu^*} \times$$

$$\times \int_0^H \exp\left[\frac{G(\Omega)}{|\mu_0|}t - \frac{1}{\mu^*}\int_0^t \sigma_K(t', \Omega, \Omega^*)\, dt'\right]dt, \quad (16)$$

$$R_{1s}(\Omega^*) = r_sG(\Omega_0) \times$$

$$\times \exp\left[-\frac{G(\Omega_0)}{|\mu_0|}H - \frac{1}{\mu^*}\int \sigma_K(t', \Omega, \Omega^*)\, dt'\right]. \quad (17)$$

Now let us consider statistical estimates of $\partial R/\partial\alpha$ for specific values of the parameters α.

a) The albedo of the underlying surface, r_s. It is evident that only R_{1s} depends on r_s, hence $W_0 = 1/r_s$ and

$$\frac{\partial R_1}{\partial r_s} = \int Q_0\psi_0W_0 =$$

$$= G(\Omega_0)\exp\left[-\frac{G(\Omega_0)}{|\mu_0|}H - \frac{1}{\mu^*}\int \sigma_K(t', \Omega, \Omega^*)\, dt'\right].$$

b) The reflection and transmission coefficients of the leaves, r_L and t_L. Only the reflection phase function f of the leaves, which enters into the formula for $P(\Omega - \Omega^*)$, depends on these parameters. Hence,

$$W_0 = \frac{\psi_\alpha'}{\psi} = \frac{P_\alpha'(\Omega_0 - \Omega^*)}{P(\Omega_0 - \Omega^*)} = \frac{f_\alpha'(\Omega' - \Omega, \Omega)}{f(\Omega' - \Omega, \Omega)},$$

$$W_{0r} = \begin{cases} |\Omega\Omega_L|/\pi, & (\Omega\Omega_L)(\Omega'\Omega_L) < 0, \\ 0, & (\Omega\Omega_L)(\Omega'\Omega_L) > 0, \end{cases}$$

$$W_{0r} = \begin{cases} 0, & (\Omega\Omega_L)(\Omega'\Omega_L) < 0, \\ |\Omega\Omega_L|/\pi, & (\Omega\Omega_L)(\Omega'\Omega_L) > 0, \end{cases}$$

c) The parameters b and c, which characterize the density of the distribution of the orientations of the leaf normals. Both of the functions ψ and Q depend on these parameters

$$\frac{Q_\alpha'}{Q} = \frac{G_\alpha'}{G} - \frac{t}{\mu}G_\alpha' ;$$

$$\frac{\psi_\alpha'}{\psi} = \frac{P_\alpha'(\Omega_0 - \Omega^*)}{P(\Omega_0 - \Omega^*)} - \int_0^t \left[-\frac{1}{\mu}\sigma_K(t', \Omega_0, \Omega^*)\right]_\alpha' dt'$$

Finally, substituting one of the parameters b or c for a yields

$$W_{0\alpha} = Q_\alpha'/Q + \psi'_\alpha/\psi,$$

where the integral in the second term is calculated analytically.

d) The parameter H — the leaf surface index of the SBC. From Eq. (15) it is clear that the upper limit of the integration as well as the quantity σ_K depends on H. Differentiating R_{1L} with respect to the parameter H, we obtain the analytical expression

$$\frac{\partial R_{1L}}{\partial H} = \pi \frac{G(\Omega_0)}{|\mu|} \frac{P(\Omega_0 - \Omega^*)}{\mu^*} \times$$

$$\times \left\{ \exp\left[-\frac{G(\Omega_0)}{|\mu_0|}H - \frac{1}{\mu^*}\int\limits_0 \sigma_\kappa(t', \Omega_0, \Omega^*)\, dt' \right] - \right.$$

$$- \frac{\partial}{\partial H}\left[-\frac{1}{\mu^*}\int\limits_0 \sigma_\kappa(t', \Omega_0, \Omega^*)\, dt' \right] -$$

$$\left. - \int\limits_0^H \exp\left[-\frac{G(\Omega_0)}{|\mu_0|}t - \frac{1}{\mu^*}\int \sigma_\kappa(t', \Omega_0, \Omega^*)\, dt' \right] dt \right\}.$$

Thus, the derivative of R_{1L} with respect to H is equal to the sum of two terms. The first of these is a fixed analytical expression, and the second is the mathematical expectation of the statistical estimate of R_{1L}, multiplied by the weight factor

$$W = -\frac{\partial}{\partial H}\left[-\int\limits_0^t \sigma_\kappa/\mu^* \right].$$

Analogously,

$$\frac{\partial R_{1s}}{\partial H} = R_{1s}\left\{ \frac{G(\Omega_0)}{|\mu_0|} + \frac{\sigma_\kappa(H, \Omega, \Omega^*)}{\mu^*} + \right.$$

$$\left. + \int\limits_0^H \frac{\partial}{\partial H}\left[-\sigma_\kappa/\mu^* \right] dt' \right\}.$$

e) The parameter κ, which characterizes the leaf sizes. Only the contribution function ψ depends on it

$$W = \frac{\psi'_\alpha}{\psi} = \int\limits_0^t \frac{\partial}{\partial \kappa}\left[\sigma_\kappa(t', \Omega_0, \Omega^*) \right] dt'.$$

6. CALCULATIONAL RESULTS

Here we give results of some model calculations for retrieving the parameters of the vegetation layer. The general scheme of the calculation is as follows: first we calculate the SBC of the transmitted and reflected radiation along different directions for some model medium. Second, the calculated values of the SBC's are used to successively retrieve the model parameters. The choice of variants of the geometry of observation depends on the parameters being retrieved. The location of the detector and the direction of observation are chosen such that 1) the fraction of singly scattered radiation should be as high as possible, and 2) the retrieval parameter (or parameters) can be restored from the values of the singly scattered radiation.

In the calculations the solar zenith angle $\theta_\odot$ was 40°, and the azimuthal angle of observation (excluding the variant for retrieving the phase function parameters b and c) φ was equal to 0°. The parameters of the detector (the height z and the zenith angle of observation θ) are given below:

1) H: $z = 0$; $\theta = 50°$, the transmitted radiation is being observed, the angle θ is close to $\theta_\odot$ ($\theta \neq \theta_\odot$, to eliminate direct radiation from the Sun on the detector).

2) κ: $z = T$; $\theta = 210°$, the reflected radiation is being observed (it is "more sensitive" to values of κ than is the transmitted radiation), the angle θ is close to $\theta_\odot + 180°$ (when $\theta = \theta_\odot + 180°$, the singly scattered radiation is independent of κ, see the formula for σ_κ (Eq. (2)).

3) r_s: $z = T$; $\theta = 180°$.

4) r_L, t_L: two detectors; $z_1 = z_2 = T$; $\theta_1 = 230°$, $\varphi_1 = 0°$; $\theta_2 = 230°$, $\varphi_2 = 90°$. There were some attempts to use different directions of observations, however, without positive results.

The standard scheme or a modified scheme is used depending on the selected variant. For fair restoration, the number of iterations is chosen in such a way that it becomes possible to obtain two significant figures in the retrieval parameter.

To start with, we considered a vegetation layer with leaf surface index $H = 4$. The retrieval parameters of the model and the results of their restoration are given in Table I. The retrieval parameters are given in the first row, the initial approximations of these parameters are given in the second row; the number of iterations of the modified scheme needed for retrieval of the parameter with the accuracy given below are given in the third row; the numbers of iterations of the standard scheme are given in the fourth row; and the relative error of restoration in percents are given in the fifth row. It was possible to retrieve the parameter κ rapidly using the modified scheme, hence the standard scheme was not used here; retrieval of the parameters b and c by the standard scheme failed; therefore the table gives only the numbers of the iterations for the modified scheme.

TABLE I

H	κ	r_s	r_L	t_L	b	c
4	0.08	0.2	0.46	0.46	1	1
2	0.04	0.1	0.20	0.20	0	0
5	2	5	3	3	5	5
2	—	2	3	3	—	—
2%	1%	2%	1%	2%	20%	30%

An analogous calculation for retrieval of the parameters b and c is performed for $H = 2$. We succeeded in retrieving these parameters with an error of approximately 10% using five iterations.

Thus, as was expected, the above problem is solved more easily with smaller values of the index H. For large values of H, the proposed algorithms do not allow one to retrieve the parameters b and c accurately and quickly. In connection with this, attempts are being undertaken to develop a more effective modification of the described algorithm.

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