Modeling of longitudinal discharge plasma resistance used in pumping repetitively pulsed gas-discharge lasers

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The group of invariant transformations for a system of differential equations describing evolution processes of plasma of the high-current phase of repetitively pulsed gas-discharge lasers is found. Transformation invariants are determined and used to find the dependence of discharge resistance on time and discharge parameters. It is shown that at certain invariant values the determined theoretical time dependence of the discharge plasma resistance is in good agreement with the experimental one.

Numerous publications are devoted to the study of low-temperature nonisothermal pulsed nanosecond gas discharges. Such discharges are widely used in various fields of science and technology. Despite the existence of authoritative monographs on the problem,[1–3] opinions on the methods of modeling plasma parameters and dynamics at different periods of development of such discharges are still contradictory.

Voltage-time characteristics of discharges of pulsed gas-discharge lasers (GDL) with different active media are known to be similar in time, if identical pumping schemes are used. The presence of this group sign is indicative of the possibility of using group analysis methods for the search of invariant (scale) transformations of the discharge plasma and the following determination of typical time dependences of plasma parameters for definite phases of the discharge evolution.

Group analysis of the system of differential equations describing kinetics of gas-discharge plasma

Consider the plasma at the high-current stage of repetitively pulsed longitudinal low-temperature gas discharge with constant chemical composition under conditions of a high prepulse electron concentration and a pulse length of several hundreds of nanoseconds. It is known[3] that the evolution of such plasma is due to the volume ionization of normal and excited states of gas atoms by electrons accelerated by an external pulse of the electric field. The processes of plasma decay due to ambipolar diffusion and collisional-radiative recombination proceed much slower and can be neglected in the case of pulse currents shorter than several hundreds of nanoseconds.

Write the system of differential equations describing the kinetics of plasma of the positive column of pulsed discharge in a gas in the dimensionless form:

$$\begin{cases} \dfrac{dn_e^*}{d\tau} = n_e^* \sum_{k=0}^{m} \nu_{ki}^*, & (1) \\ \dfrac{dn_m^*}{d\tau} = n_e^* \left(\sum_{k=0}^{m-1} \nu_{km}^* - \nu_{mi}^* \right), & (2) \\ \dfrac{d(\varepsilon^* n_e^*)}{d\tau} = \dfrac{e^2 n_e^*}{m_e \varepsilon_{0i} \nu_i^0} \dfrac{E^2(\tau)}{\nu_{el}} - n_e^* \sum_{k=0}^{i-1} \varepsilon_{ki} \nu_{ki}^*, & (3) \end{cases}$$

where $\tau = t\nu_i^0$ is the reduced time of the discharge development; ν_i^0 is the total ionization frequency of some easily ionized admixture at the moment of beginning of the discharge high-current stage; $\nu_{ki}^* = \nu_{ki}/\nu_i^0$ is the reduced frequency of single ionization of atoms in admixture by electrons from the state k; $\nu_{el} = (\sum_{k=0}^{i-1} \nu_{ki} + \nu_l)$ is the total frequency of inelastic collisions of electrons with admixture atoms and elastic collisions with buffer gas molecules, respectively; $n^* = n_0/n_e^0$ is the concentration of atoms of an easily ionized admixture reduced to the electron concentration at the beginning of the discharge high-current stage; $n^* = n_e^* + \sum_{k\ge 0}^{m} n_k^*$, $n_e^* = n_e/n_e^0$; and $n_k^* = n_k/n_e^0$ are the reduced concentrations of gas atoms, plasma electrons, and atoms excited to the level k, respectively; $\varepsilon^* = \varepsilon/\varepsilon_{0i}$ is the reduced mean energy of plasma electrons; ε_{ki} is the energy of ionization of gas atoms from the level k; $E(\tau)$ is the change in the electric field strength at the discharge plasma during the current pulse.

Using equations of the system (1)–(3), we can transform the equation of energy balance of the electron gas (3):

$$\frac{d\varepsilon^*}{d\tau} + \varepsilon^* \sum_{k=0}^{m} \nu_{ki}^* = \frac{e^2}{m_e \varepsilon_i \left(\nu_i^0\right)^2} \frac{E^2(\tau)}{\nu_{el}^*} - \sum_{k=0}^{i-1} \varepsilon_{ki} \nu_{ki}^*. \quad (3a)$$

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It is seen that the variation of ε^* in time does not depend explicitly on the electron concentration, but is determined by its initial value and by the law of the electric field variation. Similarly to Eq. (3a), other kinetic equations of the system are first-order linear differential equations with coefficients depending on the electron energy and the concentration of excited atoms, that is, are indirectly time dependent.

Determine the invariant transformations allowed for the system of differential equations of plasma kinetics. The reduced total ionization frequency of a gas $\sum_{k=0}^{m}\nu^*_{ki}$ is designated as ν^*_i; and the reduced frequency of excitation of the mth level $\sum_{k=0}^{m-1}\nu^*_{km} - \nu^*_{mi}$ is designated as ν^*_m. Using the algorithm of searching for group transformations of a system of differential equations,[4] we obtain that the system of kinetic equations permits the transformation group:

$$\tau' = \tau e^a,\ \ n'_e = n^*_e e^a,\ \ n'_m = n^*_m e^a,$$

$$\nu'_i = \nu^*_i e^{-a},\ \ \nu'_m = \nu^*_m e^{-a},$$

where $m = 0, 1 \ldots i-1$, with the group operator

$$X = \tau\frac{\partial}{\partial\tau} + n^*_e\frac{\partial}{\partial n^*_e} + n^*_m\frac{\partial}{\partial n^*_m} - \nu^*_m\frac{\partial}{\partial \nu^*_m} - \nu^*_i\frac{\partial}{\partial \nu^*_i}.$$

Invariants of this transformation for the kinetic equations of discharge plasma for the system (1) and (2) are:

$$I_1 = \tau\nu^*_i,\ \ I_2 = \frac{\nu^*_m}{\nu^*_i},\ \ I_3 = \frac{n^*_e}{n^*_m}. \qquad (4)$$

It should be noted that when deriving the invariants for equations (1) and (2), we impose no conditions on the mean electron energy and the form of the electron distribution function. Similarly, we can find the group of invariant transformations for all equations of the system:

$$\tau' = \tau e^a;\ \ \nu'_i = \nu^*_i e^{-a};\ \ \nu'_m = \nu^*_m e^{-a};\ \ n'_e = n^*_e \cdot e^a;$$

$$n'_m = n^*_m e^a;\ \ \varepsilon' = \varepsilon^* e^{-a},$$

and the group operator

$$X = \tau\frac{\partial}{\partial\tau} + n^*_e\frac{\partial}{\partial n^*_e} + n^*_m\frac{\partial}{\partial n^*_m} - \nu^*_i\frac{\partial}{\partial \nu^*_i} - \nu^*_m\frac{\partial}{\partial \nu^*_m} - \varepsilon^*\frac{\partial}{\partial \varepsilon^*}.$$

All possible changes of variables τ, n^*_e, n^*_k, ν^*_i, ν^*_k, ε^*, permitted by the system, form the Lie group. Automodel changes form its one-parameter stretching subgroup. The set of first integrals of the equation [invariants of this transformation for kinetic equations of the discharge plasma for the system (1)–(3)] $\tilde{X}\varphi = 0$ is the following:

$$I_1 = \tau\nu^*_i;\ \ I_2 = \tau\nu^*_m;\ \ I_3 = \frac{n^*_e}{\tau};\ \ I_4 = \frac{e^2E^2(\tau)\tau}{m_e\nu^*_{el}\varepsilon_i};\ \ I_5 = \varepsilon^*\tau. \qquad (5)$$

It should be noted that the values of the invariants $I_2 - I_4$ of transformation of the system (5) coincide with those of the invariants of transformation of the Boltzmann equations for different plasma particles,[5,6] while the invariants

$$I_1 = \tau\nu^*_i,\ \ I_5 = \varepsilon^*\tau$$

determine the dynamic similarity of evolution of the discharge plasma.

The automodel solution of the equations of system (1)–(3) permitting the stretching group has the form

$$u(\tau^*, \varepsilon^*, n^*_e, E) = \tau^{\gamma/\beta}\psi\left(\frac{\varepsilon^*}{\tau^{\alpha/\beta}}, \frac{n_e}{\tau^{\chi/\beta}}, \frac{E^2}{\nu_{el}\tau^{\xi/\beta}}\right),$$

ψ is the new sought function.

The form of the sought function ψ depends on the pumping scheme, which forms the pump pulse and determines the variation of the electron temperature during the discharge evolution. For the same pumping scheme (I_4 = idem), ψ is identical, but the dependences of the plasma parameters described by the automodel solutions of the equations undergo the stretching in time and in amplitude proportionally to the change in the scales (initial conditions): ν^0_i, n^0_e, n^0_k, ν^0_k, ε_{0i}.

Law of discharge plasma resistance variation

Further, concentrate our attention on the discharge with a low degree of ionization and neglect the electron heating of the gas. It is known[4] that if a differential equation has invariants, then it can be solved analytically.

From Eq. (1) we can find the law of the discharge gap resistance variation during the current pulse using the invariant $I_1 = \tau\nu_i$. Integrate Eq. (1)

$$\int_{n'_e}^{n_e}\frac{dn_e}{n_e} = I_1\int_{\tau'}^{\tau}\frac{d\tau}{\tau} \qquad (6)$$

and obtain

$$n_e = n'_e\left(\frac{t\nu^0_i}{\tau'}\right)^{I_1}, \qquad (7)$$

where $\tau' = t'\nu^0_i$ is the delay of the beginning of the high-current (arc) discharge phase after the moment of the voltage applying to the discharge gap ($t = 0$). Then the plasma resistance of the cylindrical discharge with the length l and the cross section S can be represented as

$$R = \frac{m_e\nu_{el}l}{e^2Sn_e} = \frac{R'\nu^*_{el}}{(t/t')^{I_1}}, \qquad (8)$$

where ν_{el}/ν'_{el} is denoted as ν^*_{el}; R' is the plasma resistance at the moment t'.

In the absence of the buffer gas, $\nu^*_{el} = \nu^*_i$ and dependence (8) takes the form

$$R = \frac{R' I_1}{(t/t')^{I_1+1}}.$$

Comparison with experimental results

Figure 1 shows the typical voltage-time characteristics of the longitudinal discharge of repetitively pulsed gas-discharge barium-vapor lasers with a peaking-capacitor pump source.[3,7]

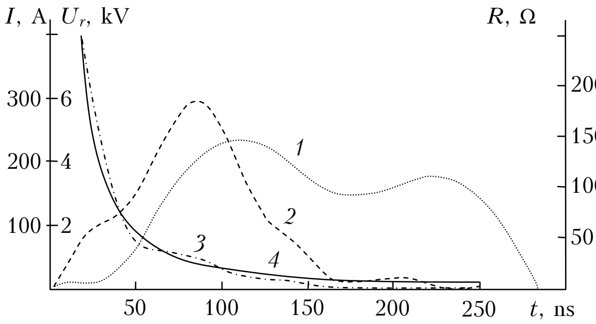

Fig. 1. Oscillograms of the current pulse (*1*), voltage drop across the plasma (*2*), active resistance $R(t_i)$ (*3*) and its approximation (*4*).

A discharge tube with a discharge length of 40 cm and an inner diameter of 20 mm was studied. A TGII-1000/25 hydrogen thyratron was used as a switch. The inductance of the discharge circuit was 0.7 μH. The pressure of the buffer gas neon was 15 Torr, the capacity of the reservoir capacitor amounted to 2500 pF, and that of the peaking capacitor was 1000 pF. The pulse repetition frequency was 8 kHz. The technique of measurement of the current I and the voltage across the discharge gap U_c was similar to that described in Ref. 1. The voltage across the discharge gap U_r was calculated as a difference between the voltage, measured across the peaking capacitor, and the calculated voltage of the inductance coil U_L. The slope of the current variation at any time was calculated through differentiation of the current curve and extrapolation by a polynomial based on seven experimental points. For this purpose, oscillograms of current pulses, as well as those of voltage, were divided into time intervals, each containing seven points. Instantaneous values of discharge resistance in each interval at the high-current stage were calculated as

$$R(t_i) = U_r(t_i)/I(t_i),\ i = 7.$$

The result of approximation of the discharge resistance by Eq. (8) is shown in Fig. 1 by solid curve *4*. The best agreement with the experiment is observed at $R' = (650 \pm 60)\ \Omega$, $t' = (10 \pm 0.1)$ ns, and $I_1 = 1.5$. As the current pulse repetition frequency increases up to 10 kHz, the value of R' keeps within the error of approximation at $t' = 9.3$ ns. The study of resistance of the GDL plasma with other active media of the copper-vapor laser and the strontium laser with the buffer gas neon has shown that the best approximation of experimental results by Eq. (8) is achieved at $I_1 = 1.5–1.4$. The calculated values of $I_1 = t'\nu^0_i$ for the studied GDL active media are close to unity.

The variation of ν^*_{el} for neon during a pulse is proportional to $\sqrt{\varepsilon}$ [Ref. 8]. Taking into account Eq. (5), this gives the change in time as $1/\sqrt{t}$. Consequently, the theoretical dependence of the discharge resistance on time coincides with the experimental one. This indicates that the dynamics of plasma evolution is determined by the ionization frequency of an easily ionized admixture of the active medium.

Conclusions

Thus, the experimentally observed variation of the plasma resistance in repetitively pulsed metal-vapor GDLs during the pump pulse is described by the theoretical time dependence of the discharge plasma determined from the condition of invariance of the system of kinetic equations to the scale transformation (5).

Since the theoretical time dependence of the discharge resistance depends only on the integral of the I_1 equation for the electron concentration, it can be used for modeling of electric characteristics of discharge and for modeling of GDLs with different pump sources.

References

1. G.A. Mesyats, *Generation of High-Power Nanosecond Pulses* (Sov. Radio, Moscow, 1974), 256 pp.
2. Yu.D. Korolev and G.A. Mesyats, *Physics of Pulsed Breakdown of Gases* (Nauka, Moscow, 1991), 224 pp.
3. V.M. Batenin, V.V. Buchanov, M.A. Kazaryan, I.I. Klimovskii, and E.I. Molodykh, *Self-Contained Atomic Metal Lasers* (Nauchnaya Kniga, Moscow, 1998), 544 pp.
4. L.V. Ovsyannikov, *Group Properties of Differential Equations* (SB AS USSR Publishing House, Novosibirsk, 1962), 240 pp.
5. A.A. Rukhadze, N.N. Sobolev, and V.V. Sokovikov, Usp. Fiz. Nauk **161**, No. 9, 195–199 (1991).
6. V.V. Konyukhov, Zh. Tekhn. Fiz. **XL**, No. 8, 1649–1655 (1970).
7. A.A. Isaev, "*Efficient Repetitively Pulsed Copper-Vapor Lasers*," Doct. Phys.–Math. Sci. Dissert., Moscow (1988), 339 pp.
8. Yu.P. Raizer, *Principles of Modern Physics of Gas-Discharge Processes* (Nauka, Moscow, 1980), 416 pp.