JOINT ESTIMATION OF THE EFFECT OF THE OPTICAL TRANSFER FUNCTIONS OF THE ATMOSPHERE AND OPTICAL SYSTEM ON THE CHARACTERISTICS OF DETECTION OF EXTENDED OBJECTS

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The joint effect of the optical transfer functions (OTF's) of the atmosphere and optical system on the characteristics of detection of the extended objects is numerically estimated on the basis of the theory of detection of the spatially extended objects using the technique of the optical transfer functions. The atmospheric OTF is estimated in the small–angle approximation.

It is shown that in the case of determining the characteristics of the detection of small (100–200 m) *objects in the horizontally homogeneous scattering atmosphere one should take into account only the OTF of the optical system and neglect the atmospheric spatial–frequency characteristics. In the case of detection of the objects whose linear dimensions are of the order of* 500 m *under the same conditions the additional contribution of the scattered radiation results in the change of the detection characteristics by 20% and more, whereas one can neglect the effect of the OTF of the optical system on the detection characteristics.*

It is well known[1] that the medium of signal propagation, the characteristics of the opto–electronic means (OEM), and the observed object itself affect the detection (discrimination) of the distant objects.

The modern approach to the estimation of this effect is based on the theory of linear filtration. Now the optical transfer functions (OTF's) of the OEM and scattering and turbulent media are studied in detail.[1,2] However, the joint effect of the OTF's of the atmosphere and OEM on the characteristics of the detection of extended objects has still received only insufficient study. This makes it impossible to estimate the extent to which we must take into account the OTF's of the atmosphere, optical system, and objects of observation when modeling the processes of the detection of the distant objects in the course of solving different problems.

The conditions in which the OTF of the atmosphere must be taken into account in the course of detection of the distant objects as functions of the characteristics of the OEM, atmosphere, and object dimensions are determined in the present paper on the basis of the obtained numerical estimations of the joint effect of the OTF's of the atmosphere and OEM.

Generalizing the results of the theory of the optimum signal detection to the case of the detection of the spatially extended objects, one can show that the detection parameter q, which has the meaning of the signal–to–noise ratio,[3] can be calculated from the formula

$$q = AQK_{\mathrm{r}} , \tag{1}$$

where A is the energy parameter depending on statistics of the received signal, Q is the shape coefficient of the spatially extended object ($0 \le Q \le 1$), K_{r} is the maximum value of the real object contrast.

The shape coefficient depends on the spatial–frequency characteristics of the object, the atmosphere, and the optical system and is determined from the formula

$$Q^2 = \int\limits_{-S} R(\rho)\, \Phi_{\mathrm{R}}(\rho)\, \mathrm{d}\rho , \tag{2}$$

where

$$R(\rho) = \int\limits_{-\infty}^{\infty} \Delta J(\mathbf{r})\, \Delta J(\mathbf{r} - \rho)\, \mathrm{d}\mathbf{r}$$

is the correlation function of the difference (under the alternative hypotheses H_1 and H_0) image $\Delta J(\mathbf{r})$ in the plane of the real scene, $\mathbf{r}$ and ρ are the radius vectors specifying the examined point in the object plane and in the plane of the input pupil, respectively,

$$\Phi_{\mathrm{R}}(\rho) = \int\limits_{-\infty}^{\infty} |T_{\Sigma}(\rho)|^2 \exp(-2\pi j \rho \nu)\, \mathrm{d}\nu , \tag{3}$$

where $\nu = \rho/\lambda R$ is the spatial frequency, λ is the wavelength of radiation, R is the distance to the object,

$$T_{\Sigma}(\rho) = T_{\mathrm{a}}(\rho)\cdot T_{\mathrm{o}}(\rho) , \tag{4}$$

$T_{\mathrm{a}}(\rho)$ is the OTF of the atmosphere or the second–order coherence function of the field of the spherical wave,[4]

$$T_{\mathrm{o}}(\rho) = \pi a_{\mathrm{o}}^2 \exp(-\rho^2/a_{\mathrm{o}}^2)\,(1 - \tau_{\mathrm{s}} + \tau_{\mathrm{s}} \exp(-\rho^2/a_{\mathrm{c}}^2)) \tag{5}$$

is the OTF of the objective with the light scattering by its material, surface roughness, and particles of dust[5] taken into account, a_{o} is the effective radius of the input pupil, τ_{s} is the scattering coefficient of the objective, and a_{c} is the radius of the blur circle.

Now several approaches to the determination of the OFT of the atmosphere[1,2,6] associated with the peculiarities of the problems being solved are developed as applied to the

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scattering atmosphere. The small–angle approximation[6,7] is widely used for solving the problem of the image transfer under conditions of the atmospheric haze and comparatively thin cloud layers. For the media with strongly elongated scattering phase functions under assumption that the optical characteristics of the medium are independent of the depth of the layer, the expression for the OTF of the atmosphere is determined as[6]

$$T_a(\rho) = \exp\left[-\tau_o + \Lambda \int\limits_0^{\tau_o} \kappa(\rho/\lambda\cdot\xi)\, d\xi \right], \qquad (6)$$

where τ_o is the optical thickness of the layer, Λ is the single scattering albedo,

$$\kappa(\rho/\lambda\xi) = \frac{1}{2} \int\limits_0^{\infty} x(\beta) J_0(\rho/\lambda\cdot\xi\cdot\beta)\cdot\beta\, d\beta \qquad (7)$$

is the zero–order Hankel transform of the scattering phase function $x(\beta)$.

The form of $T_a(\rho)$ depends to a strong degree on the approximation for the scattering phase function. Using the approximation of the form[6]

$$x(\beta) = 2/b^2 \exp(-\beta^2/2b^2), \qquad (8)$$

where b is the approximation parameter, we derive for the OTF of the atmosphere

$$T_a(\rho) = \exp\left[-\tau_o + \Lambda \sqrt{\pi}/2\lambda(\rho b)^{-1} \mathrm{erf}(\rho b \tau_o/\lambda\sqrt{2}) \right], \qquad (9)$$

where

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \int\limits_0^{z} \exp(-t^2)\, dt$$

is the error integral.

The dependences $T_a(\rho)$ for the purely scattering atmosphere (a) and the atmosphere with absorption (b) are shown in Fig. 1 for $b = 0.5$.

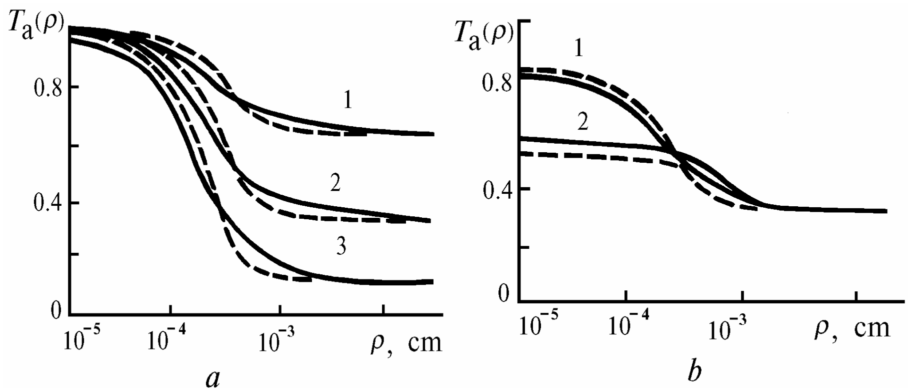

FIG. 1. The OTF of the atmosphere and its approximation (dashed curves): a) $\Lambda = 1$; *1* – $\tau_o = 0.4$; *2* – $\tau_o = 1$; *3* – $\tau_o = 2$; *b)* $\tau_o = 1$; *1* – $\Lambda = 0.8$; *2* – $\Lambda = 0.5$.

It follows from the analysis of dependence (9) that the transfer function of the scattering atmosphere can be fitted by the function convenient for numerical analysis (dashed curve)

$$T_a(\rho) = C + \Lambda(1 - C^{\Lambda}) \exp(-\rho^2/a_a^2), \qquad (10)$$

where $a_a = \dfrac{2.55\lambda}{b\sqrt{\Lambda\tau_o}}$ is the so–called scattering radius of the atmosphere and

$$C = \exp(-\tau_o).$$

The average approximation error for the considered conditions is 5...10%.

Using the obtained approximation for the OTF of the atmosphere in the form of Eq. (10) and Eq. (5) for the OTF of the optical system, we may write down the correlation function of difference image (3), trace the effect of the characteristics of the scattering atmosphere on shape coefficient (1), and estimate the joint effect of the atmosphere and optical system on the detection of the distant spatially extended objects from the variation of the shape coefficient. Substituting Eqs. (10) and (5) into Eq. (3) and neglecting light scattering by the objective, we derive

$$\Phi_\Sigma(\rho) = C^2 \Phi_{\Sigma_0}(\rho) + \Lambda^2(1 - C^{\Lambda})^2 \Phi_{\Sigma_1}(\rho) + 2C\Lambda(1 - C^{\Lambda}) \Phi_{\Sigma_2}(\rho);$$

$$\Phi_{\Sigma_i}(\rho) = \int\limits_{-\infty}^{\infty} \exp(-2\rho^2/a_i^2) \exp(-2\pi j \rho \nu)\, d\nu, \ (i = 0, 1, 2); \qquad (11)$$

$$a_1 = \left(\frac{a_o^2 a_a^2}{a_o^2 + a_a^2} \right)^{1/2}; \quad a_2 = \left(\frac{2a_o^2 a_a^2}{a_o^2 + 2a_a^2} \right)^{1/2}.$$

On account of Eq. (11), the shape coefficient can be represented in the form

$$Q^2 = C^2 Q_0^2 + \Lambda^2(1 - C^{\Lambda})^2 Q_1^2 + 2C\Lambda(1 - C^{\Lambda}) Q_2^2, \qquad (12)$$

where Q_0 is the shape coefficient depending on the OTF of the optical system and Q_1 and Q_2 are the shape coefficients on joint account of the atmosphere and the optical system.

Let us study the effect of the OTF's of the atmosphere and optical system on the characteristics of the detection of the objects of the simple shape. The calculations made for the rectangle with the sides l_x and l_y yield the following results:

$$Q_i^2 = f_i(x)\, f_i(y), \ (i = 0, 1, 2), \qquad (13)$$

where

$$f_i(x) = 2F(\sqrt{2x_i}) - 1 - \frac{1 - \exp(-x_i)}{\sqrt{\pi x_i}};$$

$$f_i(y) = 2F(\sqrt{2y_i}) - 1 - \frac{1 - \exp(-y_i)}{\sqrt{\pi y_i}};$$

$$x_i = \frac{(\pi a_i)^2 l_x^2}{2(\lambda R)^2};\ y_i = \frac{(\pi a_i)^2 l_y^2}{2(\lambda R)^2};\ F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-t^2/2)\,dt$$

is the Laplace integral

The results of the calculation of the shape coefficient for the detection of the square at the distance R = 3 km on the horizontal near–ground path at τ_o = 0.4; 1 and Λ = 0.5; 1 for the effective radii of the optical system a_o = 0.1 and 1 cm at the wavelength λ = 0.5 μm are shown in Fig. 2.

The values of Q_0^2 (curves *1* in Fig. 2) depending on the OTF of the optical system, and $C^2Q_0^2$ (curves *3* and *6*), obtained on account of the effect of the atmospheric extinction properties, are shown for comparison.

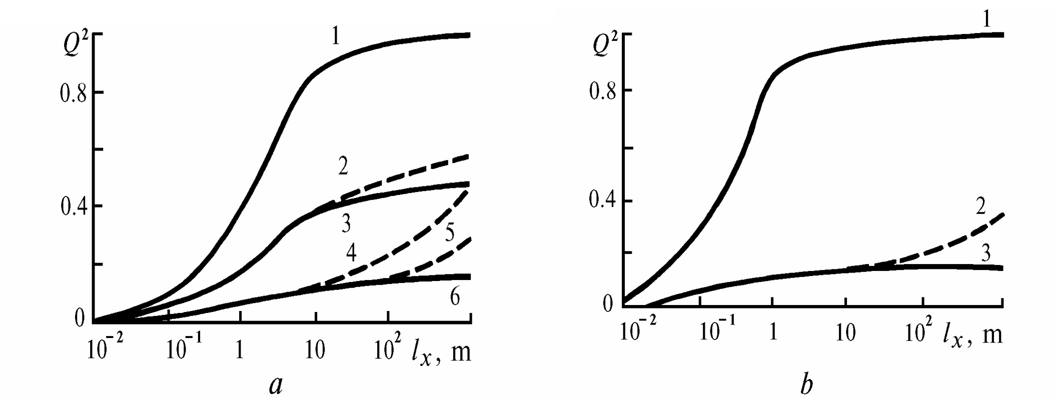

FIG. 2. The shape coefficient as a function of linear dimensions of the object: a) a_o = 0.1 cm; *1* – τ_o = 0, Λ = 0, (Q_0^2); *2* – τ_o = 0.4, Λ = 1; *3* – τ_o = 0.4, Λ = 0, ($C^2Q_0^2$); *4* – τ_o = 1; Λ = 1; *5* – τ_o = 1; Λ = 0.5; *6* – τ_o = 1, Λ = 0 ($C^2Q_0^2$); *b)* a_0 = 1 cm; *1* – τ_o = 0.4, Λ = 0, (Q_0^2); *2* – τ_o = 1, Λ = 0.5; and *3* – τ_o = 1, Λ = 0, ($C^2Q_0^2$).

Figure 2 shows that $Q^2 \sim C^2Q_0^2$, and only the extinction of radiation in the atmosphere affects the detection of the spatially extended object whose linear dimensions are smaller than 100 m (l_x < 70 ... 100 m), i.e., correct account of the OTF of the atmosphere makes it possible to refine the shape coefficient given by Eq. (12) by no more than a few percents. At the same time, the resolution of the optical system affects strongly the shape coefficient in the case of detection of such objects. The increase in the effective radius of the optical system from 0.1 to 1 cm results, on the average, in the increase of Q^2 (the signal–to–noise ratio) by 30 % (curves *5* and *2* in Figs. 2*a* and *b*, respectively).

The scattered radiation starts to affect the shape coefficient as the object dimensions increase, and it becomes necessary to take into account the OTF of the atmosphere as a function of the spatial–frequency characterictics.

The contribution of the scattered radiation depends on the optical thickness of the layer and the single scattering albedo. The contribution is about 8 % for the object whose linear dimensions are about 200 m at τ_o = 0.4, and increases up to 50 % for Λ = 1 (purely scattering atmosphere) (curves *2*, *3*, *4*, and *6* in Fig. 2*a*). It decreases from 50 to 30 % for the atmosphere with absorption (Λ = 0.5) (curves *5*, *6*, *4*, and *6*).

In addition, the change in the resolution of the optical system results in the variation of the shape coefficient of such objects by not more than a few percents.

Thus, the estimation of the joint effects of the OTF's of the atmosphere and optical system on the characteristics of detection of the extended objects shows the following. One should take into account the OTF of the optical system only when determining the characteristics of small objects in the horizontally homogeneous scattering atmosphere, and in this case one can neglect the spatial–frequency characteristics of the atmosphere, since the spatial spectrum of such objects lies in the asymptotic region of variation of the function $T_a(\rho)$.

One can neglect the effect of the OTF of the optical system on the detection characteristics and take into account only the OTF of the atmosphere in the case of detection of the objects whose dimensions are larger than 100–200 m under these conditions.

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