DISTRIBUTION OF IRRADIANCE OVER THE IMAGE PLANE OF A LIDAR RECEIVER FOR SOUNDING OF A RANDOMLY ROUGH SURFACE WITH A COMPLEX REFLECTANCE THROUGH THE ATMOSPHERE

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A spatial distribution of irradiance over an image plane of a lidar receiver is studied for the case of bistatic arrangement of atmospheric sounding of a randomly rough surface with a complex reflectivity of locally flat elements of the surface. Expressions are derived for the average irradiance, image size, and its shift with respect to a photodetector center in sounding of a surface with a local phase function having diffuse and quasispecular components through an optically dense aerosol atmosphere. It is shown that an irradiance structure strongly depends on the diffuse–to–quasispecular component ratio of the scattering phase function of the surface, on statistical characteristics of the surface roughness, and on the atmospheric conditions as well.

The spatial structure of irradiance in the image plane of a lidar receiver was studied in Ref. 1 for the case of a flat surface with a complicated phase function sounded through the atmosphere. Below we consider the irradiance distribution over the image plane for the case of sounding with a lidar of randomly rough surface with a combined phase function of locally flat elements composed of quasispecular and diffuse components through the atmosphere.

The brightness of radiation reflected from an elementary area is written as follows[2]

$$I(\mathbf{R}, \mathbf{m}) = \frac{E(\mathbf{R})}{\alpha \dfrac{2\pi}{n+2} + \beta\pi\Delta^2} \times$$

$$\times\left[\alpha \cos^n\theta + \beta \exp\left\{-\frac{(\theta-\theta_0)^2\cos^2\theta_0 + (\varphi-\varphi_0)^2\sin^2\theta_0}{\Delta^2}\right\}\right], \quad (1)$$

where $E(R) = A\,E_s(R)$, $E_s(R)$ is the irradiance of the area produced by radiation coming from a source, A is the reflectance, R is the spatial coordinate characterizing the position of the elementary scattering area, α and β are the coefficients determining relative contributions coming due to diffuse and quasispecular reflections, n and Δ are the parameters characterizing the angular width of diffuse and quasispecular components of the reflection function (formula (1) is derived at $\Delta \ll 1$), (θ, θ_0) and (φ, φ_0) are the zenith and azimuth angles of the observation direction and of the direction towards the maximum of reflected radiation (of the quasispecular component of reflection) in the local system of coordinates related to an elementary reflecting element of the surface. The angles θ_0 and φ_0 are related to the corresponding angles which characterize the direction of incident radiation by the laws of geometric optics.

Using the results obtained in Refs. 2–6 and assuming the distribution of heights and slopes of the sounded surface to be Gaussian, as in Ref. 1, we can derive the integral expression for the average (over an ensemble of surfaces) irradiance $\overline{E}(R_\phi)$ in the image plane of the lidar receiver for the general case of bistatic arrangement when the source and receiver are spaced. In so doing we assume that the receiver is far enough from the scattering surface, so that the angle ψ at which the receiving aperture is seen from the points lying on the scattering surface is much less than the angular width of the quasispecular component of the phase function, characteristic scale of variation in angles of the surface slope, and of the angular resolution of the receiver. In clear atmosphere this angle is $\psi \sim r_r/L_r$, where r_r is the effective size of the receiving aperture, and L_r is the distance measured from the receiver to the illuminated spot on the scattering surface along slant path. In a particular case of homogeneous scattering atmosphere characterized by a strongly forward–peaked phase function[4,5] the expression for $\overline{E}(R_\phi)$ has the form (assuming the surface to be smoothly rough $\overline{\gamma}_0^2 \ll 1$; small angle approximation to be taken for the source and receiver; the source, receiver, and their axes to be lying in the same plane XOZ; and, coming from integration over the rough surface to the integration over its projection S_0 onto the plane $z = 0$)

$$\overline{E}(\mathbf{R}_\phi) \simeq \frac{A}{\pi}\frac{\nu_r^{-1/2}}{\sigma\dfrac{2}{n+2} + \beta\Delta^2} \times$$

$$\times\left[\alpha\,\omega(\gamma_0)\int\limits_{S_0} d^2R_0\, E_s(\mathbf{R}_0')\, E_r(\mathbf{R}_0'', \mathbf{R}_\phi)\, K(\mathbf{R}_0, \mathbf{R}_\phi) + \beta\, G\, \mu^{-1} \times\right.$$

$$\times\int\limits_{S_0} d^2R_0\, E_s(\mathbf{R}_0')\, E_r(\mathbf{R}_0'', \mathbf{R}_\phi)\, K(\mathbf{R}_0, \mathbf{R}_\phi) \times$$

$$\left.\times \exp\left\{-\frac{\tilde{S}^2 R_{0y}^2 + \left(q_x + R_{0x}\,\tilde{t}\right)^2}{\Delta^2\,\mu}\right\}\right], \quad (2)$$

where

$$\nu_r = 1 + 2\sigma_0^2 \left(\frac{\sin^2\theta_s}{4B_s^2} + \frac{\sin^2\theta_r}{4B_r^2} \right);$$

$$\omega(\gamma_0) = m_{rz}^n \left(2\gamma_0^2\right)^{-n/4} \times$$

$$\times \exp\left(\frac{1}{4\gamma_0^2}\right)\left[(2\gamma_0^2)^{-1/4} W_{-\frac{(n+1)}{4}, -\frac{(n-1)}{4}}\left(\frac{1}{2\gamma_0^2}\right) + \right.$$

$$\left. + \frac{m_{sx}\, m_{rx}(n+1)}{2\, m_{rz}\, m_{sx}} (2\gamma_0^2)^{1/4} W_{-\frac{(n+3)}{4}, -\frac{(n-3)}{4}}\left(\frac{1}{2\gamma_0^2}\right)\right];$$

$$\mu = 1 + \frac{2\gamma_0^2 q_z^2}{\Delta^2}; \quad G = \frac{(m_{sz} - m_{sx}\gamma_{mx})(m_{rz} - m_{rx}\gamma_{mx})}{m_{sz}\, m_{rz}};$$

$$\gamma_{mx} = -\frac{q_x q_x}{\frac{\Delta^2}{2\gamma_0^2} + q_z^2}; \quad q_z = -(\cos\theta_r + \cos\theta_s); \quad q_x = (\sin\theta_s - \sin\theta_r);$$

$$\mathbf{R}_0' = \{R_{0x}\cos\theta_s, R_{0y}\}; \quad \mathbf{R}_0'' = \{R_{0x}\cos\theta_s, R_{0y}\};$$

$$K(\mathbf{R}_0, \mathbf{R}_u) =$$

$$= \exp\left\{ \frac{2\sigma_0^2}{\nu_r} \left[R_{0x} \frac{\sin\theta_s\cos\theta_s}{4B_s^2} + \frac{\sin\theta_r \left(R_{0x}\cos\theta_r - R_{\phi x}\frac{L_r}{F}\right)}{4B_r^2} \right]^2 \right\},$$

σ_0^2, γ_0^2 are the variances of the heights and slopes of the randomly rough surface,

$$\tilde{s} = A_r/B_r + A_s/B_s; \quad \tilde{t} = \frac{A_r\cos\theta_r}{B_r} + \frac{A_s\cos^2\theta_s}{B_s};$$

$$A_{s,r} = 0.5\left(\alpha_{s,r}^2 + s\langle s^2\rangle L_{s,r}\right)^{1/2};$$

$$B_{s,r} = \frac{0.5\, L_{s,r}\left(\alpha_{s,r}^2 + 0.5\sigma\langle s^2\rangle L_{s,r}\right)}{\left(\alpha_{s,p}^2 + \sigma\langle s^2\rangle L_{s,r}\right)^{1/2}};$$

R is the vector in the image plane of the lidar receiver; $E_s(\mathbf{R})$ and $E_r(\mathbf{R}, \mathbf{R}_u)$ are the irradiances produced by an actual and virtual sources, respectively,[3,4] on the surface sounded; L_s and L_r are the distances along the slant path, from the source and receiver to the surface sounded, $2\alpha_s$ and $2\alpha_r$ are the divergency angles of a beam coming from the source and the angular resolution of the receiver; σ is the scattering coefficient of the atmosphere, $\langle s^2\rangle$ is the variance of the beam deviation angle during an elementary scattering event in the atmosphere, θ_s and θ_r are the angles between the normal to the plane $z = 0$ and directions to the source and the receiver, respectively; $\mathbf{m}_s = \{m_{sx}, m_{sz}\}$ and $\mathbf{m}_r = \{m_{rx}, m_{rz}\}$ are the unit vectors along the directions of radiation incidence onto the surface and towards the receiver; $W_{n,m}(x)$ is the Whittaker function; and, F is the focal length of the receiving lens.

By calculating the integrals in Eq. (2) we can derive the following analytical relation for the average irradiance in the image plane of the lidar receiver for the case of sounding of randomly rough surface with a combined local phase function through the atmosphere:

$$E(\mathbf{R}_\phi) \simeq c\left[c_1 \exp\left\{-R_{\phi x}^2 b_{1x} - R_{\phi y}^2 b_{1y}\right\} + \right.$$

$$\left. + c_2 \exp\left\{-R_{\phi y}^2 b_{2y} - \left(R_{\phi x} + \hat{\delta}\right)^2 b_{2x}\right\}\right]; \qquad (3)$$

where

$$c = \frac{\nu_r^{-1/2}}{\alpha\frac{2}{n+2} + \beta\Delta^2} \frac{A\, r_r^2\, \alpha_r^2 \cos\theta_s\cos\theta_r P_0 \exp\{-(\varepsilon - \sigma)(L_s + L_r)\}}{16\pi B_s^2 B_r^2 r_\kappa^2};$$

$$c_1 = \alpha\, \omega(\gamma_0)\, q^{-1/2}\, \bar{p}^{-1/2}; \quad b_{1y} = \left(\frac{L_r}{F}\right)^2 \frac{q_r q_s}{q};$$

$$b_{1x} = \left(\frac{L_r}{F}\right)^2 q_r \left[1 - \frac{2\sigma_0^2}{\nu_r}\sin^2\theta_r q_r - \frac{q_r \bar{G}^2}{\bar{p}}\right];$$

$$c_2 = \beta\, G\, \mu^{-1} \left[q + \frac{\tilde{s}^2}{\Delta^2\mu}\right]^{-1/2} \hat{p}^{-1/2} \exp(\hat{k});$$

$$b_{2y} = \left(\frac{L_r}{F}\right)^2 q_r \frac{\left(q_r + \frac{\tilde{s}^2}{\Delta^2\mu}\right)}{\left(q + \frac{\tilde{s}^2}{\Delta^2\mu}\right)};$$

$$b_{2x} = \left(\frac{L_r}{F}\right)^2 q_r \left[1 - \frac{2\sigma_0^2}{\nu_r}\sin^2\theta_r q_r - \frac{q_r \bar{G}^2}{\hat{p}}\right];$$

$$\hat{\delta} = \frac{L_r}{F}\frac{q_r G q_x \tilde{t}}{\hat{p}\, b_{2x}\Delta^2\mu};$$

$$\hat{k} = -\frac{q_x^2}{\Delta^2\mu}\frac{\bar{p}}{\hat{p}} + \hat{\delta}^2 b_{2x};$$

$$\hat{p} = \bar{p} + \frac{\tilde{t}^2}{\Delta^2\mu}; \quad \bar{p} = p - \hat{G};$$

$$\hat{G} = \frac{2\sigma_0^2}{\nu_r}\left(\sin\theta_s\cos\theta_s q_s + \sin\theta_r\cos\theta_r q_r\right)^2;$$

$$\bar{G} = \cos\theta_r - \sin\theta_r \frac{2\sigma_0^2}{\nu_r}\left(\sin\theta_s\cos\theta_s\theta_s + \sin\theta_r\cos\theta_r q_r\right);$$

$$q_r = \left(4B_r^2\right)^{-1}; \quad q_s = \left(4B_s^2\right)^{-1};$$

$$q = q_r + q_s; \quad p = q_r\cos^2\theta_r + q_s\cos^2\theta_s;$$

P_0 is the power emitted by the source, r_r is the effective size of the receiving aperture, r_k is the effective aberration circle of the receiving optical system, and ε is the extinction coefficient of the atmosphere.

In the case of vanishing σ_0 and γ_0 formula (3) is identical to the formula for the irradiance when a flat surface with a combined phase function is sounded through the atmosphere.[1] At $\beta = 0$, $n = 0$, $\langle s^2\rangle = 0$, and $\sigma = 0$ formula (3) transforms into the expression for the average irradiance for the case of sounding of a randomly rough locally Lambertian surface[7] through the clear atmosphere. At $\alpha = 0$, $\Delta \to 0$, $\langle s^2\rangle = 0$, and $\sigma = 0$ formula (3) transforms into the expression for the average irradiance produced by a randomly rough locally specular surface in clear atmosphere.[8]

The calculational results on spatial distribution of irradiance over the image plane of the receiver at different values of the parameter β/α and the statistical characteristics of a rough surface are shown in Figs. 1 and 2. The quantities $\bar{E}(R_{\phi x}, R_{\phi y} = 0)/\bar{E}(R_{\phi x} = 0, R_{\phi y} = 0)$ were calculated by formula (3) for the following values of the parameters: $n = 0$, $\theta_s = 60°$, $\theta_r = 55°$, $L_s = 10^4$ m, $L_r = 10^2$ m, $\alpha_s = 10^{-2}$, $\alpha_r = 10^{-2}$, $\Delta = 0.3$, $\sigma\langle s^2\rangle = 0$, (Fig. 1), $\sigma\langle s^2\rangle = 10^{-6}$, (Fig. 2). Curves *1* and *3* correspond to $\beta/\alpha = 0$, *2* and *4* – to $\beta/\alpha = 0.3$. Curves *1* and *2* are calculated for $\sigma_0^2 = 0°$, $\gamma_0^2 = 0°$, *3* and *4* – for $\sigma_0^2 = 2°\text{m}^2$, $\gamma_0^2 = 10^{-2}$.

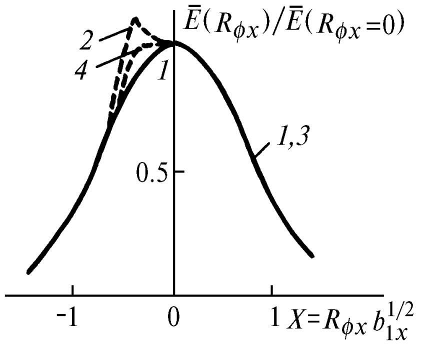

FIG. 1. Irradiance distribution over the image plane of a lidar receiver in clear atmosphere ($\sigma\langle s^2\rangle = 0$).

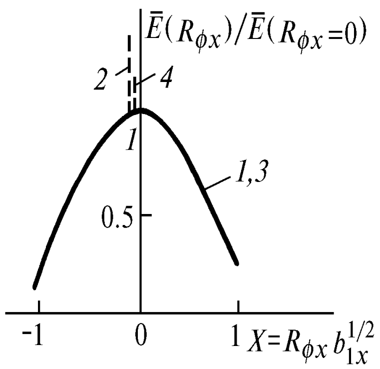

FIG. 2. Irradiance distribution over the image plane of a lidar receiver in the optically dense atmosphere ($\sigma\langle s^2\rangle = 10^{-6}$).

As is seen from figures relative contributions of quasispecular and diffuse components of the phase function of the sounded surface (the parameter β/α) strongly affects the irradiance structure in the image plane at the reception angles which are close to the specular ones both in clear (Fig. 1) and turbid (Fig. 2) atmospheres. The random roughness of the surface results in weakening of this effect what is caused by spreading of the quasispecular component of the phase function of the surface.

Atmospheric turbidity leads to a relative (compared to the size of the entire image) decrease in the domain of the image where the contribution from quasispecular component of the phase function is sufficient. This is associated with an increase in the absolute size of the image first of all due to spreading of the laser beam in the atmosphere and an increase in the size of the illuminated spot on the scattering surface.

Let us estimate now the size of the image ΔR_x and its shift δR_x along the X–axis with respect to the photodetector center.

Let us define the quantities δR_x and ΔR_x as follows:

$$\delta R_x = \frac{\int\limits_{-\infty}^{\infty} R_{\phi x}\,\bar{E}(R_{\phi x}, R_{\phi y} = 0)\,\mathrm{d}R_{\phi x}}{\int\limits_{-\infty}^{\infty} \bar{E}(R_{\phi x}, R_{\phi y} = 0)\,\mathrm{d}R_{\phi x}};$$

$$\Delta R_x^2 = \frac{\int\limits_{-\infty}^{\infty} (R_{\phi x} - \mathrm{d}\,R_x)^2\bar{E}(R_{\phi x}, R_{\phi y} = 0)\,\mathrm{d}R_{\phi x}}{\int\limits_{-\infty}^{\infty} \bar{E}(R_{\phi x}, R_{\phi y} = 0)\,\mathrm{d}R_{\phi x}}. \qquad (4)$$

Calculations by formula (3) give:

$$\delta R_x = -\frac{\hat{\delta}\,c_2\,b_{2x}^{-1/2}}{c_1\,b_{1x}^{-1/2} + c_2\,b_{2x}^{-1/2}}; \qquad (5)$$

$$\Delta R_x^2 = \frac{c_1\,b_{1x}^{-1/2}\left[0.5\,b_{1x}^{-1} + \mathrm{d}\,R_x^2\right] + c_2\,b_{2x}^{-1/2}\left[0.5\,b_{2x}^{-1} + (\hat{\delta} + \delta\,R_x)^2\right]}{c_1\,b_{1x}^{-1/2} + c_2\,b_{2x}^{-1/2}}. \qquad (6)$$

To obtain δR_x and ΔR_x^2 in formulas (5) and (6) we should take $\theta_s = \theta_r = 0$.

The dependence of the shift δR_x on the parameter β/α is shown in Fig. 3. Calculations were carried out by formula (5) for the following values of the parameters: $n = 0$, $\theta_s = 60°$, $\theta_r = 55°$ $L_s = 10^4$ m, $L_r = 10^2$ m, $\alpha_s = 10^{-2}$, $\alpha_r = 10^{-2}$, $\Delta = 0.3$, $r_r = 10^{-2}$ m, $F = 10^{-1}$ m. Curves *1* and *3* correspond to $\sigma\langle s^2\rangle = 0$, *2* and *4* – $\sigma\langle s^2\rangle = 10^{-6}$. Curves *1* and *2* are calculated for $\sigma_0^2 = 0$, $\gamma_0^2 = 0$, *3* and *4* – for $\sigma_0^2 = 2$ m^2, $\gamma_0^2 = 10^{-2}$.

It can be seen from the figures that the quasispecular component of the phase function of the sounded surface can result in a sufficient shift of the energy center of the spatial distribution of irradiance over the image plane.

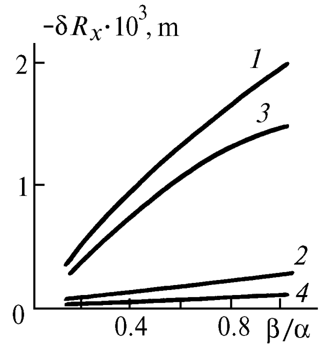

FIG. 3. Dependence of the image shift with respect to the photodetector center on the parameter β/α.

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