EFFICIENCY OF DETECTION OF SPATIALLY EXTENDED OBJECT IMAGES AGAINST THE SPOTTED BACKGROUND

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Algorithm for detection of spatially extended objects shading the background of spotted structure has been synthesized on the basis of statistic decision theory in the case of simulation of the intensity of the object and background images by Gaussian random fields. The efficiency of detection of a rectangular object is evaluated in the case of exponential correlation function for the intensities of the object and background images. The information content of the stablest textural discriminating parameters has been determined.

1. In most the known papers (see Refs. 1–5 and others) the efficiency of detection of spatially extended objects (SEO) is investigated as applied to evristic algorithms of image processing. In this case the basic characteristics of the SEO image detection are studied insufficiently in the variety of real situations. The investigation of the possibilities of the SEO image detection by the well–developed and widely spread methods of the statistical decision theory allows us to reveal the reserves and ways to increase the efficiency of the available image processing algorithms.

In this paper the characteristics of the SEO image detection against the spotted background are considered and the optimum (against the criterion of maximum likelihood) detection algorithms are synthesized for spatially extended objects shading the background in the case of simulation of the object and background images by Gaussian fields. In view of the increased complexity of the analysis of the obtained algorithms the characteristics of the SEO detection are determined for the objects of rectangular shape in the case of the exponential correlation function for the intensities of the object and background images.

2. The image being processed is represented by a discrete function of the potential relief $Y = \{Y(\mathbf{r}_1), Y(\mathbf{r}_2), ..., Y(\mathbf{r}_n)\}$, where $\mathbf{r}_i$ is the vector of coordinates of the ith element of the zone of observation D. According to the hypothesis H_0 (see in Ref. 6), the background image $\mathbf{F} = \{\mathbf{r}_i\}$, $\mathbf{r}_i \in D$, whose intensity is described by the Gaussian field with mathematical expectation $\mathbf{M}_{\text{bg}} = \{M_{\text{bg}}(\mathbf{r}_i)\}$, $\mathbf{r}_i \in D$ and by the interelement correlation tensor R_{bg}, is presented in the entire examined zone D. The spatial extension of the observed objects causes the background shading by the object in most practical cases. Taking this into account, the matrix $\mathbf{Y}$ in the zone of possible object location is formed, according to the hypothesis H_1, by the image intensity of the detectable SEO $S = \{S(\mathbf{r}_i)\}$, $\mathbf{r}_i \in G$. The latter is the realization of a Gaussian random field with mathematical expectation $\mathbf{M}_{\text{s}} = \{M_{\text{s}}(\mathbf{r}_i)\}$, $\mathbf{r}_i \in G$ and with the interelement correlation tensor $\mathbf{R}_{\text{s}} = \{R_{\text{s}}(\mathbf{r}_i, \mathbf{r}_k)\}$, $(\mathbf{r}_i, \mathbf{r}_k) \in G$. A fragment of background intensity realization $\{\mathbf{F}(\mathbf{r}_i)\}$, $\mathbf{r}_i \in G$ is observed in the zone being complementary to G. Using the object coordinate function $\mathbf{V} = \{V(\mathbf{r}_i)\}$ and taking $V(\mathbf{r}_i) = 1$ in the zone of possible object location $(r_i \in G)$ and $V(\mathbf{r}_i) = 0$ outside this zone $(r_i \in \overline{G})$, the intensity of the image being processed may be represented as

$$Y(\mathbf{r}_i) = \begin{cases} V(\mathbf{r}_i)S(\mathbf{r}_i) + (1 - V(\mathbf{r}_i))F(\mathbf{r}_i)\,, & \mathbf{r}_i \bigcirc D : H_1\,, \\ F(\mathbf{r}_i)\,, & \mathbf{r}_i \bigcirc D : H_0\,. \end{cases}$$

For the above–described model the intensity of the image $\mathbf{Y}$ being processed is, according to both hypotheses, the realization of the Gaussian random process with mathematical expectation

$$M_1(\mathbf{r}_i) = V(\mathbf{r}_i)\cdot M_{\text{s}}(\mathbf{r}_i) + (1 - V(\mathbf{r}_i))\cdot M_{\text{bg}}(\mathbf{r}_i)\,, \quad \mathbf{r}_i \in D,$$

$$\mathbf{M}_0(\mathbf{r}_i) = M_{\text{bg}}(\mathbf{r}_i)\,, \quad \mathbf{r}_i \in D,$$

and with the components of interelement correlation tensors

$$K_1(\mathbf{r}_i, \mathbf{r}_k) = V(\mathbf{r}_i)R_{\text{s}}(\mathbf{r}_i, \mathbf{r}_k)V(\mathbf{r}_k) + (1 - V(\mathbf{r}_i))R_{\text{bg}}(\mathbf{r}_i, \mathbf{r}_k)(1 - V(\mathbf{r}_k)),$$

$$K_0(\mathbf{r}_i, \mathbf{r}_k) = R_{\text{bg}}(\mathbf{r}_i, \mathbf{r}_k)\,, \quad (\mathbf{r}_i, \mathbf{r}_k) \in D$$

in accordance with the hypotheses H_1 and H_2, respectively. In what follows that the tensors R_{s} and R_{bg} are positively defined.

Based on the method described in Ref. 7, the decision rule for the object detection may be written as

$$L(\mathbf{Y}) = \frac{1}{2}\sum_i^D \sum_k^D (Y(\mathbf{r}_i) - M_0(\mathbf{r}_i))\Theta_0(\mathbf{r}_i, \mathbf{r}_k)(Y(\mathbf{r}_k) - M_0(\mathbf{r}_k)) -$$
$$- \frac{1}{2}\sum_i^D \sum_k^D (Y(\mathbf{r}_i) - M_1(\mathbf{r}_i))\Theta_1(\mathbf{r}_i, \mathbf{r}_k)(Y(\mathbf{r}_k) - M_1(\mathbf{r}_k)) +$$
$$+ \frac{1}{2}\ln\frac{\det(\mathbf{K}_0)}{\det(\mathbf{K}_1)} \overset{H_1}{\underset{H_0}{\gtrless}} C\,, \tag{1}$$

where $\Theta_l(\mathbf{r}_i, \mathbf{r}_k)$, $l = 0, 1$ are the elements of the tensor Θ_l being inverse to the interelement correlation tensor $\mathbf{K}_l$. They are the solutions of the equation $\sum_j^D \Theta_l(\mathbf{r}_i, \mathbf{r}_j)K_l(\mathbf{r}_i, \mathbf{r}_k) = \delta(\mathbf{r}_i, \mathbf{r}_k)$, $(\mathbf{r}_i, \mathbf{r}_k) \in D$, where $\delta(\mathbf{r}_i, \mathbf{r}_k)$ is

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Kronecker's delta symbol and C is the decision threshold. The summation in Eq. (1) is taken over the region indicated above the sum. The determinants are found for the matrices obtained from the corresponding tensors by means of ordered scanning of the examined zone.

Since the intensity of the image being processed according to the hypothesis H_1 is the realization of the compound Gaussian field, it is difficult to obtain the relations for determination of the elements of the tensor Θ_1. On account of the property of the positive definiteness of the tensors R_s and R_{bg}, it can be shown that the tensor Θ_1 has the structure similar to that of the interelement correlation $\mathbf{K}_1$:

$$\Theta_1(\mathbf{r}_i, \mathbf{r}_k) = V(\mathbf{r}_i)\Theta_{11}(\mathbf{r}_i, \mathbf{r}_k)V(\mathbf{r}_k) +$$
$$+ (1 - V(\mathbf{r}_i))\Theta_{10}(\mathbf{r}_i, \mathbf{r}_k)(1 - V(\mathbf{r}_k)),$$
$$(\mathbf{r}_i, \mathbf{r}_k) \in D \; . \qquad (2)$$

The coefficients $\Theta_{11}(\mathbf{r}_i, \mathbf{r}_k)$ and $\Theta_{10}(\mathbf{r}_i, \mathbf{r}_k)$ in Eq. (2) are the elements of the inverse interelement correlation tensors Θ_{11} and Θ_{10} in the image zones G and $\bar{G}$, respectively. They are the solutions of the equations

$$\sum_j^{G} \Theta_{11}(\mathbf{r}_i, \mathbf{r}_j)R_s(\mathbf{r}_j, \mathbf{r}_k) = \delta(\mathbf{r}_i, \mathbf{r}_k) \; , \quad (\mathbf{r}_i, \mathbf{r}_k) \in G \; ;$$
$$\sum_j^{\bar{G}} \Theta_{10}(\mathbf{r}_i, \mathbf{r}_j)R_{bg}(\mathbf{r}_j, \mathbf{r}_k) = \delta(\mathbf{r}_i, \mathbf{r}_k) \; , \quad (\mathbf{r}_i, \mathbf{r}_k) \in \bar{G} \; . \qquad (3)$$

Using the structure of the tensor Θ_1 (see Eq. (2)), Eq. (1) after simple transformations leading to change of the threshold C^*, can be written as

$$L(\mathbf{Y}) = \frac{1}{2}\sum_i^G \sum_k^G [(Y(\mathbf{r}_i) - M_{bg}(\mathbf{r}_i))\Theta_0(\mathbf{r}_i, \mathbf{r}_k)(Y(\mathbf{r}_k) - M_{bg}(\mathbf{r}_k)) -$$
$$- (Y(\mathbf{r}_i) - M_s(\mathbf{r}_i))\, \Theta_{11}(\mathbf{r}_i, \mathbf{r}_k)(Y(\mathbf{r}_k) - M_s(\mathbf{r}_k))] +$$
$$+ \sum_i^G \sum_k^{\bar{G}} \; (Y(\mathbf{r}_i) - M_{bg}(\mathbf{r}_i))\, \Theta_0(\mathbf{r}_i, \mathbf{r}_k)(Y(\mathbf{r}_k) - M_{bg}(\mathbf{r}_k)) +$$
$$+ \frac{1}{2}\sum_i^{\bar{G}} \sum_k^{\bar{G}} \; (Y(\mathbf{r}_i) - M_{bg}(\mathbf{r}_i))(\Theta_0(\mathbf{r}_i, \mathbf{r}_k) - \Theta_{10}(\mathbf{r}_i, \mathbf{r}_k) \times$$
$$\times (Y(\mathbf{r}_k) - M_{bg}(\mathbf{r}_k)) \underset{H_0}{\overset{H_1}{\gtrless}} C^* \; . \qquad (4)$$

This relation describes the algorithm for optimum processing of images of spatially extended objects shading the background in the case of simulation of the textures of the object and background by the Gaussian random fields. According to Eq. (4), the optimum detector implements the weighting summation of pairwise products of observed image intensity readings and the comparison of the obtained value with the threshold. The principal part of the algorithm consists in appropriate selection of the coefficients of weighting summation. Their determination is based on the inversion of the interelement correlation tensors of the background image in the entire examined zone and in the zone without the object as well as of the interelement correlation tensor of the object.

3. In order to determine the stablest textural discriminating parameters of the SEO, we subsequently assume the mathematical expectation of the intensities of image and background to be equal to zero, i.e., $\mathbf{M}_s = \mathbf{M}_{bg} = 0$. To investigate the physical meaning of the processing algorithm let us represent Eq. (4) in the other form having written down the inverse correlation tensors Θ_{11} and Θ_{10} as a sum of regular $\mathbf{U}_{1\infty}$, $\mathbf{U}_{0\infty}$ and singular $\mathbf{U}_{1\delta}$, $\mathbf{U}_{0\delta}$ solutions of Eqs. (3):

$$\Theta_{11}(\mathbf{r}_i, \mathbf{r}_k) = U_{1\infty}(\mathbf{r}_i, \mathbf{r}_k) + U_{1\delta}(\mathbf{r}_i, \mathbf{r}_k) \; , \quad (\mathbf{r}_i, \mathbf{r}_k) \in G,$$
$$\Theta_{10}(\mathbf{r}_i, \mathbf{r}_k) = U_{0\infty}(\mathbf{r}_i, \mathbf{r}_k) + U_{0\delta}(\mathbf{r}_i, \mathbf{r}_k) \; , \quad (\mathbf{r}_i, \mathbf{r}_k) \in \bar{G} \; . \qquad (5)$$

In this case the regular component coincides with the inverse correlation tensor of the corresponding Gaussian field specified at the nodes of the infinite grid, while the singular component describes the effects due to bounded zone of observation of this field. Therefore, the form of the tensor $\mathbf{U}_{0\delta}$ depends on the shape of the boundary of the zone of possible object location G. Its elements are nonzero only on both sides of this boundary determined by the correlation length. Since in the image processing in the zone G the difference between the tensors Θ_0 and Θ_{10} is used

$$\Theta_0(\mathbf{r}_i, \mathbf{r}_k) - \Theta_{10}(\mathbf{r}_i, \mathbf{r}_k) = - \; U_{0\delta}(\mathbf{r}_i, \mathbf{r}_k) \; , \quad (\mathbf{r}_i, \mathbf{r}_k) \in \bar{G} \; , \qquad (6)$$

and the singular components on the boundary of the examined zone D are identical to these tensors, it is assumed below that the tensor $\mathbf{U}_{0\delta}$ describes the boundary effects only on the external side of the boundary of the zone G.

It should be noted that the weighting coefficients $\Theta_0(\mathbf{r}_i, \mathbf{r}_k)$ entering into the first and second terms of Eq. (5) are primarily determined by the regular component of the tensor Θ_0 given that the dimensions of the examined zone are many times larger than that of the object and the correlation length of the background

$$\Theta_0(\mathbf{r}_i, \mathbf{r}_k) \approx \Theta_{0\infty}(\mathbf{r}_i, \mathbf{r}_k) \; , \quad \mathbf{r}_i \in G \; , \quad \mathbf{r}_k \in D \; . \qquad (7)$$

Using assumptions (5)–(7), decision rule (4) may be reduced to a form:

$$L(\mathbf{Y}) = \frac{1}{2}\sum_i^G \sum_k^G Y(\mathbf{r}_i)(\Theta_{0\infty}(\mathbf{r}_i, \mathbf{r}_k) - U_{1\infty}(\mathbf{r}_i, \mathbf{r}_k))Y(\mathbf{r}_k) +$$
$$+ \Bigg[\sum_i^G \sum_k^{\bar{G}} Y(\mathbf{r}_i)\mathrm{H}_{0\infty}(\mathbf{r}_i, \mathbf{r}_k)Y(\mathbf{r}_k) -$$
$$- \frac{1}{2}\sum_i^G \sum_k^G Y(\mathbf{r}_i)U_{1\delta}(\mathbf{r}_i, \mathbf{r}_k)Y(\mathbf{r}_k) -$$
$$- \frac{1}{2}\sum_i^{\bar{G}} \sum_k^{\bar{G}} Y(\mathbf{r}_i)U_{0\delta}(\mathbf{r}_i, \mathbf{r}_k)Y(\mathbf{r}_k) \Bigg] \underset{H_0}{\overset{H_1}{\gtrless}} C^* \; . \qquad (8)$$

In accordance with Eq. (8), the algorithm for the optimum detection of the object against the spotted background comprises the procedures of the discrimination of the object and background images by means of the analysis of correlation properties (of the texture) of the image intensity realization in the zone of possible object location[1] (the first term) and the selection of the object image boundary (the term in the brackets). The latter, in its turn, is provided with deterioration in the probability characteristics (intensity correlation) of the elements of the recorded realization located on the internal and external sides of this boundary.

When the correlation properties of the intensities of the images of the object are identical to those of the background, the equality $\Theta_{0\infty} = U_{1\infty}$ follows from $R_s(\mathbf{r}_i, \mathbf{r}_k) = R_{bg}(\mathbf{r}_i, \mathbf{r}_k)$ and the first term in Eq. (8) becomes equal to zero. The object detection in this case is provided only at the expense of the selection of its boundary with the background.

4. Owing to the significant difficulties of obtaining the analytical relations for distribution of the likelihood functional $L(\mathbf{Y})$ in the form of Eq. (8), the quality of the spatially extended object detection against the spotted background was evaluated by the Monte Carlo method.

The detection characteristics were calculated for the images of the rectangular SEO for exponential correlation functions

$$R_s(\mathbf{r}_i, \mathbf{r}_k) = \sigma_1^2 \rho_1^{|\alpha-\beta|} \rho_1^{|\gamma-\delta|} ; \quad (9)$$

$$R_{bg}(\mathbf{r}_i, \mathbf{r}_k) = \sigma_0^2 \rho_0^{|\alpha-\beta|} \rho_0^{|\gamma-\delta|} \quad (10)$$

for the intensities of the object and background images, respectively. Here σ_1^2 and σ_0^2 are the variances of the intensity readings, ρ_1 and ρ_0 are the coefficients of the intensity correlation for the adjacent elements of images, and α, γ, β, and δ are the rectangular coordinates of the frame elements. Based on the approach described in Ref. 6, the expressions for the elements of the tensors $\Theta_{0\infty}$ and $U_{1\infty}$ were found in the form of the nine–element operators, characterizing the realization processing in the vicinity of each image intensity reading, and the values of weighting coefficients $U_{1\delta}(\mathbf{r}_i, \mathbf{r}_k)$ and $U_{0\delta}(\mathbf{r}_i, \mathbf{r}_k)$ were determined. The obtained operator of object boundary selection acts only in the image zone adjacent to this boundary (bold dots in Fig. 1). The clumsy operator elements are not given in the paper.

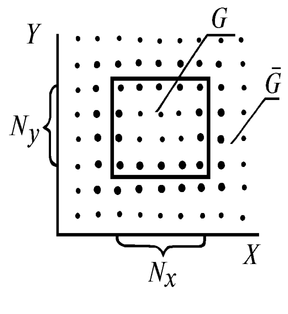

FIG. 1. The information–bearing zone of the image for the object boundary selection.

The algorithm for determining the detection characteristics was as follows. First, according to the well–known methods described in Ref. 8, the realizations of the uniform Gaussian field, simulating the background intensity with zero mathematical expectation and correlation function given by Eq. (10), were formed. Each of these realizations was processed in accordance with the algorithm in the form of Eq. (8). Based on the obtained values of the probability functional, the detection threshold C^* was determined against the Neumann–Pearson criterion by the method of extremal statistics.[9] Then the realizations of the compound centered Gaussian field were simulated with the correlation function in the form of Eq. (9) (in accordance with the hypothesis H_1) in the zone occupied by the object and with the background correlation function given by Eq. (10) in the external zone. The obtained intensities of the images were processed in accordance with algorithm (8) and the result of processing was compared with the threshold C^* to detect the presence or absence of the object and to estimate the detection probability.

The calculations have shown that the quality of the SEO image detection depends on the interelement correlation coefficients ρ_1 and ρ_2, on the ratio of variances of fields describing the object and background textures $\varepsilon^2 = \sigma_1^2/\sigma_0^2$ as well as on the object dimensions along the X and Y axes (in pixels), N_x and N_y (see Fig. 1). The dependences of the object detection probability P_D on these parameters with erroneous alarm $P_F = 10^{-3}$ and the confidence interval 10 ... 15 % are shown in Figs. 2–4.

Figure 2 shows the object detection probability P_D as a function of the interelement correlation coefficient ρ being equal for the intensities of the object and background images (i.e., $\rho_1 = \rho_0 = \rho$) given that the variances of the intensity readings of the object and background are equal ($\varepsilon^2 = 1$). The object dimensions are the variable parameters. As has already been noted, the SEO detection in this case is provided with distinguishing the boundary between the object and background in the image. It follows from the analysis of the curves that the efficiency of the SEO detection depends strongly on the value of the interelement correlation coefficient ρ. In this case the small detection probability corresponds to small ρ because the intensity difference on the background–object boundary becomes comparable to that between the adjacent readings of the object and background images. For higher degree of correlation of the intensities of the object and background images (as ρ increases), the boundary effects are more clearly pronounced, that leads to the corresponding increase in the SEO detection probability. The longer object image perimeter due to increase in its area or, when the area is constant, due to the change of the proportion in the lengths of its sides, leads to increase of the detection probability for any ρ.

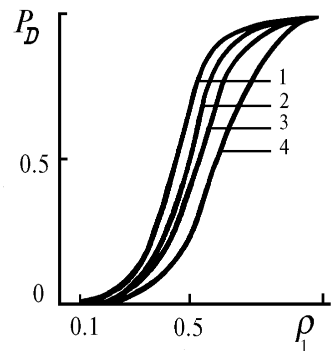

FIG. 2. The SEO detection probability as a function of the interelement correlation coefficient $\rho = \rho_0 = \rho_1$ *for equal variances of the intensities of the object and background images* ($\varepsilon^2 = 1$) *and different object dimensions: 1)* 10×10; *2)* 12×2; *3)* 5×5; *and, 4)* 3×3.

The dependences of the SEO detection probability on the coefficient of the interelement correlation of the object image intensities ρ_1 for different object dimensions, ratio of variances ε^2, and the coefficient of interelement correlation of the object image intensity ρ_0 are shown in Fig. 3. As could be expected, the detection curves, regardless of the object image area, reach their minimum when the object and background intensity correlation coefficients are equal $\rho_1 = \rho_0$ and monotonically increase with the difference $|\rho_1 - \rho_0|$. The improvement of detection quality is primarily explained by the increased reliability of distinguishing the SEO and background images in the zone of possible object location based on the image texture (correlation functions).

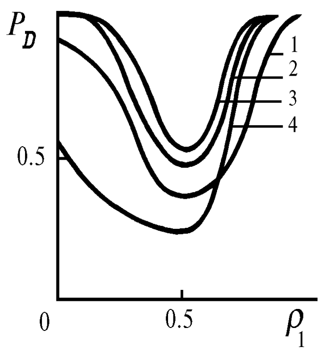

FIG. 3. The SEO detection probability as a function of the coefficient of the object interelement correlation ρ_1 *for object dimensions* 7×7 *(curves 2, 3) and* 5×5 *(1, 4); the ratio of the variances of the intensity of the object and background images is* $\varepsilon^2 = 1$ *(1, 3),* 0.8 *(2), and* 0.5 *(4); the background interelement correlation coefficient is* $\rho_0 = 0.5$

Figure 4 shows the dependence of the SEO detection probability on the ratio of the variances of the object and background image intensity readings ε^2 when the interelement correlation coefficients are equal, i.e., $\rho_1 = \rho_0 = \rho$, for different fixed values of ρ and object dimensions.

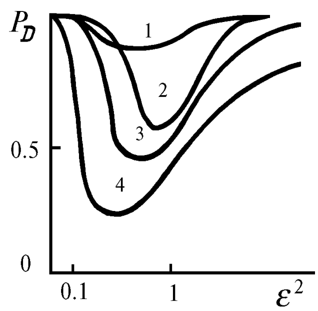

FIG 4. The SEO detection probability as a function of the ratio of the variances of the intensities of the object and background images ε^2 *for different values of the interelement correlation coefficients* $\rho = \rho_0 = \rho_1 = 0.7$ *(1) and* 0.5 *(2–4) and the object dimensions* 5×5 *(1, 4),* 7×7 *(3), and* 10×10 *(2).*

The comparative analysis of these curves allows us to draw a conclusion that the detection curves reach their minimum at the point in which $\varepsilon_*^2 \ll 1$. Moreover, the value of ε_*^2 depends strongly on the number of degrees of freedom (i.e., on the uncorrelated intensity readings) of images in the zone of possible object location, which is determined by the relation between the object area $N_x \times N_y$ and the correlation coefficient ρ. Therefore, for the 5×5 object when $\rho = 0.5$ the detection probability reaches its minimum at $\varepsilon_*^2 = 0.5$, whereas when $\rho = 0.7$ the minimum is achieved at $\varepsilon_*^2 = 0.75$. This phenomenon is explained by different effect of the texture and the boundary of the object image on the quality of the object image selection. With the decrease of ε^2 in the interval $[\varepsilon_*^2, 1]$, the quality of the SEO detection on its boundary with the background decreases markedly due to the decrease of the difference between the intensities of the object and background images on the boundary, whereas the difference between the object and background textures is slightly pronounced. With further decrease of ε^2 below the threshold ε_*^2 the discrimination based on the object and background correlation properties becomes predominate in the processing that leads to the increase of the SEO detection probability.

Thus, the absence of correlation between the image textural characteristics in the zone of possible SEO location as well as the difference between the intensities of the image on the boundary of this zone are the informative parameters of the presence of the spatially extended object against the spotted background. The contribution of each component depends on the values of the SEO and background parameters and disregarding one of them may lead to principal errors in the evaluation of the quality of the SEO image detection against the spotted background.

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