AN ALGORITHM FOR OPTIMUM PROCESSING OF NONMONOCHROMATIC SIGNALS IN AN ADAPTIVE SYSTEM OF AUTOFOCUSING

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The present paper is devoted to the synthesis and analysis of an optimal processing algorithm in an adaptive system of autofocusing. The actual width of the signal spectrum is taken into consideration, resulting in the necessity of including a new operation in the algorithm, specifically, time inversion of the complex signal amplitude.

At present the development of optical locational-informational systems using an atmospheric channel is closely connected with the wider application of adaptive systems based on wavefront reversal (WFR). This is due to the fact that these systems allow one to forego the use of special systems for operational sounding of the atmosphere since compensation for the distorting influence of the atmosphere is achieved as a result of wavefront reversal of the light signal scattered by the object.[1,2] Investigations in this field, devoted to an analysis of the quality of wavefront correction when implementing WFR by controllable adaptive mirrors[3,4] and the influence of the limited dynamic range of the adaptive mirror profile correction system on the efficiency of focusing,[5,6] are known.

In all of this the assumption of the monochromaticity of the signal radiation lies at the foundation of the developed algorithms and related investigations. However, such ideal signals do not exist in nature. In this connection, synthesis of an algorithm for optimal processing of the signal in an adaptive autofocusing system which takes account of the real spectrum of the processed signal is of interest.

In general it is important for the effective operation of an adaptive autofocusing system that the total energy reemitted by it should be completely intercepted by the object. In so doing it is not important for the realization of, for example, the energy transfer how the energy is distributed over the object surface.

Therefore, the problem of constructing signal processing algorithms can be solved in, at least, three different ways depending on whether one assumes that

1) the maximum radiated energy flux on the object is obtained by reproducing the initial field distribution on it; or that

2) the radiated energy is concentrated on some limited area of the object; or that

3) the total radiated energy is randomly distributed over the object.

The first way of solving the stated problem, which has a unique solution because there exists only one way of reproducing the initial field distribution, is considered below. In addition, this approach allows us to simultaneously elucidate the optimal algorithm for signal processing in adaptive systems for reconstructing an undistorted image of the object, which is of interest for optical locational—informational systems.

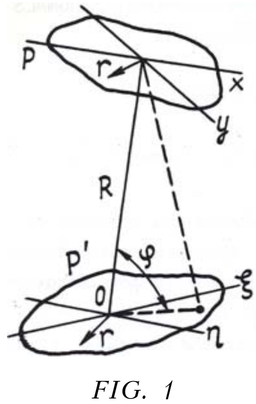

FIG. 1

Let us suppose that the field scattered by the object P (see Fig. 1) passes through a nonuniform medium — the turbulent atmosphere — and is received by the aperture p', and the centers of the receiving aperture and the image plane of the object define a line forming an angle of $\pi/2 - \varphi$ with the normal to the receiving aperture plane. In this case the distribution $E(r,\ t)$ in the image plane is assumed to be a sampling function of the normal random field. Let us find the algorithm of optimal processing of the field $E(r',\ t)$ at the receiving aperture that enables one best to reconstruct the original field distribution $E(r,\ t)$ in the image plane of the object P.

As our criterion of optimality we shall use the time-averaged variance of the error of reconstructing the true field distribution in the image plane:

$$\xi(r) = \int\limits_{\tau_p} \langle |E(r,\ t) - \hat{E}(r,\ t)|^2 \rangle\, dt, \tag{1}$$

where

$$\hat{E}(r,\ t) = \int\limits_{p'}\int\limits_{\Delta\omega} E_r(r,\ \omega) h_f(r',\ r,\ \omega)\exp(-j\omega t)\, d\omega dr'$$

is the estimate of the field distribution in the image plane obtained by passing the received spectral component $E_r(r', \omega)$ through a filter with impulse response $h_f(r', r, \omega)$, and $\Delta\omega$ and τ_p are the spectral width and duration of the signal, respectively.

In its turn

$$E_r(r', \omega) = \int_P E(r, \omega)\, h_m(r, r', \omega)\, dr + E_n(r', \omega), \tag{2}$$

where $h_m(r, r', \omega)$ is the impulse response of the "frozen" medium at the frequency ω; and,

$$E(r, \omega) = \int_{\tau_p} E(r, t) \exp(j\omega t)\, dt$$

is the spectral component of the signal at the frequency ω at the point r of the image plane of the object. $E_n(r', \omega)$ is the spectral component of the noise background at the frequency ω at the point r' of the receiving aperture. Let us suppose that each spectral component of the noise background is a normal random field with zero mean and a spatial correlation function of the form $K_\omega(r', r_1', \omega) = N_0(\omega)\delta(r' - r_1')$, where $N_0(\omega)$ is the power spectral density of the noise.

On the basis of the Parseval theorem we rewrite Eq. (1) in the form

$$\xi(r) = \int_\omega \xi(r, \omega)\, d\omega = \int_{\Delta\omega} \langle |E(r, \omega) - \hat{E}(r, \omega)|^2 \rangle\, d\omega, \tag{3}$$

where

$$\hat{E}(r, \omega) = \int_{P'} E_r(r', \omega)\, h_f(r', r, \omega)\, dr'$$

is the estimate of the spectral component with frequency ω at the point r of the image plane of the object.

The physical meaning of expression (3) lies in the fact that optimization of the estimate can be achieved by minimizing the error of reconstructing the field in each spectral component.

As a result of such a minimization of $\xi(r, \omega)$ by the method described in Ref. 7 and a number of mathematical transformations and assumptions, one can show that the expression for the optimal estimate of $\hat{E}(r, \omega)$ reads

$$\hat{E}(r, \omega) = \frac{J_0(\omega)}{N_0(\omega) + J_0(\omega)} \times \int_{P'} E_r(r', \omega)\, h_f^*(r', r, \omega)\, dr', \tag{4}$$

where $J_0(\omega)$ is the spectral density of the signal at frequency ω.

Before determining the content of the optimal processing algorithm, let us list some considerations that enable one to make this more evident. We note that the quantity $\xi(r)$, calculated according to Eq. (3), does not change when $\left|E(r, \omega) - \hat{E}(r, \omega)\right|$ is replaced by $\left|E^*(r, \omega) - \hat{E}^*(r, \omega)\right|$.

The physical nature of the optimatility criterion allows one to compare not only the field distribution in the image plane of the object $E(r, \omega)$ with the estimate $\hat{E}(r, \omega)$, but also their complex-conjugate values, since the intensity distribution in the image plane of the object does not change. This is important for autofocusing. Therefore to formulate the physical nature of the optimal processing algorithm, it is permissible to use the complex-conjugate expression

$$\hat{E}^*(r, \omega) = \frac{J_0(\omega)}{N_0(\omega) + J_0(\omega)} \times \int_{P'} E_r^*(r', \omega) h_f(r', r, \omega)\, dr' \tag{5}$$

instead of expression (4).

Thus, the physical nature of the optimal processing algorithm for the spectrum of the received signal consists in the following: each of its spectral components must be subjected to phase conjugation and reemission onto the object.

Let us take the inverse Fourier transform of both sides of Eq. (5) assuming that the signal spectrum is uniform, and the noise component is characterized by the same power at all frequencies. Note that for the optical signal[8]

$$h_f(r', r, \omega) = \frac{h_{at}(r', r)}{j\lambda_0 R} \exp(j\omega t_m(r', r)), \tag{6}$$

where $h_{at}(r', r)$ is the impulse response of the atmosphere, and

$$t_m(r', r) = \frac{R}{C}\left[1 + \frac{|r' - r|^2}{2R^2}\right].$$

As a result of taking the Fourier transform of Eq. (5) we obtain

$$\hat{E}^*(r, -t) = \frac{1}{j\lambda_0 R} \cdot \frac{J_0}{N_0 + J_0} \times \int_{P'} E_r^*(r', t_m(r', r) - t)\, h_{at}(r, r')\, dr'. \tag{7}$$

In conclusion we note that the optimal algorithm for processing a quasi-monochromatic signal in adaptive systems of autofocusing, in contrast to the optimal algorithm of processing for a monochromatic signal, includes (in addition to phase conjugation) the time inversion of the complex amplitude of the signal, i.e., formation and reemission onto the object of a time-reversed phase-conjugated copy of the received signal.

It follows from an analysis of the known methods for wavefront reversal that it is possible to implement this signal processing algorithm on the basis of non-stationary effects with a four-wave shift.[2]

REFERENCES

1. Ya. Hermann, *Adaptive Optics* [Russian translation] (Mir, Moscow, 1980).
2. B.Ya. Zel'dovich, N.F. Pilipetskii, and V.V. Shkunov, *Wavefront Reversal* (Nauka, Moscow, 1985).
3. V.E. Kirakosyants, V.A. Loginov, and V.V. Slonov, Kvantovaya Elektron. **10**, No. 12, 2485 (1983).
4. M.A. Vorontsov, A.V. Koryabin, and V.I. Shmal'gauzen, *Controllable Optical Systems* (Nauka, Moscow, 1988).
5. V.P. Lukin and V.L. Mironov, Kvantovaya Elektron. **12**, No. 9, 1959 (1985).
6. V.G. Taranenko, Izv. Akad. Nauk SSSR, Ser. Fiz. **48**, No. 7, 1419 (1984).
7. H.L. Van Trees, *Detection, Estimation, and Modulation Theory*, Pt. 1 (Wiley, New York, 1968).
8. I.N. Matveev, V.V. Protopopov, I.N. Troitskiĭ, and N.D. Ustinov, *Laser Location*, ed. by N.D. Ustinov (Mashinostroenie, Moscow, 1984).