DISCUSSION OF THE ERRORS IN LIDAR MEASUREMENTS OF THE ATTENUATION COEFFICIENT OF THE ATMOSPHERE DUE TO THE VARIABILITY OF THE SCATTERING PHASE FUNCTION

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The results of model calculations of the errors in measurements of the average attenuation coefficient of the atmosphere owing to the variability of the scattering phase function g_π in the case when a cloud is present in the sounding path are analyzed. Three methods for fixing the path on which the average attenuation coefficient is measured are studied. It is shown that the best method for minimizing the indicated errors is to determine the path length from the decrease of the backscattering signal by a fixed factor.

The solution of the lidar equation for the attenuation coefficient $\mu(z)$ for an inhomogeous atmosphere is an improperly posed problem, even if there is no absorption of the sounding radiation and in the single-scattering approximation, because a second unknown function is present – the phase-function parameter $g_\pi(z)$ characterizing the backscattering. Information about the attenuation coefficient in different ranges of variation of the parameters of the atmosphere being sounded is extracted with the help of a priori assumptions about the character of the function $g_\pi(z)$. Any deviation of the real dependence $g_\pi(z)$ from these assumptions leads to the appearance of corresponding methodical errors. In addition, depending on wheter the deviation along the sounding path is distributed randomly or systematically and whether or not it is known beforehand, the errors will be random, systematic, or uneliminated systematic errors.

Because the problem is difficult to analyze in a general form, in this paper the measurement errors are analyzed for a specific lidar, the Elektronika-03 lidar, in which data processing is performed by the integral method.[1] The model of the inhomogeneous atmosphere and the range of variation of the atmospheric parameters were chosen based on the fact that this lidar is used for providing meteorological information for aircraft flights under conditions of dense haze and fog with sounding angles of up to 15° relative to the horizon.[1,2]

To analyze systematic errors, which are larger than the random errors, which can be reduced by averaging, we shall study a model of a stationary atmosphere. We shall characterize the model atmosphere by a constant attenuation coefficient μ_0 in the layer near the ground up to an altitude of H_L where $\mu(H) = \mu_0\exp(k'(H - H_L))$ which corresponds to an increase in the attenuation coefficient in the layer beneath the cloud and at the lower boundary of the cloud. This situation, in which $\mu(H)$ is constant or increases with altitude, is the most typical situation for conditions of dense haze and fog,[3] while an exponential law describes satisfactorily the increase in the attenuation coefficient at the lower boundary of a cloud.[4]

In accordance with the data of Refs. 5 and 6 we shall employ the power-law dependence

$$g\pi = k_1\mu^{k_2-1} \tag{1}$$

and we shall write for the model of the atmosphere employed here an expression for the backscattered signal, corrected for the square of the distance, in the form

$$S(z)=\begin{cases} S_0k_1\mu_0^{k_2}\exp^{-2\mu_0(z-z_0)} \text{ at } z<z_H; \\ S_0k_1\mu_0^{k_2}\exp^{-2\mu_0(z_H-z_0)}e^{k_2k(z-z_H)-2\mu_0/k(e^{k(z-z_H)}-1)} \end{cases} \tag{2}$$

where $S_0 = S(z_0)$; z_0 is the distance from which the signal processing starts; $k = k' \cdot \sin\nu_3$; ν_3 is the sounding angle relative to the horizon; and, z_H is the distance along the sounding path up to the altitude H_L.

The relation between the points z_0 and z_1 on the sounding path for calculating the average attenuation coefficient by the integral method has the form

$$\bar{\mu}(z_0, z_1) = \frac{1}{2(z_1-z_0)}\cdot\ln(I_m/I_1), \tag{3}$$

where $I_m = \int_{z_0}^{z_m} S(z)dz$; $I_1 = \int_{z_1}^{z_m} S(z)dz$; z_m is the distance at which the signal drops to the noise level.

An important part of the processing using the formula (3) is to determine the distance z_1 at which the lidar measurement baseline terminates. The problem of determining the distance z_1 automatically with the recording system of the lidar can be solved in three ways:

1) by fixing the distance $z_1 - z_0$;

2) by fixing the ratio of the integrals in (3); and,

3) by fixing the ratio of the amplitudes $S(z_0)/S(z_1) = n = \text{const}$.

Each method has its own advantages and disadvantages. The instrumentation is simplest in the case of the first method, and the values of $\bar{\mu}$ obtained agree best with the data from photometric monitoring devices with a baseline of $z_1 - z_0$. The second method permits fixing the optical thickness of the probed layer $\bar{\mu}(z_0, z_0)\cdot(z_1 - z_0)$. In this case the range of variation of the integrals recorded is minimized. The difficulty in setting up the instrumentation in this case with analog signal integration, as in Ref. 1, stems from the need to have information about the magnitude of the asymptotic integral up to the moment at which z_1 is fixed in real time. The third method for determining the distance z_1 can be easily implemented by using a comparator, which records the moment at which the condition $S(z_0)/S(z_1) = n$ is satisfied, but in this case the dynamic range of variation of the integrals under the conditions of an inhomogeneous atmosphere increases.

The errors in measuring the average attenuation coefficient $\bar{\mu}$ on the measuring baseline chosen by one of the methods enumerated above were calculated using the formula

$$\delta\bar{\mu} = \frac{\bar{\mu}_{meas} - \bar{\mu}_{source}}{\pi_{source}},$$

where $\bar{\mu}_{meas}$ was determined from the relation (3) by numerical integration of the signals from Eq. (2), while $\bar{\mu}_{true}$ was determined from the model true profile $\mu(z)$ in the range from z_0 to z_1. In so doing z_m was taken to be the distance at which the backscattering signal dropped to a level 250 times lower than $S(z_0)$. The value of z_m was determined by solving the transcendental equation obtained from Eq. (2) under the assumption that $S(z_m)/S(z_0) = 250$:

$$2\mu_0(z_H - z_0) - k_2\cdot k\cdot(z_m - z_H) + \frac{2\mu_0}{k}\left[\exp^{k(z_m - z_0)} - 1\right] = \\ = \ln 250. \quad (4)$$

The quantity z_1 was determined analogously for the fixed amplitude ratio method. In this case the quantity Inn on the right side of (4) was written for situations when $z_1 > z_H$ (see Fig. 1) and $z_1 = z_0 + \ln n/2\mu_0$ for $z_1 \le z_H$.

In order to use the relations (2) to calculate the errors the parameter k_2 and the ranges of variation of the parameters of the three-parameter model of the atmosphere must be determined.

The values of k_2 ranging from 1.2 to 1.6 are presented in Refs. 5 and 6 for conditions of dense haze and fog. The value $k_2 = 1.3$, which in our opinion is most likely, was employed in the calculations.

The parameters of the model atmosphere were chosen as follows. The range of values of the attenuation coefficient was chosen to correspond to dense haze and tenuous fog at the earth's surface: $0.5\ \text{km}^{-1} \le \mu_0 \le 3\ \text{km}^{-1}$. The distance z_H was chosen to be ≥ 0.1 km, which for sounding angles for the Elektronika-03 lidar ranging from 0 to 15° corresponds to H_L ranged from practically zero to altitudes at which, owing to the high optical density of the layer near the ground, clouds have no effect on the measurements. For the indicated sounding angles, based on the date from Ref. 4, according to which k' ranges from 0 to 60 km^{-1} the range of k was chosen to be 0 to 15 km^{-1}. The values of the ratios of the integrals and signals for which the distance z_1 was fixed were set equal to 10.

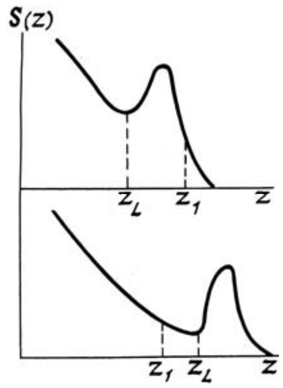

FIG. 1. The backscattered signal S as a function of the distance to the scattering volume z: $z > z_H$; (a) and $z < z_H$ (b).

The errors in measuring the attenuation coefficient were calculated for the entire indicated three-dimensional region of parameters of the model atmosphere. Figures 2–4 show sections of surfaces of 15% error, which can be regarded as the maximum admissable error for the Elektronika-03 lidar, assuming that the error under study is the dominant error (the total measurement error of this lidar should not exceed 25%). With the exception of curve 5 in Fig. 2, the curves presented correspond to errors $\delta\bar{\mu} = -15\%$, which is due to the "large" contribution of clouds to the scattered signal owing to the increase in $g_\pi(z)$. As a result, in the expression (3), derived under the assumption that the so-called average integral scattering functions on the sections where the integrals are calculated are equal,[7] the value of I_1 Is higher than the true value to a much greater degree than I_m. This results in the fact that the ratio of the Integrals and correspondingly the average attenuation coefficient are underestimated. This effect is all the larger the lower the value of μ_0 (the more transparent the atmosphere near the ground) and the higher the value of k (the higher the rate of growth of μ as the depth of penetration into the cloud increases). Therefore in the figures the regions of the parameters μ_0 and k below and to the right of the curves correspond to even larger errors. In this case the problem consists of minimizing in the space of likely parameters of the model atmosphere the region with errors above a given level.

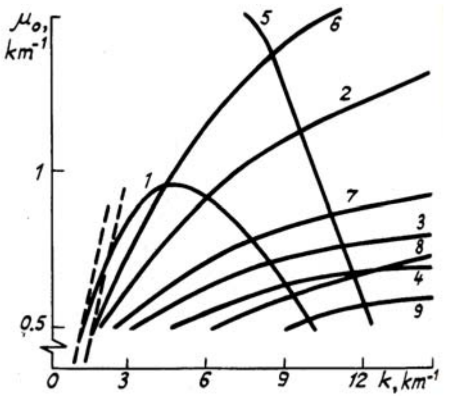

FIG. 2. Sections of surfaces of constant error $(\delta\bar{\mu} = \pm 15\%)$ *by the planes* z_H *= const in the coordinates k and* μ_0. z_1 *is fixed.* $\delta\bar{\mu} = -15\%$ *(1÷4.6÷9), +15% (5);* z_1 *= 0.8 km (1÷5), 1.2 (6÷9);* z_H *= 0.8 km (1, 5, 6), 1.2 (2, 7), 1.6 km (3, 8), and 2 (4, 9).*

To describe the degree of inhomogeneity of the atmosphere in this model it is convenient to introduce a parameter characterizing the degree of inhomogeneity $k/2\mu_0$, which fixes the angular position of the straight lines passing through the origin of coordinates in the figures (dashed lines). It is obvious from the figures that the characteristics of the signal from the cloud, such as the relative amplitude and the position of the maximum relative to z_H, depend on the values of the parameter $k/2\mu_0$. The computational results can be compared with the help of this parameter, if it is assumed that the greater the inhomogeneity of the atmosphere, in which the errors do not exceed a fixed level, the better the result is.

From the viewpoint of this criterion the results presented in Fig. 2 for negative error, equal to −15%, and the values of z_1 (0.8 and 1.2 km) differ insignificantly ($k/2\mu_0 = 1.45$ and 1.25, respectively). However for a baseline 1.2 km the figure contains explicitly a deficiency characteristic for the case when z_1 is fixed – the boundary effect becomes significant as $\bar{\mu}(z)$ decreases, when the value of z_m approaches z_1 and therefore I_1 is determined with lower accuracy. This results in overestimation of $\bar{\mu}_{meas}$ and in the appearance of a region ofpositive errors exceeding 15%, relative to curve 5. In this case the absolute value of $\delta\bar{\mu}$ can exceed the fixed level even in a homogeneous atmosphere ($k = 0$).

The results presented in Fig. 3 are preferable. Here the limiting value of $k/2\mu_0$ is 1.75, and the "boundary effect" is negligible.

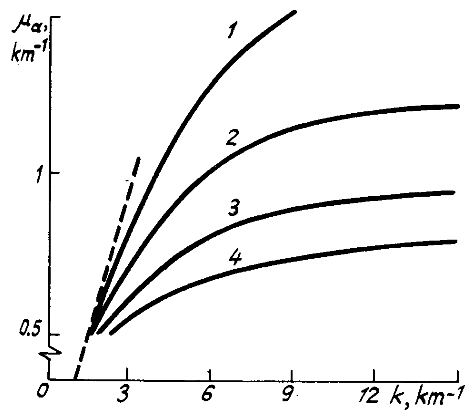

FIG. 3. Sections of the surface of constant error $(\delta\bar{\mu} = -15\%)$ *by planes* z_H *= const in the coordinates k and* μ_0. *The ratio of the* integrals *is fixed.* z_H *= 0.8 km (1), 1.2 (2), 1.6 (3), and 2 (4).*

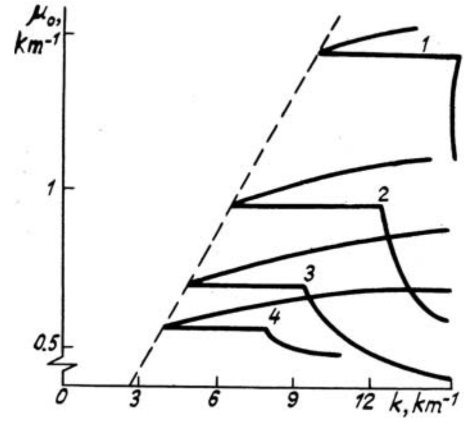

FIG. 4. Sections of the surface of constant error $(\delta\bar{\mu} = -15\%)$ *by the planes* z_H *= const in the coordinates k and* μ_0. *The amplitude ratio is fixed.* z_H *= 0.8 km (1), 1.2 (2), 1.6 (3), and 2 (4).*

The results presented in Fig. 4 are of greatest interest. Here the parameter $k/2\mu_0$, equal to 3.5, has the highest value of the three cases studied. In addition the upper parts of the graphs (upto cutoff) are close to the corresponding graphs in Fig. 3, since in this case z_1 is recorded on the starting section of the signal from a homogeneous atmosphere, and the effect of the signal from the cloud is relatively small. Therefore in a completely homogeneous atmosphere for these two cases z_1 will be the same. The break in the graphs is caused by the jump-like increase in z_1 (increasing penetration into the cloud), when $S(z_H)$ becomes greater than $S(z_0)/n$. As a result the range of near the maximum of the pulse from the cloud is not realized, and the error under study is maximum precisely for this range.

Thus from the .viewpoint of increasing the probability of measurements with a fixed error owing to the variability of the phase function parameter g_π, the preferred method is to implement in the lidar the integral method with the measuring baseline fixed according to a decrease of the scattering signal by a fixed factor.

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