Step-by-step technique for taking into account thermal radiation scattering by aerosol formations in the atmosphere. Part 2. Account of polarization. Computational algorithm. Aerosol models

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A computational scheme with polarization for thermal radiation scattering by aerosol formations in the atmosphere is described, and an algorithm for these computations is suggested. Main requirements to aerosol models used in solution of the inverse problems of atmospheric optics for the IR and microwave spectral regions are considered. According to these requirements, some models of clouds, precipitation, and aerosol layers are proposed.

Introduction

In Part 1 of this paper (see this issue of *Atmospheric and Oceanic Optics*) we considered the step-by-step technique for calculation of radiative transfer in the plane horizontally homogeneous atmosphere and discussed its capabilities and specific features as applied to the problems of thermal radiation scattering by aerosol formations in the IR and microwave (MW) spectral regions. This part completes the general description of the method, namely, takes into account the radiation polarization at scattering and reflection from the surface, as well as discusses the features of its particular computer implementation and the corresponding models of clouds, precipitation, and aerosol layers.

Polarization of radiation

The polarization of radiation must be taken into account in the problem under study first, because it manifests itself in the processes of radiation scattering and reflection and, consequently, makes their physical and mathematical description without polarization only approximate. The second reason is that the majority of instrumentation currently in use can measure polarization characteristics of radiation. The polarized radiation is described[1] using the vector of Stokes parameters $\mathbf{L}$ including four components: $\mathbf{L} \equiv (I, Q, U, V)$ (the Stokes vector is a column vector, but hereinafter it will be written as a row for saving place). The first Stokes parameter I is the radiation intensity. For rotation of the coordinate system in the plane normal to the radiation propagation direction by the angle β, the Stokes parameters are re-calculated as $\mathbf{L}(\beta) = \mathbf{M}(\beta)\mathbf{L}(0)$ using the rotation matrix[1–3]

$$\mathbf{M}(\beta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\beta & \sin 2\beta & 0 \\ 0 & -\sin 2\beta & \cos 2\beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (1)$$

In many atmospheric processes, extinction of radiation that does not alternate its direction is independent of polarization. Such processes are, in particular, radiation extinction by homogeneous spherical aerosol particles and, ignoring the Zeeman effect in the Earth's magnetic field, radiation absorption by atmospheric gases. Then with no scattering, the equation for each of the Stokes vector components is solved independently using Bouguer law. Thus the concepts of the transmission function $P(\nu, z_1, z_2, \vartheta)$, optical depth $\tau(\nu, z)$, and optical thickness of the atmosphere $\tau_0(\nu, z)$ keep unchanged (see Part 1 of this paper, where the following designations are introduced: ν is the frequency (wavelength, wave number) of monochromatic radiation, z is the height in the atmosphere, ϑ is the nadir angle of the radiation propagation direction, φ is the azimuth of radiation propagation).

Since the natural radiation is unpolarized [for it $\mathbf{L} = (I, 0, 0, 0)$] and if scattering is ignored, polarization can arise only at radiation reflection from a surface. Transfer of such radiation is described by the Bouguer law, and no new difficulties arise in calculation of the thermal radiation with the allowance for polarization, but with neglect of scattering (see equation in Part 1 of this paper).

The account of scattering [2,3] leads to the transfer equation for the thermal radiation under LTE (local thermodynamic equilibrium) conditions with the allowance for polarization

$$\frac{d\mathbf{L}(\nu, z, \vartheta, \varphi)}{dz}\cos\vartheta =$$

$$= -\alpha(\nu, z)\mathbf{L}(\nu, z, \vartheta, \varphi) + \frac{\sigma(\nu, z)}{4\pi}\int_0^\pi \sin\vartheta' d\vartheta' \times$$

$$\times \int_0^{2\pi} d\varphi' \mathbf{M}(\beta)\mathbf{X}(\nu, z, \gamma)\mathbf{M}(\beta')\mathbf{L}(\nu, z, \vartheta', \varphi') + \kappa(\nu, z)\mathbf{E}_0[\nu, T(z)],$$

$$\tag{2}$$

where $\sigma(\nu, z)$ is the volume scattering coefficient, $\mathbf{X}(\nu, z, \gamma)$ is the normalized scattering phase matrix depending on the scattering angle $\gamma = \angle[(\vartheta,\varphi),(\vartheta',\varphi')]$, its element X_{11} is the scattering phase function of unpolarized radiation $x(\nu, z, \gamma)$ obeying the normalization condition $\frac{1}{2}\int_0^\pi x(\gamma)\sin\gamma d\gamma = 1$; $\kappa(\nu, z)$ is the volume absorption coefficient; $\mathbf{E}_0[\nu, T(z)]$ is the column vector $\{B_e[\nu, T(z)], 0, 0, 0\}$, in which T is the air temperature; $B_e(\nu, T)$ is the Planck's function. Spherical trigonometry[2,3] for the rotation angles β, β' and the scattering angle γ gives

$$\cos\gamma = \cos\vartheta\cos\vartheta' + \sin\vartheta\sin\vartheta'\cos(\varphi - \varphi'),$$

$$\cos\beta = \frac{\sin\vartheta\cos\vartheta' - \cos\vartheta\sin\vartheta'\cos(\varphi - \varphi')}{\sin\gamma},$$

$$\sin\beta = \frac{\sin\vartheta'\sin(\varphi - \varphi')}{\sin\gamma}, \tag{3}$$

Equations for $\cos\beta'$ and $\sin\beta'$ follows from Eqs. (3) through formal permutation of primed and non-primed variables.

Then Eq. (2) is converted just similarly to the case of transfer of unpolarized radiation.[3]

Step-by-step technique of the account of polarization

Repeating literally our reasoning from Part 1, we obtain the same formal expansions in terms of the number of scattering and reflection events, but already for the Stokes vector rather than for the intensity. The account of polarization here can be reduced to the following substitutions:

1) The radiation field is characterized by the Stokes vector $\mathbf{L}$ rather than the intensity $\mathbf{I}$.

2) The field of source of the primary radiation $\mathbf{B}_0$ is characterized by the vector, whose first component coincides with the case of unpolarized radiation, and the other three components are zero (certainly, if the natural radiation of the surface is not polarized).

3) The transfer operator for the direct radiation $\mathbf{T}_0$ is equal to the product of this operator for the unpolarized radiation and unit matrix.

4) The transfer operator for the single scattered radiation $\mathbf{T}_1$ is

$$\mathbf{T}_1(\nu, \tau, \eta, \varphi, \tau', \eta', \varphi') =$$

$$= \frac{\omega_0(\nu, \tau)}{4\pi\eta'}P(\nu, \tau', \tau, \eta')\mathbf{M}(\beta)\mathbf{X}(\nu, \tau, \chi)\mathbf{M}(\beta'),$$

if $\eta' > 0$ and $\tau' \geq \tau$ or $\eta' < 0$ and $\tau' \leq \tau$;

$$\mathbf{T}_1(\nu, \tau, \eta, \varphi, \tau', \eta', \varphi') = 0, \tag{4}$$

if $\eta' > 0$ and $\tau' < \tau$ or $\eta' < 0$ and $\tau' > \tau$,

where $\omega_0(\nu, \tau)$ is the single scattering albedo, $\chi = \cos\gamma$ (see Eq. (3)).

5) The relation between the radiation incident on the surface $\mathbf{L}^\downarrow(\nu, \tau_0, \eta, \varphi)$ and that reflected from the surface $\mathbf{L}^\uparrow(\nu, \tau_0, \eta, \varphi)$ is the following:

$$\eta\mathbf{L}^\uparrow(\nu, \tau_0, \eta, \varphi) =$$

$$= -\int_0^{2\pi} d\varphi' \int_{-1}^0 \eta'\mathbf{R}(\nu, \eta, \eta', \varphi - \varphi')\mathbf{L}^\downarrow(\nu, \tau_0, \eta', \varphi')d\eta', \tag{5}$$

where the reflection matrix $\mathbf{R}(\nu, \eta, \eta', \varphi - \varphi')$ relates the Stokes vector of the radiation incident along the direction (η', φ') to the Stokes vector of radiation reflected along the direction (η, φ) [in the general case it must include re-calculation of polarization azimuth by the rotation matrices (1)]. For the ideally specular reflection one has

$$\mathbf{R}(\nu, \eta, \eta', \varphi - \varphi') = \mathbf{r}(\nu, \eta)\delta[\eta - (-\eta')]\delta(\varphi - \varphi'),$$

where $\mathbf{r}(\nu, \eta)$ is the reflection matrix determined by the Fresnel equations.[1] For isotropic reflection $\mathbf{R}(\nu, \eta, \eta', \varphi - \varphi')$ is equal to the product of the unit matrix by $\eta A(\nu)/\pi$, where $A(\nu)$ is the spectral albedo of the surface. The reflection matrix is related to the single reflection operator $\mathbf{R}_1$ having, according to Eq. (5), the following form:

$$\mathbf{R}_1(\nu, \tau, \eta, \varphi, \tau', \eta', \varphi') =$$

$$= -\eta'\mathbf{R}(\nu, \eta, \eta', \varphi - \varphi')\delta(\tau' - \tau_0)\delta(\tau - \tau_0),$$

if $\eta > 0$ and $\eta' < 0$;

$$\mathbf{R}_1(\nu, \tau, \eta, \varphi, \tau', \eta', \varphi') = 0, \tag{6}$$

if $\eta < 0$ or $\eta' > 0$.

Peculiarities of taking into account polarization in the IR and MW regions

As in the case of unpolarized radiation, the thermal radiation is considered as azimuth-isotropic. Justification of this statement is given in Part 1 along with possible ways of taking into account the azimuth anisotropy. After averaging over azimuth, nonzero parts of the single scattering (4) and reflection (6) operators can be written as

$$\mathbf{T}_1(\nu, \tau, \eta, \tau', \eta') = \frac{\omega_0(\nu, \tau)}{4\pi\eta'}P(\nu, \tau', \tau, \eta')\mathbf{p}(\nu, \tau, \eta, \eta'),$$

$$\mathbf{R}_1(\nu, \tau, \eta, \tau', \eta') = -\eta' \mathbf{r}_1(\nu, \eta, \eta') \delta(\tau' - \tau_0) \delta(\tau - \tau_0), \quad (7)$$

where

$$\mathbf{p}(\nu, \tau, \eta, \eta') = \int_0^{2\pi} \mathbf{M}(\beta) \mathbf{X}(\nu, \tau, \chi) \mathbf{M}(\beta') d(\varphi - \varphi');$$

$$\mathbf{r}_1(\nu, \eta, \eta') = \int_0^{2\pi} \mathbf{R}(\nu, \eta, \eta', \varphi) d\varphi.$$

For the Rayleigh scattering matrix, we can calculate[3]

$$\mathbf{p}(\eta, \eta') = \frac{3}{4}\pi \times$$

$$\times \begin{pmatrix} 3 + 3\eta^2(\eta')^2 - \eta^2 - (\eta')^2 & (1 - 3\eta^2)(1 - (\eta')^2) & 0 & 0 \\ (1 - 3(\eta')^2)(1 - \eta^2) & 3(1 - \eta^2)(1 - (\eta')^2) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3\eta\eta' \end{pmatrix}.$$

$$(8)$$

Such a structure of the matrix (8) means that if only scattering processes are considered, then for the initial unpolarized radiation the parameters U and V of the Stokes vector are always zero. If the matrix of reflection operator $\mathbf{r}_1(\nu, \eta, \eta')$ has such properties too (it is so for the ideal mirror and isotropic reflections), then when calculating polarization it is sufficient to take into account only two Stokes parameters (I and Q). This two-component description of polarization is rather often used in the MW spectral region.

Computational algorithm

Below we describe the computational algorithm proposed in Part 1. The algorithm is considered for the general case of polarization taken into account. Its transformations for particular cases (Rayleigh approximation, unpolarized radiation, taking into account only single reflection, etc.) are quite obvious.

1. The problem of simulating measurements for direct radiation is solved — the Stokes vector $L_d(l, \tau, \eta, \varphi)$ is calculated. Hereinafter the index l denotes the Stokes vector components and elements of the corresponding matrices; in all equations $l = 1, ..., 4$. As was mentioned in Part 1, for direct radiation it is possible to consider the spherical geometry of the atmosphere, azimuth anisotropy of the reflection, etc.

2. The quadrature equation for integration over the optical depth is selected, its nodes are $\tau(j)$, $j = 1, ..., J$. This equation must include explicitly the integration limits (the trapezoid rule is recommended). Let, for certainty, $\tau(1) = 0$, $\tau(J) = \tau_0$. Introduce the function $d(j, j_1, j_2)$ describing the weight of this quadrature equation (it can be both positive and negative) corresponding to the node $\tau(j)$ in calculating the integral from $\tau(j_1)$ to $\tau(j_2)$

$[\min(j_1, j_2) \le j \le \max(j_1, j_2)]$. Note that if in the IR and MW regions the scattered radiation is considered as a small addition to the direct radiation, then the integration grid can be taken rough enough, for example, the nodes $\tau(j)$ are not obliged to coincide with the similar nodes used in calculation of direct radiation. This significantly speeds up the computations.

3. The quadrature equation for integration over directions is selected only for the interval [0, 1]; its nodes are $\eta(k)$, $k = 1, ..., K$. This equation may have any form (usually the Gauss equation with four to six nodes is recommended, but, as will be clear below, the equation with not strictly fixed nodes may be more convenient). Assume that $\eta(-k) = -\eta(k)$. Denote the weight of this quadrature function (always positive) as Δk, and $\Delta(-k) = \Delta(k)$. For a compact presentation of the algorithm, define also the index function $J(k) = J$, if $k > 0$; $J(k) = 1$, if $k < 0$.

4. The function of the initial sources is calculated for the selected grid:

$$B_0(l, j, k) = \{1 - \omega_0[\tau(j)]\}B_e\{\nu, T[\tau(j)]\} \text{ at } l = 1,$$

$$B_0(l, j, k) = 0 \text{ at } l > 1, \ j = 1, ..., J, \ |k| = 1, ..., K.$$

The sources of thermal radiation from the surface $\eta(k)\varepsilon(l, \eta(k))B_e(\nu, T)$ are added to it at $j = J$ for all $k = 1, ..., K$; here $\varepsilon(l, \eta)$ is the surface emissivity (if there is no radiation polarization, then $\varepsilon(l, \eta) = 0$ at $l > 1$), T is the surface temperature. In the MW region, the relic cosmic radiation with the temperature $T_0 = 2.7$ K is also often taken into account at the atmospheric top. This gives the addition to $-\eta(k)B_e(\nu, T_0)$ at $l = 1$, $j = 1$, for all $k = -K, ..., -1$.

5. For the selected grid, the operator of direct radiation

$$T_0(j, j', k') = -\frac{1}{\eta(k')}\exp\left(-\frac{\tau(j') - \tau(j)}{\eta(k')}\right)$$

and the single scattering operator (4)

$$T_1(l, l', j, k, j', k') =$$

$$= -\frac{\omega_0[\tau(j)]}{4\pi}p[l, l', \tau(j), \eta(k), \eta(k')]T_0(j, j', k'),$$

$l = 1, ..., 4, \ l' = 1, ..., 4, \ j = 1, ..., J, \ |k| = 1, ..., K,$ $|k'| = 1, ..., K, \ j' = \min[j, J(k')], ..., \max[j, J(k')],$ are calculated.

For faster calculations, we can, having specified certain accuracy, assign zero values to the elements $T_0(j, j', k)$, for which the exponent is rather small, and then use the well-known sweep method. In the IR and MW regions, scattering is often considered in some atmospheric layers (for example, clouds) between the levels j_{up} and j_{dn}, rather than in the entire atmosphere. In this case, for example, at $j > j_{dn}$, $j' < j_{up}$ we have

$$T_1(l,l',j,k,j',k') = T_{1,\text{up-dn}}(l,l',j_{\text{up-dn}},k,j'_{\text{up-dn}},k') \times$$

$$\times \exp\left(-\frac{\tau(j_{\text{up}}) - \tau(j')}{\eta(k')}\right)\exp\left(-\frac{\tau(j) - \tau(j_{\text{dn}})}{\eta(k)}\right),$$

where $T_{1,\text{up-dn}}(l,l',j_{\text{up-dn}},k,j'_{\text{up-dn}},k')$ is the single scattering operator for only the scattering layer $(j_{\text{up}} \leq j_{\text{up-dn}} \leq j_{\text{dn}}, \; j_{\text{up}} \leq j'_{\text{up-dn}} \leq j_{\text{dn}})$. Similar equations can be easily written for all the rest cases of mutual arrangement of the levels j, j', j_{up}, j_{dn}, that is, in practice it is sufficient to deal with only $T_{1,\text{up-dn}}(l,l',j_{\text{up-dn}},k,j'_{\text{up-dn}},k')$.

6. Assign $L_0(l, \tau, \eta) = 0$.

7. The next iteration of the scattering cycle number n (beginning from $n = 1$). Assign $B_{n,0}(l, j, k) = B_{n-1}(l, j, k)$, $j = 1, ..., J$, $k = -K, ..., -1$. Assign $L_{r,0}(l, \tau, \eta) = 0$ and $L_{s,0}(l, \tau, \eta) = 0$.

8. The next iteration of the reflection cycle number m (beginning from $m = 1$). Calculate by Eq. (7)

$$B_{r,m}(l,k) = -\sum_{k'=-K}^{-1}\eta(k')\Delta(k')\sum_{j'=1}^{J}T_0(j',J,k')d(j',1,J)\times$$

$$\times\sum_{l'=1}^{4}r_1(l,l',k,k')B_{n,m-1}(l',j',k'), \;\; k = 1, ..., K.$$

If $m > 1$ and $\eta > 0$, we calculate direct contribution of the reflected radiation $L'(l, \tau, \eta)$ as

$$L'(l,j,k) = B_{r,m}(l,k)\sum_{j'=j}^{J}T_0(j,j',k)d(j',J,j)$$

(if $m = 1$ or $\eta < 0$, then $L'(l, \tau, \eta) = 0$). In calculation of the Stokes vector, the indices j and k are hereinafter fixed (it is convenient if the values of τ and η are nodes of the integration grid, otherwise interpolation should be used). Sum up $L_{r,m}(l, \tau, \eta) = L_{r,m-1}(l,\tau,\eta) + L'(l,\tau,\eta)$. If all the components of nonzero $L'(l, \tau, \eta)$ are insignificant within the given accuracy, then $L_{s,m}(l, \tau, \eta) = L_{s,m-1}(l, \tau, \eta)$, the reflection cycle is terminated, and the algorithm passes on to the 10th operation, otherwise (and always if $\eta < 0$) calculate

$$B_{n,m}(l,j,k) = \sum_{k'=1}^{K}\Delta(k')\sum_{l'=1}^{4}B_{r,m}(l',k')\times$$

$$\times\sum_{j''=j}^{J}T_1(l,l',j,k,j',k')d(j',J,j), \; j = 1, ..., J, \; |k| = 1, ..., K.$$

9. Calculate the contribution to the Stokes vector

$$L'(l,j,k) = \sum_{j'=\min(j,J(k))}^{\max(j,J(k))}B_{n,m}(l,j',k)T_0(j,j',k)d(j',J(k),j).$$

Sum up $L_{s,m}(l, \tau, \eta) = L_{s,m-1}(l, \tau, \eta) + L'(l, \tau, \eta)$.

If all the components of $L'(l,\tau,\eta)$ are insignificant within the given accuracy, then the reflection cycle is terminated and the algorithm passes on to 10th operation, otherwise — to the 8th operation — the next iteration of the cycle.

10. Calculate

$$B_n(l,j,k) = \sum_{|k'|=1}^{K}\Delta(k')\sum_{j'=\min[j,J(k')]}^{\max[j,J(k')]}d[j',J(k'),j]\times$$

$$\times\sum_{l'=1}^{4}T_1(l,l',j,k,j',k')B_{n-1}(l',j',k'), \; j = 1, ..., J, \; |k| = 1, ..., K.$$

Calculate the contribution to the sought Stokes vector $L'(l, \tau, \eta)$:

$$L'(l,j,k) = \sum_{j'=\min[j,J(k)]}^{\max[j,J(k)]}B_n(l,j',k)T_0(j,j',k)d[j',J(k),j].$$

Sum up

$$L_n(l,\tau,\eta) = L_{n-1}(l,\tau,\eta) + L_{r,m}(l,\tau,\eta) + L_{s,m}(l,\tau,\eta) + L'(l,\tau,\eta).$$

If all the components of $L'(l, \tau, \eta)$ are insignificant within the given accuracy, then the scattering cycle is terminated and the algorithm passes on to the 11th operation, otherwise — to the 7th operation — the next iteration of the scattering cycle.

11. Calculate the sought Stokes vector

$$L(l,\tau,\eta,\varphi) = L_d(l,\tau,\eta,\varphi) + L_n(l,\tau,\eta).$$

Aerosol models in inverse problems in the IR and MW spectral regions

As was already mentioned, the account of radiation scattering by aerosol formations in the IR and MW spectral regions is of interest mostly for interpretation of field measurements, that is, for solution of inverse problems of the atmospheric optics, namely, reconstruction of surface and atmospheric parameters from the data of such measurements. Within the current aerosol optics, it is possible to propose rather complicated and well-developed aerosol models (in particular, cloud and precipitation models), allowing for the composition of particles, their shape, etc. However, the more complex is a model, the larger is the number of variables needed for its mathematical description.

In a combined approach to solution of inverse problems, all these parameters are to be reconstructed, and this deteriorates the accuracy of determination of each of them. A contradiction arises between the need of using aerosol models adequate to reality and, at the same time, describing them by as small as possible number of variables (this also applies to the surface models). The traditional way of resolving this contradiction is successive complication of a model: first the simplest model with the minimum number of parameters is selected, and then, whenever necessary, it is complicated as long as such complication makes sense, starting from the accuracy of particular measurements.

The simplest model of aerosol is the model of homogeneous spherical particles. If the same (mean, effective) chemical composition of particles is assumed, then this model is described using only the particle size-distribution function $C(r)$, where r is the particle radius, and the complex refractive index (CRI) $m(\nu)$ of the particulate matter. The distribution function can be specified analytically, that is, described by a small number of parameters. The CRI of the particulate matter is either assumed known for a particular model (for example, for clouds it is CRI of water or ice) or is to be reconstructed, and then its spectral dependence should be first presented as a function of some parameters. The algorithms for calculation of optical characteristics of this model of homogeneous spherical particles are now well known and described thoroughly (see, for example, Refs. 1, 4–6). Below we propose such "minimum" models for clouds, precipitation, and aerosol layers.

Cloud model

The distribution function for cloud particles is approximated rather accurately by the Khrgian–Mazin distribution.[7,8] For practical use, some elementary transformations can help representing this distribution through cloud water content W (Ref. 7) and the modal radius of the distribution function r_m as $C(r) = 2Wr^2 \exp(-2r/r_m)/(5\rho r_m^6)$, where ρ is the density of cloud particles (water or ice). In mixed clouds, the parameter d is introduced as a fraction (in number) of ice particles among all particles in a unit volume. Water droplet and crystal clouds can be considered as a particular case of mixed clouds at $d = 0$ and $d = 1$, respectively. There is a model[9] of the dependence of d on the air temperature: $d = 1 - 1.058[1 - \exp(-x^2)]$, where $x = (T - 232)/24.04$; if $T < 232$, then $d = 1$; if $T > 273$, then $d = 0$. The optical characteristics are calculated separately for the water-droplet and ice fractions, and then they are summed with the weights of $(1 - d)$ and d, respectively. The modal radii of the fractions can be thought different, but, to minimize the number of parameters, they can also be assumed the same, then the density of cloud particles is determined as a weighting function $\rho = (1 - d)\rho_w + d\rho_i$, where ρ_w and ρ_i are the water and ice density, respectively. The values of CRI for water and ice are now well known. Note that for the MW region some models of water and ice CRI dependence on the frequency and temperature are proposed.[10,11] The characteristic ranges of model parameters for clouds of different type can be found in Refs. 7 and 8.

Precipitation model

For precipitation particles, the Marshall–Palmer distribution[8] $C(r) = 0.08 \exp\left(-82J^{-0.21}r\right)$, where J is the precipitation intensity, in mm/h and r is the particle radius, in cm, gives a good agreement with the experiment. Thus, in the simplest model, precipitation is described by a single parameter J. Model variations of this parameter depending on the type of precipitation are presented in Refs. 7 and 8.

Model of aerosol layers

For non-water aerosols, the single-parameter distribution function can hardly be a good approximation (the Junge distribution does not help in this case, because it requires specification of the minimum radius, which is, in fact, the second parameter). Take the most "standard" lognormal distribution, which lies in the foundation of many aerosol models, see, for example, Refs. 12 and 13, and write it through the modal radius

$$C(r) = \frac{C}{rs\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\ln\left(\frac{r}{r_m}\right)/s - s\right)^2\right].$$

Here C is the number density of aerosol particles; s is the distribution parameter. The simplest model of $m(\nu)$ can be the piecewise linear approximation on a given grid of ν values, but if the range of the measuring device is relatively narrow and CRI of aerosol particles has no singular points in this range, then the model CRI can merely be taken constant.

Approximation of small particles

If the size of aerosol particles is much smaller than the radiation wavelength, then the approximation of small particles is true.[1] This approximation can likely be used in the IR region for models of not very large aerosol particles and in the MW region for clouds and precipitation (it is violated only for very large precipitation particles). In this approximation, the scattering phase matrix is determined by Eq. (8). For the extinction and scattering factors, we have explicit analytical equations,[1] using which we can also easily derive the equations for the volume extinction and absorption coefficients of aerosol:

$$\alpha(\lambda) = \lambda^{-1}A_1 M_3[C(r)] + \lambda^{-3}A_3 M_5[C(r)] + \lambda^{-4}A_4 M_6[C(r)],$$

$$\sigma(\lambda) = \lambda^{-4}B_4 M_6[C(r)], \qquad (9)$$

where λ is the radiation wavelength; $M_n[C(r)] = \int_0^\infty C(r)r^n \mathrm{d}r$ are the moments of the distribution function.

The coefficients A_1, A_3, A_4, B_4 depend only on CRI of the aerosol matter m and are equal to[1]

$$A_1 = -8\pi^2 \,\mathrm{Im}\left(\frac{m^2 - 1}{m^2 + 2}\right),$$

$$A_3 = -\frac{32}{15}\pi^4 \,\mathrm{Im}\left[\left(\frac{m^2 - 1}{m^2 + 2}\right)^2 \frac{m^4 + 27m^2 + 38}{2m^2 + 3}\right],$$

$$A_4 = \frac{128}{3}\pi^5 \operatorname{Re}\left[\left(\frac{m^2 - 1}{m^2 + 2}\right)^2\right], \quad B_4 = \frac{128}{3}\pi^5 \left|\frac{m^2 - 1}{m^2 + 2}\right|^2.$$

Since the moments of all the above model distribution functions are expressed explicitly through their parameters, after substitution of them into Eq. (9) we get the equations expressing the optical characteristics of clouds, precipitation, and aerosol through the model parameters. For brevity, we omit these equations that can be easily derived, but note that in the MW region for the cloud model the first term in $\alpha(\nu)$ takes the following form:

$$\alpha(\nu, T) = 18\pi \frac{W}{\rho} \frac{\nu}{c} \frac{k_2(\nu, T)}{[n_2(\nu, T) + 2]^2 + k_2^2(\nu, T)}, \quad (10)$$

where $m^2(\nu, T) = n_2(\nu, T) - i k_2(\nu, T)$. If all other terms, including $\sigma(\nu)$ are neglected, that is, only direct radiation without the scattered one is considered, then for calculation by Eq. (10) it is sufficient to specify only the cloud water content W, which is the single model parameter. This approximation is used in the algorithm in Ref. 14.

Conclusion

The method and algorithm described allow solution of various problems on simulating measurements of the intensity (Stokes parameters) in the IR and MW regions. Besides, they allow us to study easily different simplifications and approximations and to estimate their accuracy. In fact, the explicit equations for the sought parameters permit taking derivatives of them with respect to the surface and atmospheric parameters, what is necessary both for analyzing the information content of measurements and for their interpretation in solving inverse problems. The described "minimum" models of clouds, precipitation, and aerosol layers are also intended for use in solving inverse problems

taking into account the interaction of radiation with the objects mentioned above.

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