FORMATION OF HOLOGRAPHIC SHEAR INTERFEROGRAMS PRODUCED BY DIFFUSELY SCATTERED LIGHT FIELDS FOR CONTROL OF AN OPTICAL TELESCOPE

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A shear interferometer is analyzed based on a double–exposure hologram formed with the help of a Galilean telescope. It is shown theoretically and experimentally that the spatial filtering in the hologram plane enables checking the telescope over the field. Spatial filtering in the image plane of a mat screen makes it possible to record the interference pattern characterizing the phase distortions introduced in the reference wave by the aberrations of the optical system forming it.

The method of differential interferometry using diffusely scattered light fields for checking quasispherical wavefront based on double–exposure recording of the Fourier lensless hologram was implemented in Refs. 1 and 2. In its turn, a double–exposure recording of the Fresnel hologram[3,4] can be implemented for checking the quasiplanar wavefront formed with a telescope. In the both above–indicated cases the holograms were recorded when the objective speckle fields were superimposed in the plane of the medium, in which the hologram was recorded. As shown in Ref. 5, a double–exposure recording of the image of a mat screen when this image was focused with the help of the Kepler telescope and the subjective speckle fields in the hologram plane were superimposed results in the formation of shear interferograms in fringes of infinite width for checking the telescope over the field.

In this paper a method of the hologram recording by means of superimposition of the subjective speckle fields of two exposures for checking the Galilean telescope over the field is analyzed.

As follows from Fig. 1, the mat screen *1* is illuminated with a quasiplanar wave, and a coherent diffusely scattered light is transmitted through the collimating system of lenses L_1 (objective) and L_2 (eyepiece). The hologram is recorded during the first exposure on the photographic plate *2* by a quasiplanar reference wave *3*. Prior to the second exposure, the mat screen is displaced along the x axis at a distance a and the photographic plate is displaced along the same direction at a distance b.

Ignoring the constant coefficients, we may represent in Fresnel's approximation the distribution of the complex amplitude of the field produced during the first exposure in the plane (x_4, y_4) of the photographic plate in the form

$$u_1(x_4, y_4) \sim \iiint\!\!\!\iiint_{-\infty}^{\infty} t(x_1, y_1)\exp iz_0(x_1, y_1)\exp\left\{ik[(x_1-x_2)^2+(y_1-y_2)^2]/2l_1\right\} p_1(x_2, y_2)\times$$
$$\times \exp iz_1(x_2, y_2)\exp[-ik(x_2^2+y_2^2)/2f_1]\exp\left\{ik[(x_2-x_3)^2+(y_2-y_3)^2]/2\Delta\right\} p_2(x_3, y_3)\exp i\varphi_2(x_3, y_3)\times$$
$$\times \exp[ik(x_3^2+y_3^2)/2f_2]\exp\left\{ik[(x_3-x_4)^2+(y_3-y_4)^2]/2l_2\right\}dx_1dy_1dx_2dy_2dx_3dy_3, \quad (1)$$

where k is the wave number, $t(x_1, y_1)$ is the complex transmission amplitude of the mat screen and a random function of the coordinates, $\varphi_0(x_1, y_1)$ is the deterministic phase function characterizing the front distortions introduced in the illuminating wave by the wave aberrations of the optical system forming it, $p_1(x_2, y_2)\exp i\varphi_1(x_2, y_2)$ is the generalized pupil function of the objective[6] with the focal length f_1 which takes into account the wave axial aberrations, $p_2(x_3, y_3)\exp i\varphi_2(x_3, y_3)$ is correspondingly the generalized pupil function of the eyepiece with the focal length f_2 of the Galilean telescope with optical baseline $\Delta = f_1 - f_2$, l_1 is the distance between the mat screen and the principal plane of the objective, and l_2 is the distance between the principal plane of the eyepiece and the plane of the photographic plate.

Expression (1) can be represented in the form of convolution of the functions

$$u_1(x_4, y_4) \sim \exp[ik(x_4^2+y_4^2)/2l_2]\left\{\exp[-ik(x_4^2+y_4^2)M/2l_2^2]\left\{\exp[-ik(x_4^2+y_4^2)NM^2/2l_2^2D^2]\iint_{-\infty}^{\infty} t(x_1, y_1)\exp i\varphi_0(x_1, y_1)\times\right.\right.$$
$$\left.\left.\times\exp\left[ik(x_1^2+y_1^2)(l_1-N)/2l_1^2\right]\exp[-ik(x_1x_4+y_1y_4)NM/l_1l_2\Delta]dx_1dy_1 \otimes P_1(x_4, y_4)\right\}\otimes P_2(x_4, y_4)\right\}, \quad (2)$$

where $\frac{1}{M} = \frac{1}{\Delta} + \frac{1}{f_2} + \frac{1}{l_2}$, $\frac{1}{N} = \frac{1}{l_1} - \frac{1}{f_1} + \frac{1}{\Delta} - \frac{M}{\Delta^2}$, $\otimes$ denotes the convolution operation, and

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$$P_1(x_4, y_4) = \iint\limits_{-\infty}^{\infty} p_1(x_2, y_2)\exp i\varphi_1(x_2, y_2)\exp[-ik(x_2x_4 + y_2y_4)M/l_2\Delta]\mathrm{d}x_2\mathrm{d}y_2\ ;$$

$$P_2(x_4, y_4) = \iint\limits_{-\infty}^{\infty} p_2(x_3, y_3)\exp i\varphi_2(x_3, y_3)\exp[-ik(x_3x_4 + y_3y_4)/l_2]\mathrm{d}x_3\mathrm{d}y_3$$

are the Fourier transforms of the generalized pupil functions of the objective and eyepiece, respectively.

Since, following Goodman,[7] the width of the function $P_1(x_4, y_4)$ is equal to $\lambda l_2\Delta/Md_1$, where λ is the wavelength of the coherent light source used for recording and reconstructing the hologram and d_1 is the diameter of the objective pupil, the phase of the spherical wave with curvature radius $l_2^2\mathrm{D}^2/NM^2$ within the domain of existence of the function $P_1(x_4, y_4)$ does not exceed π within the region of the plane of the photographic plate whose diameter $D_1 \le d_1l_2D/NM$. Now, in expression (2) let us take the multiplier $\exp[-ik(x_4^2 + y_4^2)NM^2/2l_2^2\Delta^2]$, characterizing the distribution of the spherical wave phase, outside the integral of convolution with the function $P_1(x_4, y_4)$. In an analogous way, if within the domain of existence of the function $P_2(x_4, y_4)$ determined by the value $\lambda l_2/d_2$, where d_2 is the diameter of the eyepiece pupil, the phase change of the spherical wave with curvature radius $l_2^2\Delta^2/(M\Delta^2 + NM^2)$ does not exceed π, that will be satisfied in the plane of the photographic plate within the region whose diameter $D_2 \le d_2l_2\Delta^2/(M\Delta^2 + NM^2)$, then we may take the exponent $\exp[-ik(x_4^2 + y_4^2)(M\Delta^2 + NM^2)/2l_2^2\Delta^2]$ outside the integral with the function $P_2(x_4, y_4)$. If the diameters of the illuminated spots of the objective and eyepiece are determined by the path of the aperture ray ($d_1 = d_2f_1/f_2$) (see Ref. 8), then within the region of the photographic plate whose diameter $D_2 \le d_2[f_1^2(l_2 + f_2) + f_2^2(l_1 - f_1)]/f_2(f_1^2 - f_1f_2 + l_1f_2)$ the distribution of the field amplitude in the plane of the photographic plate has the form

$$u_1(x_4, y_4) \sim \exp[ik(x_4^2 + y_4^2)(l_2\Delta^2 - M\Delta^2 - NM^2)/2l_2^2\Delta^2]\{F[kx_4NM/l_1l_2\Delta,\ ky_4NM/l_1l_2\Delta]\otimes P_1(x_4, y_4)\otimes P_2(x_4, y_4)\}\ , \tag{3}$$

where

$$F[kx_4NM/l_1l_2\Delta,\ ky_4NM/l_1l_2\Delta] = \iint\limits_{-\infty}^{\infty} t(x_1, y_1)\exp i\varphi_0(x_1, y_1)\exp[ik(x_1^2 + y_1^2)(l_1 - N)/2l_1^2]\exp[-ik(x_1x_4 + y_1y_4)NM/l_1l_2\Delta]\mathrm{d}x_1\mathrm{d}y_1$$

is the Fourier transform of the function $t(x_1, y_1)\exp i\varphi_0(x_1, y_1)\exp[ik(x_1^2 + y_1^2)(l_1 - N)/2l_1^2]$.

We will write the of distribution of the objective field complex amplitude recorded during the second exposure, in the plane (x_4, y_4) in the form

$$u_2(x_4, y_4) \sim \iiiint\!\!\iint\limits_{-\infty}^{\infty} t(x_1 + a, y_1)\exp i\varphi_0(x_1, y_1)\exp\{ik[(x_1 - x_2)^2 + (y_1 - y_2)^2]/2l_1\}p_1(x_2, y_2)\ \times$$
$$\times \exp i\varphi_1(x_2, y_2)\exp[-ik(x_2^2 + y_2^2)/2f_1]\exp\{ik[(x_2 - x_3)^2 + (y_2 - y_3)^2]/2\Delta\}p_2(x_3, y_3)\exp i\varphi_2(x_3, y_3)\ \times$$
$$\times \exp[ik(x_3^2 + y_3^2)/2f_2]\exp\{ik[(x_3 - x_4 + b)^2 + (y_3 - y_4)^2]/2l_2\}\mathrm{d}x_1\mathrm{d}y_1\mathrm{d}x_2\mathrm{d}y_2\mathrm{d}x_3\mathrm{d}y_3\ , \tag{4}$$

Based on the well–known properties of the Fourier transforms we can derive the following expression for the amplitude distribution in the plane of the photographic plate:

$$u_2(x_4, y_4) \sim \exp[ik(x_4^2 + y_4^2)(l_2\Delta^2 - M\Delta^2 - NM^2)/2l_2^2\Delta^2]\exp[-ikx_4b(l_2\Delta^2 - M\Delta^2 - NM^2)/l_2^2\Delta^2]\ \times$$
$$\times \{[\exp(ikNMax_4/l_1l_2\Delta)F[kx_4NM/l_1l_2\Delta,\ ky_4NM/l_1l_2\Delta]\otimes\Phi_1(x_4, y_4)]\otimes P_1(x_4, y_4)\otimes P_2(x_4, y_4)\}\ , \tag{5}$$

where

$$\Phi_1(x_4, y_4) = \iint\limits_{-\infty}^{\infty}\exp i[\varphi_6(x_1, y_1) - \varphi_0(x_1 + a, y_1)]\exp[-ik(x_1x_4 + y_1y_4)NM/l_1l_2\Delta]\mathrm{d}x_1\mathrm{d}y_1$$

is the Fourier transform of the function $\exp i[\varphi_0(x_1, y_1) - \varphi_0(x_1 + a, y_1)]$.

If the distances of the displacement of the mat screen and photographic plate prior to the second exposure satisfy the condition $a = bf_1/f_2$, then making use of the identity

$$\exp(-ikNMax_4/l_1l_2\Delta)\{[\exp(ikNMax_4/l_1l_2\Delta)F[kx_4NM/l_1l_2\Delta,\ ky_4NM/l_1l_2\Delta]\otimes\Phi_1(x_4, y_4)\otimes P_1(x_4, y_4)\otimes P_2(x_4, y_4)\} =$$
$$= F[kx_4NM/l_1l_2\Delta,\ ky_4NM/l_1l_2\Delta]\otimes\Phi_1(x_4, y_4)\otimes\exp(-ikNMax_4/l_1l_2\Delta)[P_1(x_4, y_4)\otimes P_2(x_4, y_4)]$$

which is proved by means of representation of the convolution in an integral form and by substitution of the corresponding values of the Fourier transforms, the complex amplitude of the field, recorded during the second exposure in the plane of the photographic plate takes the form

$$u_2(x_4, y_4) \sim \exp\left[ik\left(x_4^2 + y_4^2\right)\left(l_2\Delta^2 - M\Delta^2 - NM^2\right)/2l_2^2D^2\right] \left\{F\left[kx_4NM/l_1l_2\Delta,\ ky_4NM/l_1l_2\Delta\right]\otimes\right.$$

$$\left.\otimes\Phi_1(x_4, y_4) \otimes \exp(-\ ikNMax_4/l_1l_2\Delta) \left[P_1(x_4, y_4) \otimes P_2(x_4, y_4)\right]\right\} . \qquad (6)$$

Let a double–exposure hologram thus recorded be reproduced by a copy of the reference wave, for which the distribution of the field in the plane (x_4, y_4) corresponds, e.g. , to the first exposure recording. In this case the diffraction field in the hologram plane is given by the expression

$$u(x_4, y_4) \sim \exp\left[ik\left(x_4^2+y_4^2\right)\left(l_2\Delta^2-M\Delta^2-NM^2\right)/2l_2^2\Delta^2\right]\left\{F\left[kx_4NM/l_1l_2\Delta,\ ky_4NM/l_1l_2\Delta\right]\otimes P_1(x_4, y_4)\otimes P_2(x_4, y_4) + \exp i\left[\varphi_3(x_4, y_4) -\right.\right.$$

$$\left.\left. -\ \varphi_3(x_4 + b, y_4)\right] \left\{F\left[kx_4NM/l_1l_2\Delta,\ ky_4NM/l_1l_2\Delta\right]\otimes\varphi_1(x_4, y_4)\otimes\exp\left(-\ ikNMax_4/l_1l_2\Delta\right)\left[P_1(x_4, y_4)\otimes P_2(x_4, y_4)\right]\right\}\right\} , \qquad (7)$$

where $\varphi_3(x_4, y_4)$ is the deterministic phase function characterizing the distortions introduced in the reference wavefront by the aberrations of the optical system forming it.

As follows from expression (7), in the hologram plane the speckle fields of two exposures with a typical size of the subjective speckle, being determined by the width of the function $P_1(x_4, y_4)\otimes P_2(x_4, y_4)$, are superimposed, in addition, the exponent $\exp(-ikNMax_4/l_1l_2\Delta)$ characterizes the relative angle $\alpha = NMa/l_1l_2\Delta$ of the tilt between the two speckle fields. As a result, the interference pattern characterizing the distortions introduced in the reference wavefront by the wave aberrations of the optical system forming it, is localized in the hologram plane. If the opaque screen P_3 (Fig. 1) with a circular aperture centered on the optical axis is positioned in the plane in the plane (x_4, y_4) and the condition $\varphi_3(x_4, y_4) - \varphi_3(x_4 + b, y_4) \le \pi$ is satisfied within the diameter of the aperture, i.e., the diameter of the aperture does not exceed the width of the interference band in the interference pattern localized in the hologram plane, then the diffraction field at the exit from the filtering diaphragm is given by the expression

$$u(x_4, y_4) \sim p_3(x_4, y_4) \exp[ik(x_4^2 + y_4^2)(l_2\Delta^2 - M\Delta^2 - NM^2)/2l_2^2\Delta^2] \left\{F[kx_4NM/l_1l_2\Delta,\ ky_4NM/l_1l_2\Delta]\otimes\right.$$

$$\left.\otimes[1 + \Phi_1(x_4, y_4)\otimes\exp\ (-\ ikNMax_4/l_1l_2\Delta)[P_1(x_4, y_4)\otimes P_2(x_4, y_4)]\right\} , \qquad (8)$$

where $p_3(x_4, y_4)$ is the transmission function of the screen with circular aperture.[9]

Let the positive lens L_3 with focal length f_3, for which the opaque screen P_3 with circular aperture is an aperture diaphragm, be placed in the hologram plane (Fig. 1). In this case we may write the diffraction field at the distance l_3 from the diaphragm in the form

$$u(x_5, y_5) \sim \int\limits_{-\infty}^{\infty}\int u(x_4, y_4)\exp[-\ ik(x_4^2 + y_4^2)/2f_3] \exp\left\{ik[(x_4 - x_5)^2 + (y_4 - y_5)^2]2l_3\right\}\mathrm{d}x_4\mathrm{d}y_4 . \qquad (9)$$

If the image of the mat screen is formed in the plane (x_5, y_5) with the lens L_3 and $1/f_3 = 1/l_3 + (l_2\Delta^2 - M\Delta^2 - NM^2)/l_2^2\Delta^2$, then as a result of substituting expression (8) into expression (9) we obtain

$$u(x_5, y_5) \sim \exp\left[ik(x_5^2 + y_5^2)/2l_3\right]\left\{\left\{t(-\ \mu_1x_5, -\ \mu_1y_5) \exp i\varphi_0(-\ \mu_1x_5, -\ \mu_1y_5)\exp\left[ik(x_5^2 + y_5^2)\ \mu_1^2(l_1 - N)/2l_1^2\right] \times\right.\right.$$

$$\times\ p_1(-\ \mu_2 x_5, -\ \mu_2 y_5)p_2(-\ \mu_3 x_5, -\ \mu_3 y_5) \exp i\left[\varphi_1(-\ \mu_2 x_5, -\ \mu_2 y_5) + \varphi_2(-\ \mu_3 x_5, -\ \mu_3 y_5)\right] + t(-\ \mu_1 x_5, -\ \mu_1 y_5) \times$$

$$\times\ \exp i[2\varphi_0(-\ \mu_1x_5, -\ \mu_1y_5) - \varphi_0(-\ \mu_1x_5 + a, -\ \mu_1y_5)]p_1(-\ \mu_2 x_5 - aN/l_1, -\ \mu_2 y_5)\ p_2(-\ \mu_3 x_5 - aNM/l_1\Delta, -\ \mu_3 y_5) \times$$

$$\left.\left.\times\ \exp i[\varphi_1(-\ \mu_2 x_5 - aN/l_1, -\ \mu_2 y_5) + \varphi_2(-\ \mu_3 x_5 - aNM/l_1\Delta, -\ \mu_3 y_5)]\right\}\otimes P_3(x_5, y_5)\right\} , \qquad (10)$$

where $\mu_1 = l_1l_2\Delta/NMl_3$, $\mu_2 = l_2\Delta/Ml_3$, and $\mu_3 = l_2/l_3$ are the scale factors of image transformation and

$$P_3(x_5, y_5) = \int\limits_{-\infty}^{\infty}\int p_3(x_4, y_4) \exp[-\ ik(x_4 x_5 + y_4 y_5)/l_3]\mathrm{d}x_4\mathrm{d}y_4$$

is the Fourier transform of the transmission function of the screen with circular aperture.

As follows from expression (10), if the condition $(D_0l_1/N) > d_1$, where D_0 is the diameter of the illuminated spot of the mat screen during the double–exposure record of the hologram is satisfied, then within the region of overlap of images of the eyepiece pupils the identical speckles of the two exposures are superimposed. It then follows that the interference pattern is localized in the plane (x_5, y_5). Indeed, if in expression (10) the period of variation of the function

$$\exp i[\varphi_0(-\mu_1 x_5, -\mu_1 y_5) + \varphi_1(-\mu_2 x_5, -\mu_2 y_5) + \varphi_2(-\mu_3 x_5, -\mu_3 y_5)] + \exp i[2\varphi_0(-\mu_1 x_5, -\mu_1 y_5) -$$

$$- \varphi_0(-\mu_1 x_5 + a, -\mu_1 y_5) + \varphi_1(-\mu_2 x_5, -aN/l_1, -\mu_2 y_5) + \varphi_2(-\mu_3 x_5, -aNM/l_1\Delta, -\mu_3 y_5)]$$

exceeds the size of the speckle in the plane (x_5, y_5), determined by the width of the function $P_3(x_5, y_5)$, even by an order of magnitude,[10] we may take this function in expression (10) outside the convolution integral. In this case a superposition of the correlating speckle fields being filtered results in the following illuminance distribution:

$$I(x_5, y_5) \sim \{1 + \cos[\varphi_0(-\mu_1 x_5 + a, -\mu_1 y_5) - \varphi_0(-\mu_1 x_5, -\mu_1 y_5) + \varphi_1(-\mu_2 x_5, -\mu_2 y_5) - \varphi_1(-\mu_2 x_5 - aN/l_1, -\mu_2 y_5) +$$

$$+ \varphi_2(-\mu_3 x_5, -\mu_3 y_5) - \varphi_2(-\mu_3 x_5, -aNM/l_1\Delta -\mu_3 y_5)\} \left| t(-\mu_1 x_5, -\mu_1 y_5) \exp[ik(x_5^2 + y_5^2)\mu_1^2(l_1 - N)/2l_1^2] \otimes P_3(x_5, y_5) \right|^2, \quad (11)$$

which describes the speckle structure modulated by the interference bands. The interference pattern is the shear interferogram in fringes of infinite width. The interferogram characterizes axial wave aberrations of the Galilean telescope and phase distortions introduced in the illuminating wavefront by the wave aberrations of the optical system forming it. If within the region of the image of the eyepiece pupil $\varphi_0(-\mu_1 x_5 + a, -\mu_1 y_5) - \varphi_0(-\mu_1 x_5, -\mu_1 y_5) < \pi$, the shear interferogram will characterize the wave aberrations of the telescope.

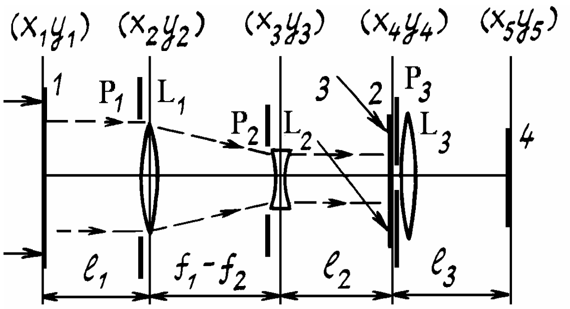

FIG. 1. The optical scheme used for recording and reconstructing a double–exposure hologram: 1) mat screen, 2) photographic plate–hologram, 3) reference beam, and 4) recording plane of the interference pattern; L_1, L_2, and L_3 are lenses; P_1, P_2, and P_3 are aperture diaphragms.

Based on expression (7), we can conclude that the information about the phase distortions introduced in the light wave by the objective and the eyepiece is contained in an individual speckle in the hologram plane. At the same time, in a small region of the hologram on the optical axis the field distribution deremined by the convolution $P_1(x_4, y_4) \otimes P_2(x_4, y_4)$ within every individual speckle is a result of diffraction of the plane wave, propagating along the optical axis, by the pupils of the objective and the eyepiece of the telescope.

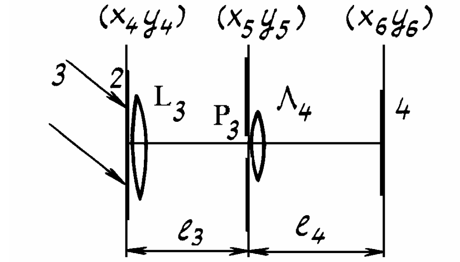

FIG. 2. Schematic diagram of recording of the interference pattern, localized in the hologram plane, with spatial filtering in the image plane of the mat screen.

Hence it follows that the aperture diaphragm in the hologram plane in Fig. 1 enables us to record independently the narrow range of spatial frequencies in the spatial spectrum of the waves scattered by the mat screen near the optical axis. The displacement of the aperture diaphragm along the x axis results in the formation of the shear interferogram characterizing a combination of the on–axis and off–axis wave aberrations introduced in the light wave by the objective and eyepiece of the telescope, since in this case the aperture diaphragm enables us to record independently the narrow range of the spatial frequencies near the spatial frequency $x_{40}(l_2\Delta^2 - M\Delta^2 - NM^2)/\lambda\Delta^2$, where x_{40} is the coordinate of the center of aperture diaphragm in the hologram plane.

In order to record the interference pattern localized in the hologram plane let us consider spatial filtering of the light field reconstructed by a double–exposure hologram in the image plane (x_5, y_5) of the mat screen in accordance with Fig. 2. Assuming that the diameter of the lens L_3 exceeds the diameter of the reference beam, based on expression (7), we may write the amplitude distribution of the diffraction field in the form

$$\sim \exp\left[ik\left(x_5^2 + y_5^2\right)/2l_3\right]\left\{t(-\mu_1 x_5, -\mu_1 y_5)\exp i\varphi_0(-\mu_1 x_5, -\mu_1 y_5)\exp\left[ik\left(x_5^2 + y_5^2\right)\mu_1^2(l_1 - N)/2l_1^2\right] \times\right.$$

$$\times p_1(-\mu_2 x_5, -\mu_2 y_5) p_2(-\mu_3 x_5, -\mu_3 y_5)\exp i\left[\varphi_1(-\mu_2 x_5, -\mu_2 y_5) + \varphi_2(-\mu_3 x_5, -\mu_3 y_5)\right] + \Phi_2(x_5, y_5) \otimes$$

$$\otimes t(-\mu_1 x_5, -\mu_1 y_5)\exp i\left[2\varphi_0(-\mu_1 x_5, -\mu_1 y_5) - \varphi_0(-\mu_1 x_5 + a, -\mu_1 y_5)\right] p_1(-\mu_2 x_5 - aN/l_1, -\mu_2 y_5)$$

$$\left.\times p_2(-\mu_3 x_5 - aNM/l_1\Delta, -\mu_3 y_5)\exp i\left[\varphi_1(-\mu_2 x_5 - aN/l_1, -\mu_2 y_5) + \varphi_2(-\mu_3 x_5 - aNM/l_1\Delta, -\mu_3 y_5)\right]\right\}, \quad (12)$$

where

$$\Phi_2(x_5, y_5) = \iint_{-\infty}^{\infty} \exp i[\varphi_3(x_4, y_4) - \varphi_3(x_4 + b, y_4)] \exp[-ik(x_4 x_5 + y_4 y_5)/l_3] dx_4 dy_4$$

is the Fourier transform of the function $\exp i[\varphi_3(x_4, y_4) - \varphi_3(x_4 + b, y_4)]$.

If within the diameter of the aperture diaphragm p_3 (Fig. 2) centered on the optical axis the condition

$$\varphi_0(-\mu_1 x_5 + a, -\mu_1 y_5) - \varphi_0(-\mu_1 x_5, -\mu_1 y_5) + \varphi_1(-\mu_2 x_5, -\mu_2 y_5) -$$
$$-\varphi_1(-\mu_2 x_5 - aN/l_1, -\mu_2 y_5) + \varphi_2(-\mu_3 x_5, -\mu_3 y_5) - \varphi_2(-\mu_3 x_5, -aNM/l_1\Delta - \mu_3 y_5) \le \pi ,$$

is satisfied, i.e., the diameter of the filtering aperture does not exceed the width of the interference band in the interference pattern localized in the image plane of the mat screen, the distribution of the field amplitude at its exit is determined by the expression

$$u(x_5, y_5) \sim p_3(x_5, y_5)\exp\left[ik\left(x_5^2 + y_5^2\right)/2l_3\right]\left\{t(-\mu_1 x_5, -\mu_1 y_5)\exp[ik(x_5^2 + y_5^2)\mu_1^2(l_1 - N)/2l_1^2]\otimes[1 + \Phi_1(x_5, y_5)]\right\} . \quad (13)$$

Let the positive lens L_4 with focal length f_4 reform the light field at the distance l_4 in the plane (x_6, y_6) so as we may write the amplitude distribution in Fresnel's approximation in the form

$$u(x_6, y_6) \sim \int\int_{-\infty}^{\infty} u(x_5, y_5)\exp[-ik(x_5^2 + y_5^2)/2f_4]\exp\left\{ik[(x_5 - x_6)^2 + (y_5 - y_6)^2]/2l_4\right\}dx_5\,dy_5 . \quad (14)$$

If in the plane (x_6, y_6) an image of the hologram is constructed and $1/f_3 = 1/l_3 + 1/l_4$, then after substituting expression (13) into expression (14) we obtain

$$u(x_6, y_6) \sim \exp\left[-ik\left(x_6^2 + y_6^2\right)/2l_4\right]\left\{F[kx_6/l_4, ky_6/l_4]\left\{1 + \exp i[\varphi_3(-\mu_4 x_6, -\mu_4 y_6) - \varphi_3(-\mu_4 x_6 + b, -\mu_4 y_6)]\right\}\otimes P_3(x_6, y_6)\right\}, \quad (15)$$

where $\mu_4 = l_3/l_4$ is the scale factor of image transformation,

$$F[kx_6/l_4, ky_6/l_4] = \int\int_{-\infty}^{\infty} t(-\mu_1 x_5, -\mu_1 y_5)\exp\left[ik\left(x_5^2 + y_5^2\right)\mu_1^2(l_1 - N)/2l_1^2\right]\exp[-ik(x_5 x_6 + y_5 y_6)/l_4]dx_5 dy_5 ;$$

and

$$P_3(x_6, y_6)\} = \int\int_{-\infty}^{\infty} p_3(x_5, y_5)\exp\left[-ik(x_5 x_6 + y_5 y_6)/l_4\right]dx_5 dy_5$$

are the Fourier transforms of the corresponding functions.

It follows from expression (15) that in the plane (x_6, y_6) the identical speckles of the two exposures are superimposed. If the period of variation of the function $1 + \exp i\left[\varphi_3(-\mu_4 x_6, -\mu_4 y_6) - \varphi_3(-\mu_4 x_6 + b, -\mu_4 y_6)\right]$ exceeds the speckle size determined by the width of the function $P_3(x_6, y_6)$, then we may take this function in expression (8) outside the integral convolution. In this case the superposition of the correlating speckle fields results in forming the illuminance distribution

$$I(x_6, y_6) \sim \left\{1 + \cos\left[\varphi_3(-\mu_4 x_6, -\mu_4 y_6) - \varphi_3(-\mu_4 x_6 + b, -\mu_4 y_6)\right]\right\}\left|F\left[kx_6/l_4, ky_6/l_4\right]\otimes P_3(x_6, y_6)\right|^2 , \quad (16)$$

which describes the speckle structure modulated by the interference bands. Here the interference shear pattern characterizes the distortions introduced in the reference wavefront by the wave aberrations of the optical system forming it.

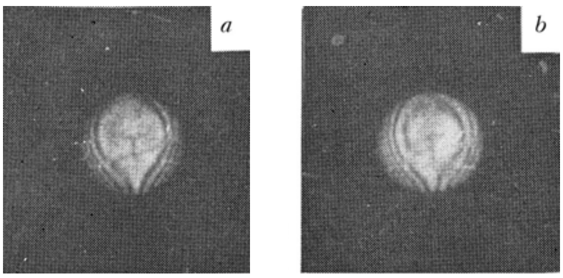

FIG. 3. Shear interferograms localized in the image plane of the mat screen and recorded when performing on–axis (a) and off–axis (b) spatial filtering.

In the experiment, the double–exposure holograms were recorded on Mikrat–VRL photographic plates using a He–Ne laser at a wavelength of 0.63 mm. At the preliminary stage, a double–exposure Fresnel hologram was recorded using the method proposed in Refs. 3 and 4 for checking the quasiplanar illuminating wavefront. The investigations have shown that the beam 27 mm in diameter formed by the chosen collimating system of lenses, was subject to the phase distortions of the wavefront which satisfied the condition $[\partial\varphi_0(x_1, y_1)/\partial x_1]a \le \pi$ for the quantity a not exceed 2 mm. Then, in accordance with Fig. 1, a double–exposure hologram was recorded with the help of the Galilean telescope with positive lens with the 140 mm focal length (the pupil diameter was 22 mm) and with the negative lens with the 70 mm focal length (the pupil diameter was 11 mm). The distances l_1 and l_2 were 60 and 230 mm, respectively. The diameter of the reference beam was 40 mm. Figure 3*a* shows the shear interferogram recorded when performing a spatial filtering on the optical axis in the hologram plane and reconstructed by a small–aperture laser beam ~2 mm in diameter.

The interference pattern is localized in the image plane of the mat screen. This is indicated by a mark "*T*", which was drawn on the mat screen and characterized the spherical aberration of the telescope with postfocal defocusing. The distances at which the mat screen, and photographic plate were displaced prior to the second exposure were $a = 1.5 \pm 0.002$ mm and $b = 0.75 \pm 0.002$ mm, respectively. The displacement of the hologram relative to the laser beam used for reconstruction of this hologram toward the displacement prior to the second exposure leads to independent recording of the interference pattern. The latter is shown in Fig. 3*b* for the case in which $x_{40}= 4.5$ mm and characterizes a combination of the on–axis and off–axis wave aberrations (Fig. 3*a*).

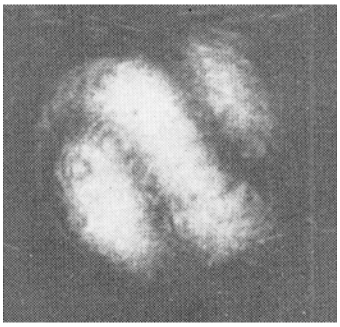

FIG. 4. Shear interferogram localized in the hologram plane.

The reconstruction of the double–exposure hologram in accordance with Fig. 2 when performing the spatial filtering of the reconstructed field on the optical axis in the image plane of the mat screen results in recording of the interference pattern localized in the photographic plate–hologram, which is shown in Fig. 4. The interference pattern characterizes the phase distortions introduced in the reference wavefront by the aberrations of the optical system forming it.

Figure 5 shows a shear interferogram recorded with the spatial filtering on the optical axis in the hologram plane based on the double exposure recording with the help of theater binoculars at 2.5X magnification ($a = 2 \pm 0.002$ mm and $b = 0.8 \pm 0.002$ mm). The interference pattern characterizes the predominant wave aberration of the coma type, which is explained by decentering of the movable correction lens, whose displacement along the system axis is used for adjustment of the focal lengths.

Thus, the recording of the double exposure hologram when transmitting the diffusely scattered light through the Galilean telescope and when the subjective speckle fields are superimposed in its plane results in the formation of the interference shear patterns. In this case, in the image plane of the mat screen an interference pattern is generally localized which takes into account the wave aberrations of the telescope and of the optical system forming the wavefront of the illuminating beam. The interference pattern characterizing the wave aberrations of the reference beam is localized in the hologram plane. Recording the interference patterns is possible when performing spatial filtering in appropriate planes.

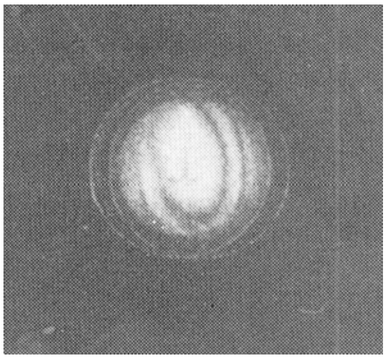

FIG. 5. Shear interferogram characterizing the wave aberrations of binoculars at 2.5X *magnification.*

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