Influence of scattering on the distribution of absorbed energy in disperse medium

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The classical statement of the problem on characteristics of a light field arising in a bounded layer of a homogeneous medium under exposure to a parallel optical beam is considered. From statistical Monte Carlo experiments, conditions are determined for appearance of a nonmonotonic dependence of the absorbed energy, as well as light fluxes scattered into both hemispheres (from the source and in the backward direction), on the depth of radiation penetration into the medium. The reaction of these characteristics and the angular distributions of the radiation scattered inside the medium to variations of the surface albedo is studied. Interpretation is given to the obtained dependences.

Introduction

The increasing interest of specialists in atmospheric optics to the problem of solar radiation absorption in the Earth's atmosphere is largely caused by the discrepancy between theoretical estimates and experimental measurements of the absorbed solar energy in the optical wavelength region. This discrepancy achieves (according to estimates given, for example, in Ref. 1) 20%, and theoretical simulation gives underestimated values of the solar flux absorbed in the atmosphere. Knowledge of the total absorption of the solar radiation by the cloud field covering the planet is needed for correct weather and climate forecasting.

There exists another class of problems connected with the formation and evolution of clouds, fogs, and hazes, in which it is important to know the spatial distribution of absorption in the medium, rather than its integral value. Thus, for example, one of the factors determining the cloud formation process in thermodynamics of the cloudy atmosphere is the dry-adiabatic gradient[2]:

$$\left(-\frac{\mathrm{d}T}{\mathrm{d}z}\right) = \gamma_\mathrm{a} = \frac{g}{c_p}\frac{T}{T'},$$

where T is the temperature of the ascending air mass; T' is the temperature of the ambient air; γ_a is the dry-adiabatic temperature gradient; c_p is the air heat capacity at constant pressure; z is the particle height above the ground; g is the acceleration due to gravity.

In particular, cooling of adiabatically ascending air mass (particle) non-saturated with water vapor is determined by this gradient. It can be supposed that the temperature of both the ascending particle and the ambient air depends, in particular, on the radiation absorbed by these components from the radiation fluxes coming both from the Earth's surface and the Sun (in daytime).

The problems concerning the spatial-angular structure of the scattered light fluxes inside a disperse system illuminated by an infinitely extended source are not new and therefore they have been studied rather thoroughly.[3,4] But the same cannot be said about the spatial distribution of a light flux absorbed in a medium. We can cite, for example, Ref. 5, which gives only qualitative analysis of this dependence based on approximate solutions of the radiative transfer equation.

The work, the results of which are considered in this paper, was aimed at the study of the effect of optical and geometrical parameters on the spatial-angular characteristics of diffuse light fluxes propagating in a medium and on the distribution of the absorbed energy along the direction of incident radiation.

Statement of the problem

Let a scattering medium be bounded by an absolutely transparent surface coinciding with the plane xOz and a reflecting (albedo $0 \geq \alpha \geq 1$) Lambertian surface $y = L$ be specified in the Cartesian system of coordinates (Fig. 1). A parallel monochromatic flux of unit energy at the radiation wavelength λ is incident along the direction $\boldsymbol{\omega}_0$ normal to the medium boundary xOz. The task is to find the spatial-angular structure of the scattered radiation intensity $I_\mathrm{forw}(\mathbf{r}, \omega)$ and $I_\mathrm{back}(\mathbf{r}, \omega)$ and the spatial distributions of the absorbed $P_\mathrm{a}(\mathbf{r})$ and scattered $P_\mathrm{f}(\mathbf{r}) = \int I_\mathrm{forw}(\mathbf{r}, \omega)\,\mathrm{d}\omega$ and $P_\mathrm{b}(\mathbf{r}) = \int I_\mathrm{back}(\mathbf{r}, \omega)\,\mathrm{d}\boldsymbol{\omega}$ fluxes, where $\mathbf{r}$ is the radius vector of a point inside the medium and at its boundary; $\boldsymbol{\omega}$ is the unit directing vector; subscripts *forw* and *f* correspond to the direction toward the plane $y = L$, and subscripts *back* and *b* correspond to the direction toward the source.

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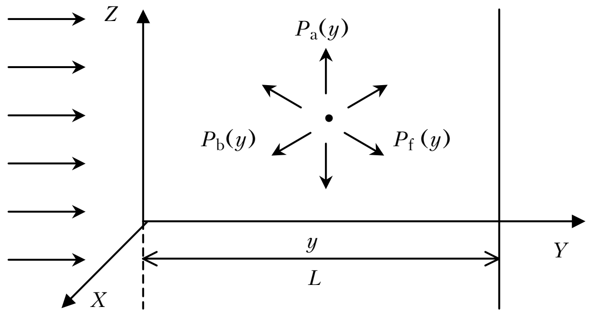

Fig. 1. Problem geometry.

The scattered fluxes (in spite of the fact that they are sufficiently studied) were estimated in this work to interpret the obtained dependences of the absorbed energy on the optical and geometrical parameters in numerical experiments. The aim of this work was not to simulate particular optical situations that can take place, for example, in the atmosphere, but to study the main regularities in the process of formation of the P_a flux. Therefore, the first step in this study was made using a simplified model of the medium. Let the scattering medium be homogeneous. Then the fluxes I_{forw}, P_f, I_{back}, P_b, and P_a are obviously dependent only on the coordinate $l = y$.

Let the optical thickness of the medium τ be in the range $1 < \tau < 10$, which lies beyond the domain of applicability of approximate solutions of the radiative transfer equations corresponding to depth conditions or to the approximation of few scattering events.

It can be supposed that the main regularities in the spatial distribution of energy absorbed in a disperse medium of a certain class (for example, polydisperse ensemble of spherical particles) manifest themselves regardless of its particular features (for example, modal radius of particles, the type of particle size distribution, etc.), although they certainly intensify or hide that or other characteristics of these regularities. As a model of the scattering properties of the medium, we took, from Ref. 6, the scattering phase function of advective fog for the radiation wavelength $\lambda = 0.89$ μm. As to the single scattering albedo χ, it was assumed that the value of this characteristic can be set arbitrarily, without restricting the consideration to the value calculated in Ref. 6 for a particular model of fog optical properties.

Methods

As a method for study, we have chosen the Monte Carlo method in the version of direct simulation without local estimates. The main difficulty in the development of the corresponding algorithm is connected with modeling of an infinitely extended source. This difficulty can be avoided, if we take into account the above assumption on the medium homogeneity. In this case it can be easily shown, using the symmetry condition, that the extended source can be replaced by a unidirectional one, and a point receiver can be replaced by an extended one.

For solving the problem formulated, we should solve the stationary transfer equation for the intensity

$$(\boldsymbol{\omega}, \text{grad } I) = -\beta_{ext} I + \beta_{sc} \int_{\Omega} I(\mathbf{r}, \boldsymbol{\omega}') \, q(\mathbf{r}, \boldsymbol{\omega}, \boldsymbol{\omega}') \, d\boldsymbol{\omega}' + \Phi_0(\mathbf{r}, \boldsymbol{\omega})$$

with the boundary conditions

$$I(y, \boldsymbol{\omega}) = 0, \ (\boldsymbol{\omega}, \mathbf{n}_2) > 0,$$

$$I(y, \boldsymbol{\omega}) = \delta(y) \, \delta(\boldsymbol{\omega} - \boldsymbol{\omega}_0), \ (\boldsymbol{\omega}, \mathbf{n}_1) < 0,$$

where β_{ext}, $\beta_{sc}(\mathbf{r})$, and $q(\boldsymbol{\omega}, \boldsymbol{\omega}', \mathbf{r})$ are the extinction and scattering coefficients and the scattering phase function, respectively; $\mathbf{n}_{1,2}$ are the external normals to the planes $y = xOz$ and $y = L$. It should be noted that with the above assumptions (spherical particles, homogeneous medium) β_{ext}, β_{sc} and $q(\boldsymbol{\omega},\boldsymbol{\omega}')$ are independent of $\mathbf{r}$. Besides, the scattering phase function is a function of one angular variable $q(\theta)$, namely, the angle between the photon directions before and after the collision with a particle, Φ_0 is the source function.

To construct the algorithm for statistical simulation of the process of radiation propagation through the medium, let us divide it into m layers by the planes $y = y_k$ ($k = 1, 2, \ldots, m$; $y_m = L$). Specify the set of directions $\{\boldsymbol{\omega}_k\}$, for which the flux intensities I_{forw} and I_{back} are determined ($k = 1, 2, \ldots, K$). The statistical estimation of the characteristics sought can be largely reduced to the following iteration procedure.

1. First, the trajectory of a photon moving from the point $\mathbf{r} = (0, 0, 0)$ along the direction $\boldsymbol{\omega} = (0, 1, 0)$ is generated.

2. The mean free path l is randomly selected.

3. Satisfiability of the boundary conditions is checked.

4. If the photon does not intersect the planes xOz and $y = L$ (Fig. 1), then at the point of its interaction with the medium (in the kth layer) the absorption probability is estimated (consequently, the flux P_a in this layer is assessed). Intersection of all previous layer boundaries before this point (if it is not the first interaction with medium particles) is considered as intersection of these boundaries by the scattered photon, and the intensity I_{forw} and the flux P_f or I_{back} and P_b (depending on the photon direction) are estimated. The new photon direction is a random select direction, and the iteration procedure is performed at the second stage.

5. If the photon intersects the plane xOz, then the intensity I_{back} and the flux P_b are estimated, and the next photon trajectory is generated (transition to the first stage of the procedure).

6. If the photon intersects the plane $y = L$, then I_{forw} and P_f are estimated. If the surface albedo $\alpha = 0$ (the surface is absolutely transparent or absolutely absorbing), then the trajectory terminates and the next trajectory is generated. If $\alpha > 0$, the direction of photon reflection from the surface is selected and continuation of the photon trajectory in the medium is constructed.

The algorithm was implemented in Turbo-Pascal, Version 7.0 (Borland International, Inc.).

Results of statistical experiments

Consider regularities in the formation of P_a, P_f, and P_b fluxes. As to the effect of the variable parameters on the shape of functions $I_{forw}(\boldsymbol{\omega}; y)$ and $I_{back}(\boldsymbol{\omega}; y)$, it should only be noted that it has no interesting peculiarities. However, it should be emphasized that the shapes of these functions become more similar, as the observation point is farther from the medium boundary nearest to the source. The longer is the optical distance from the observation point to the medium boundaries, the smaller is the difference between these functions.

Figures 2–11 depict the results of statistical simulation of the process of light flux propagation in a medium for the geometry of numerical experiments shown in Fig. 1. The standard deviations of the mean values of the characteristics estimated were controlled in the experiments; normally those did not exceed the level of 0.1% for the scattered and absorbed fluxes and 5% for the intensities I_{forw} and I_{back}.

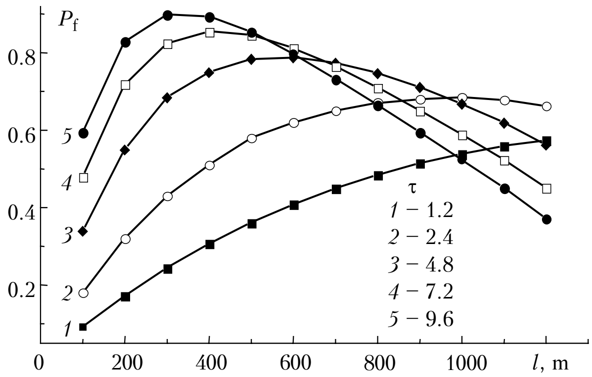

Fig. 2. Distribution of forward scattered fluxes inside a medium ($\chi = 0.999$).

The first series of numerical experiments was conducted assuming fixed geometrical thickness of the medium $L = 1200$ m and the single-scattering albedo $\chi = 0.999$.

Figure 2 depicts the distributions $P_f = P_f(l)$ for a set of optical depths of a medium layer from the range $1 < \tau < 10$ at the absolutely transparent boundary farthest from the source (it was assumed that the optical depth of the medium or β_{ext} changes due to variations of the particle concentration).

From the data shown in Fig. 2 it follows that, as the optical density increases, the distribution $P_f(l)$ of the flux scattered into the forward hemisphere inside the medium transforms from the monotonically increasing function (small particle concentrations, curve $\tau = 1.2$) to the peaked function (medium and large concentrations, curves $\tau \geq 2.4$). As this occurs, the higher is the optical density of the medium, the nearer to the medium boundary illuminated by the source are peaks of the forward scattered flux. These changes in $P_f(l)$ can be easily explained, if we take into account the following.

As the photon flux penetrates deeper into the medium, the probability of photon interaction with the medium particles increases, and, consequently, the scattered flux increases too. In this case, we observe redistribution of the flux between the directions to the forward (from the source) and backward (to the source) hemispheres. This redistribution is apparently regulated by the scattering phase function. For media with a strongly forward-peaked scattering phase function $q(\theta)$, the decrease of P_f and increase of P_b proceed more slowly than for media with a weaker forward peakedness of the scattering phase function. If we consider a scattering medium infinitely extended in the direction of the axis Oy with nonzero quantum survival probability (single scattering albedo), then it can be stated that the scattered flux $P_f \to 0$ at $l \to 0$ or $l \to \infty$ (in the latter case, because of nonzero absorption in the medium). Consequently, starting from some thickness of the scattering medium, the function $P_f = P_f(l)$ has a peak. Just this transformation in the shape of the distribution of fluxes scattered in the medium is well seen in Fig. 2.

The distribution of the fluxes scattered in the direction to the source $P_b = P_b(l)$ is shown in Fig. 3.

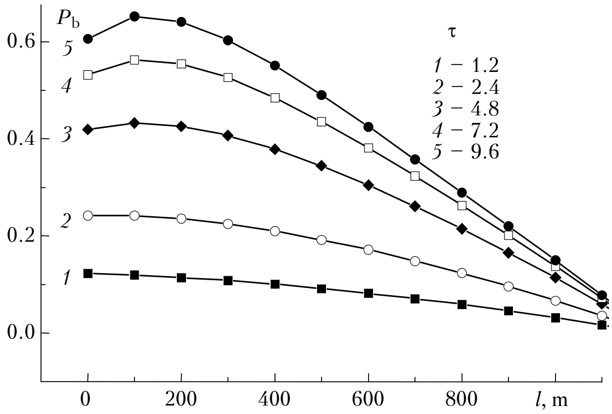

Fig. 3. Distribution of backward scattered fluxes inside the medium ($\chi = 0.999$).

As can be seen, the shape of these functions also transforms as the optical density of the medium increases. In spite of some common features (appearance of a peak near the medium boundary illuminated by the source and the increase of its amplitude, this family of curves differs from that shown in Fig. 2 by the feature that the inequality $P_b(l; \tau_1) > P_b(l; \tau_2)$ holds for any pair of values $\tau_1 > \tau_2$ at any l. The physical interpretation can be easily given to the obtained dependences by using roughly the same reasoning as for $P_f(l; \tau)$ in Fig. 2.

The distribution of the fluxes $P_a(l; \tau)$ absorbed in the medium under the same conditions is shown in Fig. 4. As could be expected (taking into account the spatial structure of the scattered fluxes shown in Figs. 2 and 3), the functions $P_a(l; \tau)$ have common features with the distributions $P_f(l; \tau)$ and $P_b(l; \tau)$. The absorbed fluxes reach maximum at $0 < l < L$, but this is characteristic already for the whole range of τ values.

The inequality $P_a(l;\,\tau_1) > P_a(l;\,\tau_2)$ holds for them, as for the family of $P_b(l;\,\tau)$, at any l, if $\tau_1 > \tau_2$.

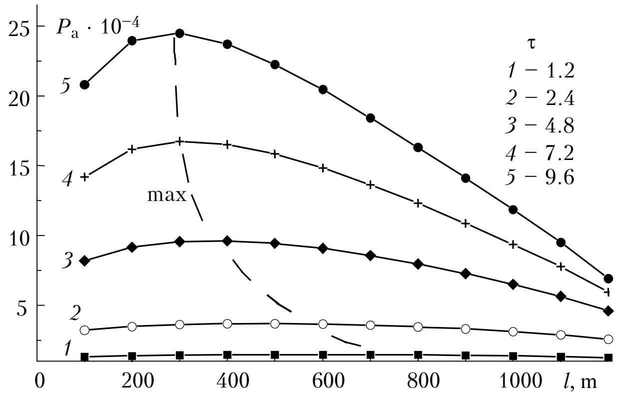

Fig. 4. Distribution of fluxes absorbed in the medium ($\chi = 0.999$).

The overall conclusion from analysis of the dependences $P_a(l;\,\tau)$ at $\chi = 0.999$ is the following. Even in a homogeneous scattering medium (in contrast to a medium that only absorbs) the peak of the absorbed energy lies inside the medium, and as the optical density (particle concentration) of the medium increases, it approaches its boundary illuminated by the source, but still keeps at a noticeable distance from it (in the case considered $l_{max} \approx 300$ m at $\tau = 9.6$).

Is this conclusion (as well as the conclusions concerning scattered fluxes) also valid for media with different absorptivity? The answer to this question is given in Fig. 5, which shows some results of the next series of statistical experiments with the quantum survival probability $\chi = 0.8$. The change of the single scattering albedo (all other conditions being the same) led to significant changes in both the shape of functions $P_a(l)$ and the relation between them at different values of τ. Actually, at $\chi = 0.999$ $P_a(l;\,\tau_1) > P_a(l;\,\tau_2)$ for any l, if $\tau_1 > \tau_2$, and at $\chi = 0.8$ this inequality holds only at $l < 400$ m. At $l > 400$ m the relation between $P_a(l;\,\tau)$ at different τ is similar to the relations characteristic of the distribution of fluxes scattered in the forward direction in the medium depth at $\chi = 0.999$ (see Fig. 2).

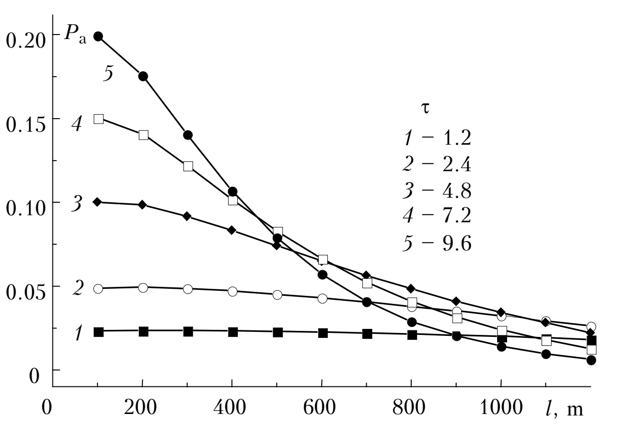

Fig. 5. Distribution of fluxes absorbed in the medium ($\chi = 0.8$).

It is obvious that this transformation of the dependence $P_a = P_a(l;\,\tau)$ is caused by changes in the spatial structure of the radiation scattered in the medium, because the decrease in the quantum survival probability inevitably leads to suppression of multiple scattering. This is illustrated in Figs. 6 and 7, which show the distributions of the fluxes scattered in the forward (Fig. 6) and backward (Fig. 7) directions in the medium for two values of the quantum survival probability and optical depth of the medium.

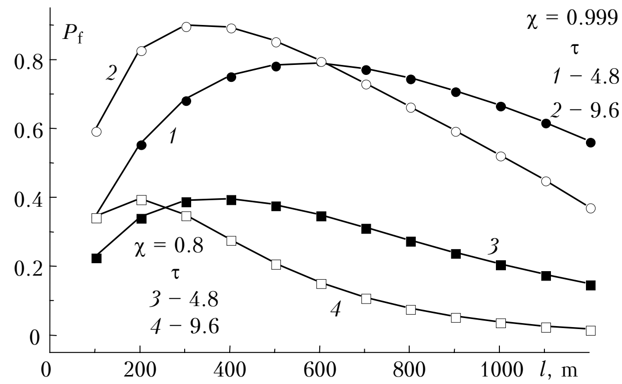

Fig. 6. Forward scattered fluxes for media with different absorption.

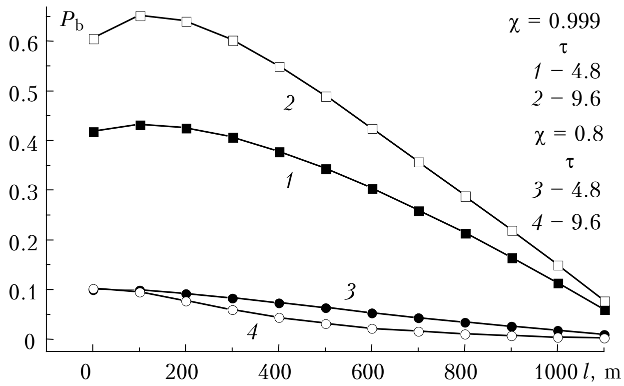

Fig. 7. Effect of the quantum survival probability on the fluxes scattered in the backward direction in the medium.

When studying the effect of the optical depth of the medium on the scattered and absorbed fluxes, we have obtained a surprising, at first sight, result. It is shown in Fig. 8. In these statistical experiments, the optical depth of the medium was increased by changing the geometrical thickness of the medium in the direction of the axis Oy (see Fig. 1) with fixed extinction, scattering, and absorption coefficients (that is, fixed composition and concentration of particles).

The calculated results show that at high values of the photon survival probability ($\chi \to 1$) addition of identical layers to the medium (or, what is the same, increase of L) leads not only to the increase in the total absorption and to the change in the distribution of the backward scattered radiation (that seems to be quite obvious), but also to the change in the spatial distribution of the forward scattered flux, and (what is rather unexpected) starting from the boundary illuminated by the source. Such a reaction of the P_f fluxes to displacement of the farthest medium boundary becomes clear, if we take into account that photons moving in the backward direction are also sources of forward scattering.

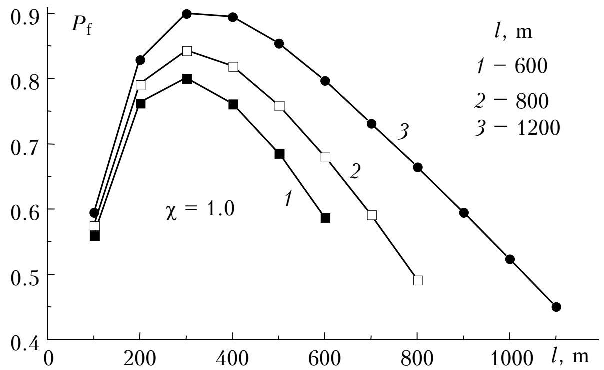

Fig. 8. Reaction of forward scattered fluxes to the displacement of the medium boundary farthest from the source.

In all the examples considered above, it was assumed that the medium boundary farthest from the source is absolutely absorbing (or absolutely transparent). Consider how the nonzero surface albedo $y = L$ affects the distribution of the absorbed flux $P_a(l)$. For this purpose, turn our attention to Figs. 9–11. The results shown were obtained for the photon survival probability $\chi = 0.999$.

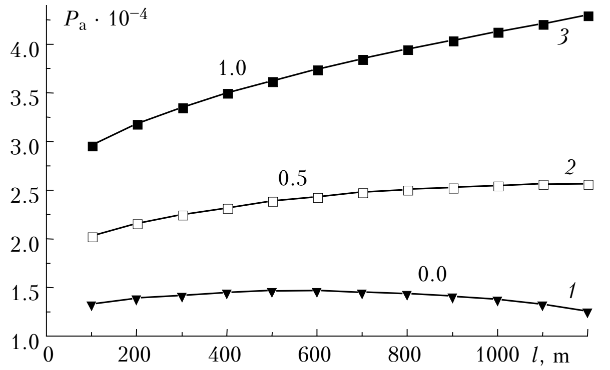

Fig. 9. Distribution of the flux absorbed in the medium ($\tau = 1.2$) at the surface albedo $\alpha = 0.0, 0.5, 1.0$ (curves *1–3*); $\chi = 0.999$.

Figure 9 shows the fluxes $P_a(l)$ absorbed in the medium for $\tau = 1.2$ at three values of albedo of the surface $y = L$ (Fig. 10 – for $\tau = 9.6$). The reflecting surface can introduce considerable changes in the dependence of the energy absorbed in the medium on the distance l. Variations of the surface albedo in the range [0, 1] lead to transformation of this dependence from having an extreme ($\alpha = 0$) to almost monotonically increasing ($\alpha = 1.0$). The closer to unity is χ and the larger is the optical depth of the medium, the more essential is this transformation. This is connected with the increasing role of re-reflections from the surface $y = L$ in the process of formation of the scattered fluxes in the medium.

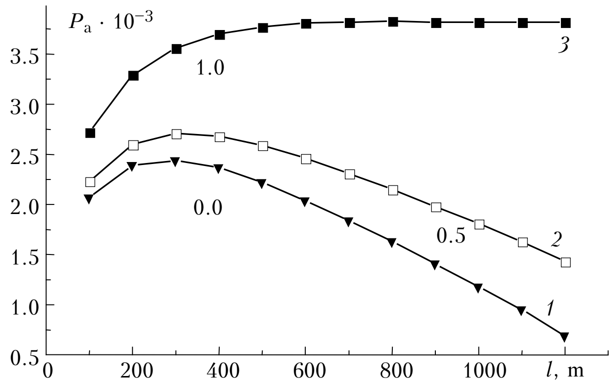

Fig. 10.

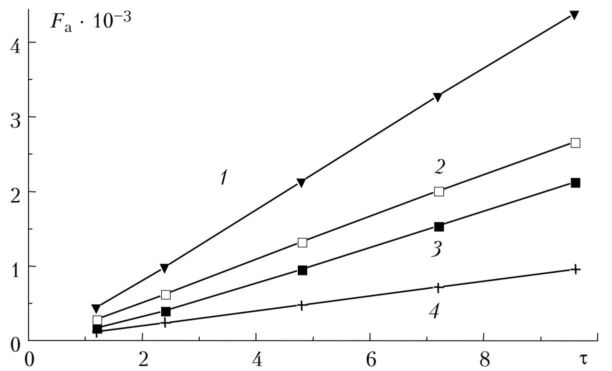

Fig. 11. Total absorption by a medium layer taking into account (curves *1–3*) and neglecting (*4*) scattering. Surface albedo $\alpha = 1.0$ (*1*), 0.5 (*2*), 0.0 (*3*); $\chi = 0.999$.

Figure 11 shows the total flux absorbed by the entire medium depth $F_a = \sum_i P_i$ (P_i is the absorption by the ith medium layer, see Fig. 1) in the τ range $1 < \tau < 10$ studied at $\chi = 0.999$ and three values of albedo of the surface $y = L$. For a comparison, it also shows the data (curve *4*) obtained assuming that medium particles have the same absorptivity, but do not scatter incident radiation, that is, $F_a = 1 - \exp(-\tau_i)$, where $\tau_i = y_i\,\beta_{abs}$, β_{abs} is the absorption coefficient.

Conclusion

Thus, within the framework of the above statement of the problem, statistical experiments have determined the following.

At stationary illumination of an isolated layer of a disperse medium with absolutely transparent boundary surfaces, the distribution of the energy absorbed inside the medium can have a peaked shape with a peak of the absorbed energy inside the medium, if the photon survival probability is close to unity and the optical depth of the medium $\tau > 2$. As the medium absorptivity increases, the distribution of the absorbed energy transforms into the monotonically decreasing function, which is maximum near the boundary closest to the source.

The increase of the optical depth of the medium due to the increase of its geometrical thickness can lead to the growth of the forward scattered fluxes starting from the boundary illuminated by the source.

At illumination of a medium adjoining a reflecting surface, the distribution of the energy absorbed in the medium is determined by the albedo of this surface. Even at the photon survival probability close to unity, changing the albedo, we can transform widely the spatial structure of the absorbed energy: from the distribution having a peak inside the medium to the almost monotonic increase of the absorbed energy as the medium boundary farthest from the source is approached. The closer to unity is the photon survival probability, the more significant is this transformation.

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