

## RADIATIVE CHARACTERISTICS OF INHOMOGENEOUS STRATOCUMULUS WITH STOCHASTIC CLOUD TOP GEOMETRY

G.A. Titov and E.I. Kas'yanov

*Institute of Atmospheric Optics,  
Siberian Branch of the Russian Academy of Sciences, Tomsk*

*Received December 25, 1996*

*Radiative characteristics (albedo, transmission, absorption, and horizontal transport) of stochastic stratocumulus are studied. The spatial distribution of the liquid water content (LWC) is simulated as a random process with a one-dimensional lognormal distribution and the power-law spectrum (inhomogeneous interval cloud structure). Stochastic cloud top geometry is simulated as a random Gaussian process with exponential correlation function. It is shown that the stochastic geometry and the inhomogeneous internal cloud structure affect nearly identically the mean and the variance of albedo and transmission. The contribution from the stochastic geometry to the variance of horizontal transport predominates. Fluctuations of cloud top altitude decrease the accuracy of cloud absorption retrieval by an order of magnitude. Simultaneous measurements of fluxes in the visible and near-IR spectral ranges can be used to investigate small-scale (~0.4 km) variations of absorption.*

### 1. INTRODUCTION

To refine the radiative codes of climate models and methods for interpretation of remote sensing data, of primary significance is adequate description of interaction between radiation and clouds and, in particular, stratocumulus (Sc). In spite of great progress in the three-dimensional theory of radiative transport, we have not yet a clear idea of roles of the stochastic cloud top geometry and inhomogeneous internal cloud structure in the formation of radiative properties of Sc.

Based on the experimental data, in Ref. 1 it is established that the horizontal distribution of liquid water content (optical thickness) of marine Sc near the south coast of California is well approximated by the two-dimensional random field with lognormal distribution and the power-law spectrum. To describe the observed distribution of optical thickness, the cascade<sup>2</sup> and spectral<sup>3</sup> cloud models were constructed. With the help of the model constructed on the basis of multiplicative cascade processes,<sup>2</sup> the sensitivity of the mean albedo of stratocumulus to the horizontal fluctuations of the liquid water content was investigated.<sup>4-6</sup> It was shown that for the most typical values of the mean  $\langle\tau\rangle$  and variance of the optical thickness, the mean albedo of the layer with inhomogeneous internal structure may be smaller than the albedo of the homogeneous layer having the optical thickness  $\langle\tau\rangle$  by ~15%. The model based on the spectral methods of simulation of random processes<sup>3</sup> (fields) was used to study the effect of horizontal fluctuations of Sc liquid water content on the horizontal transport<sup>7-9</sup> and the solar radiation absorption.<sup>8-9</sup> It

was shown that the horizontal transport, comparable by the order of magnitude with the albedo, transmission, and absorption, may substantially affect the accuracy of determining the cloud absorption.<sup>8-9</sup> An essential disadvantage of models considered above, which adequately describe the horizontal variability of optical thickness, is the simplest geometry of cloud model in the form of a plane-parallel layer.

Real clouds have irregular cloud top and therefore the cloud model must consider the cloud top fluctuations. Model of stratus with stochastic boundary<sup>10</sup> was used to study the effects of variance and correlation radius of cloud top altitude on the mean albedo and transmission. The stochastic geometry of stratus was simulated on the basis of a homogeneous Gaussian field. It was established that the difference between the mean fluxes calculated for the cloud layer with random cloud top and corresponding radiative characteristics of the plane-parallel layer may reach 25%. A disadvantage of this model is the neglect of inhomogeneous internal structure of clouds.

In the real stratocumulus the liquid water content and the cloud top altitude fluctuate *simultaneously*. The cloud model that considers the *joint* effect of cloud top stochastic geometry and horizontal variations of liquid water content was developed in Ref. 11. With the help of this model, the mean, variance, and the probability density and spectral density of the albedo and transmission of stratocumulus were studied.<sup>11</sup> In particular, it was shown that a break of the spectrum could be observed at high spatial frequencies not only for the albedo, but also for the transmission.

In this paper, a comparative analysis is made of the effects of liquid water content and cloud top fluctuations of stratocumulus on the mean and variance of the albedo, transmission, absorption, and horizontal transport as well as on the accuracy of absorption retrieval in clouds.

The paper consists of four sections. Cloud models and solution methods are considered in Sec. 2. Sensitivity of the mean and variance of radiative characteristics of stratocumulus to the inhomogeneous internal structure and stochastic cloud top geometry is discussed in Sec. 3. Section 4 is devoted to an analysis of cloud top altitude fluctuations on the accuracy of reconstruction of the absorption by clouds. In conclusion, the main results are formulated.

## 2. CLOUD MODELS AND SOLUTION METHOD

Hereafter we will use the following cloud models:

1. Plane-parallel layer with random horizontal distribution of optical thickness called WP model. Fluctuations of the optical thickness  $\tau_{WP}$  obey the one-dimensional lognormal distribution and the power-law spectrum<sup>1</sup>  $f(k) \sim k^{-5/3}$ . With the aim of decreasing the expenditures of computing time, we will consider one-dimensional model, that is, we will consider that  $\tau_{WP}$  is the random process, and the values  $\tau_{WP}$  depend only on horizontal coordinate  $x$ . We note that algorithm for modeling of the optical thickness is described in ample detail in Refs. 3, 7, 9, and 11. A *continuous* realization  $\tau_{WP}(x)$  is divided into  $N = 2^{12}$  pixels with equal horizontal extent  $\Delta x = 0.05$  km. The optical thickness of every pixel  $\tau_{WP}(x_i)$  is determined as a value of the random process at a point  $x_i = i\Delta x$ , and the extinction coefficient is calculated by the formula  $\sigma(x_i) = \tau_{WP}(x_i)/\Delta H$ ,  $i = 1, \dots, N$ , where  $\Delta H$  is the geometrical cloud layer thickness. In modeling of  $\tau_{WP}(x)$ , we used the mean  $\langle \tau_{WP}(x) \rangle$  and the variance  $D_{\tau_{WP}}$  typical of Sc.

2. Model considering the fluctuations of cloud top altitude and extinction coefficient (liquid water content) is called the GWP model. Because between microphysical and geometrical characteristics the statistical interrelations were not observed experimentally, we assume that the cloud top altitude and the extinction coefficient are the *independent* random fields. Analysis of the data of laser sensing of stratified clouds shows that in the first approximation the cloud top can be approximated by the one-dimensional isotropic Gaussian field with exponential correlation function<sup>11</sup> (correlation radius is  $\sim 3$  km). We note that in Ref. 10 the simplest correlation function of an isotropic field in the form of the Bessel function of the first kind was used. For the GWP model as well as for the WP model considered above we will study only one-dimensional

case, that is, when the cloud top altitude  $H(x)$ , the extinction coefficient  $\sigma(x)$ , and the optical thickness  $\tau_{GWP}(x)$  are random processes. The algorithm for calculating  $\tau_{GWP}(x_i)$   $i = 1, \dots, N$ , is as follows. For every pixel with the extinction coefficient  $\sigma(x_i)$ , calculated for the WP model, the cloud top altitude is additionally determined as a value of continuous realization of the random process  $H(x)$  at the point  $x_i = i\Delta x$ ,  $i = 1, \dots, N$ . The random process  $H(x)$  is model for the one-dimensional Gaussian distribution with mean value  $\langle H(x) \rangle = \Delta H$  and exponential correlation function having a correlation radius of 2.75 km. To estimate the *maximum* influence of stochastic cloud top on the radiative characteristics of continuous stratocumulus, all calculations were done for the variance of  $H(x)$  being equal to  $D_H = (\Delta H/3)^2$ . The optical thickness of the pixel for the GWP model was calculated by the formula  $\tau_{GWP}(x_i) = \sigma(x_i) H(x_i)$ ,  $i = 1, \dots, N$ . Because of the independence of the random processes  $\sigma(x)$  and  $H(x)$ , the mean value  $\langle \tau_{GWP}(x) \rangle = \langle \tau_{WP}(x) \rangle$ .

3. Plane-parallel and horizontally homogeneous layer is called PP model. We denote by  $H_{PP}$  and  $\tau_{PP}$  geometrical and optical thicknesses of the layer, respectively. To compare radiative properties of different models, we take  $H_{PP} = \Delta H$  and  $\tau_{PP} = \langle \tau_{WP}(x) \rangle = \langle \tau_{GWP}(x) \rangle$ . We note that the PP model is widely used in models of the global atmospheric circulation and in the interpretation of field measurements.

For marine stratocumulus near the south coast of California, geometrical thickness  $\Delta H$  is of the order of 0.3 km, the most characteristic limits of variations of the mean optical thickness  $\langle \tau \rangle$  are 10–15, and the variance is  $D_\tau \sim \langle \tau \rangle^2/4$  (see, for example, Refs. 1 and 4); therefore, in modeling of  $\tau_{WP}(x)$  and  $\tau_{GWP}(x)$ , we used the following values:  $\Delta H = 0.3$  km,  $\langle \tau_{WP}(x) \rangle = 13$ , and  $D_{\tau_{WP}} = 29$ . Realizations of  $\tau_{WP}(x_i)$  and  $\tau_{GWP}(x_i)$ ,  $i = 1, \dots, N$ , so constructed correspond to the sample means  $\langle \tau_{WP} \rangle = \langle \tau_{GWP} \rangle = 11.5$  and the variances  $D_{\tau_{WP}} = 22.4$  and  $D_{\tau_{GWP}} = 40.6$ . Mean, minimum, and maximum ( $H_{max}$ ) values of the realization  $H(x_i)$   $i = 1 \dots N$ , are 0.3, 0.07, and 0.57 km, respectively. During the measurements of optical, microphysical, and radiative characteristics of marine Sc, the solar zenith angle was approximately equal to 50–60°; therefore, the calculations of the radiative fluxes were done for  $\xi = 60^\circ$ . Because for  $\xi \leq 60^\circ$  the albedo of the ocean surface  $A_s$  is smaller than 0.1 (see Ref. 2), the influence of the underlying surface on the radiative characteristics of marine Sc was not considered, that is, it was assumed that  $A_s = 0$ . Radiative properties of every pixel were calculated by the Monte Carlo method with periodic boundary conditions. The method of maximum cross section<sup>13</sup> was used to increase the efficiency of algorithms. Transmission was calculated at the cloud bottom (plane  $z=0$ ),

whereas the albedo – at the cloud top (plane  $z = H_{pp}$ ) for PP and WP models and at the maximum cloud top altitude (plane  $z = H_{max}$ ) for GWP model. The scattering phase function was for C1 cloud at a wavelength of  $0.69 \mu\text{m}$  (see Ref. 14). The calculations were done for two values of the single scattering albedo of water droplets  $\omega_0 = 1$  and  $0.99$ . Under background conditions, the optical thickness of the atmospheric aerosol is approximately  $0.1$  and is much smaller than the mean optical thickness of stratocumulus; therefore, in our calculations of the radiative cloud characteristics, the influence of the atmospheric aerosol was not considered. The mean relative error in calculating radiant fluxes was  $\sim 1\%$ .

To denote the mean and the variance of albedo  $R$ , transmission  $T$ , absorption  $A$ , and horizontal transport  $E$ , we use the angular brackets  $\langle \rangle$  and symbol  $D$ , respectively. The subscript denotes the cloud model. For example,  $\langle E_{GWP} \rangle$  and  $D_{E_{GWP}}$  denote the mean and the variance of horizontal transport for GWP model.

### 3. RADIATIVE CHARACTERISTICS OF STRATOCUMULUS

Before discussing the results, recall that the dependence of the albedo, transmission, absorption and horizontal transport on the inhomogeneous internal structure of Sc (for WP model) was analyzed in ample detail in Refs. 7 and 9. In particular, it was shown that horizontal transport, equal to zero for PP model, is the reason for violation of unambiguous dependence between the optical and radiative characteristics of a given pixel.

Figure 1 shows realizations of optical thickness, albedo, transmission, and horizontal transport for WP and GWP models. It can be seen that the stochastic geometry leads to the significant increase of the fluctuation amplitudes of optical thickness and radiative characteristics.

Large values of horizontal transport  $|E_{GWP}|$  may lead to inequality  $R_{GWP} > 1$  (see Fig. 1). Values of the transmission exceeding unity were obtained in field measurements<sup>15</sup> and theoretical calculations.<sup>16</sup>

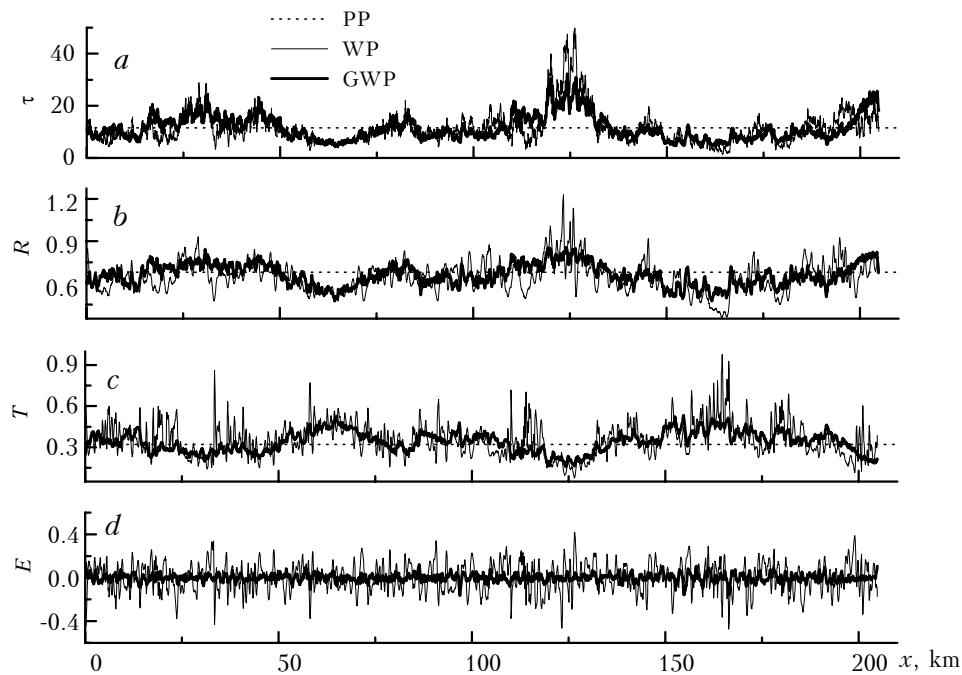


FIG. 1. Numerical realizations of the optical thickness  $\tau$  (a), albedo  $R$  (b), transmission  $T$  (c), and horizontal transport  $E$  (d) at  $\omega_0 = 1$ ,  $\xi_0 = 60^\circ$ , and  $A_s = 0$  (ocean).

Let us estimate the sensitivity of the mean albedo, transmission, and absorption to the stochastic cloud top geometry and inhomogeneous internal structure of Sc. As a measure of influence of these two factors on the mean, we introduce two differences

$$\Delta F_{WP} = F_{PP} - \langle F_{WP} \rangle, \quad \Delta F_{GWP} = F_{PP} - \langle F_{GWP} \rangle, \quad (1)$$

where  $F$  denotes the radiative characteristics  $R$ ,  $T$ ,  $A$  and  $E$ . The differences  $\Delta F_{WP}$  and  $\Delta F_{GWP}$  are the deviations of  $\langle F_{WP} \rangle$  and  $\langle F_{GWP} \rangle$  from  $F_{PP}$  caused solely by inhomogeneous internal structure and joint

fluctuations of the liquid water content and cloud top altitude, respectively. The value of  $\Delta R_{GWP}$  is nearly twice as large as  $\Delta R_{WP}$  not only in the case of conservative scattering (Fig. 2a), but also in the presence of absorption by water droplets (Fig. 2b). Hence, we can conclude that the effects of inhomogeneous internal structure and stochastic geometry on the mean albedo are comparable. This conclusion is also true for the transmission. The stochastic cloud top geometry affects slightly the mean absorption by Sc (see Fig. 2b). For a fixed value of

the photon survival probability  $\omega_0$ , the mean absorption increases with the increase of the radiant energy arriving at the upper and lower boundaries of the cloud layer and the mean scattering multiplicity. By virtue of the above assumptions, the solar radiation arrives only at the cloud top. Hence, the radiant energy arriving at the cloud layer is the same for all examined cloud models. From the foregoing, it follows that for the WP and GWP models the average values of the scattering multiplicity are approximately identical.

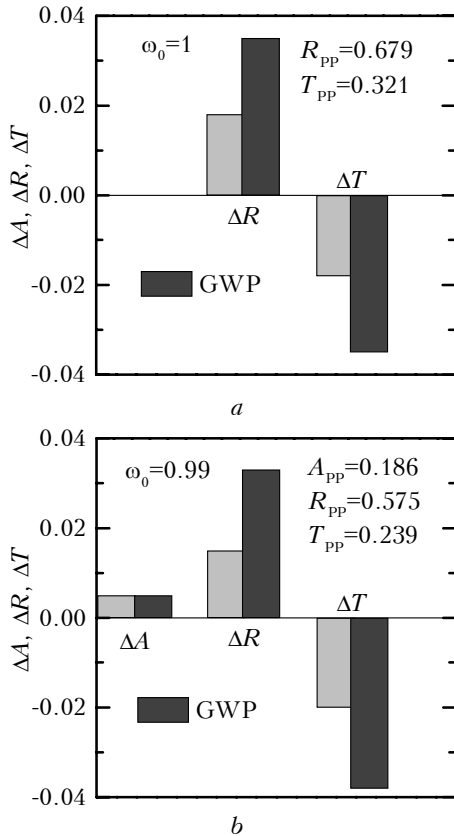


FIG. 2. Differences  $\Delta R$ ,  $\Delta T$ ,  $\Delta A$ , and  $\Delta E$  for two cloud models at  $\omega_0 = 1$  (a) and 0.99 (b),  $\xi_0 = 60^\circ$ , and  $A_s = 0$  (ocean).

The variances of the optical thickness and the radiative characteristics for WP and GWP models are compared in Fig. 3. It can be seen that for these models the variances of  $\tau$ ,  $R$ ,  $T$ , and  $A$  differ nearly twice. Hence, the contributions from the inhomogeneous internal structure and the stochastic geometry to the variances  $D_\tau$ ,  $D_R$ ,  $D_T$ , and  $D_A$  are comparable. Special attention must be given to an interesting and very important result: the variance  $E_{GWP}$  exceeds the variance  $E_{WP}$  by more than an order of magnitude. This means that the variance of the horizontal transport is determined primarily by the stochastic geometry.

Before explanation of this result, we recall how the inhomogeneous internal structure affects the horizontal transport in the plane-parallel layer. Let us examine a fragment of realization for WP model

comprising three pixels (Fig. 4), with the first and third pixels having identical small optical thicknesses (extinction coefficients) and the second optically thick pixel located between them. Heights of all three pixels are identical and equal to  $\Delta H$ .

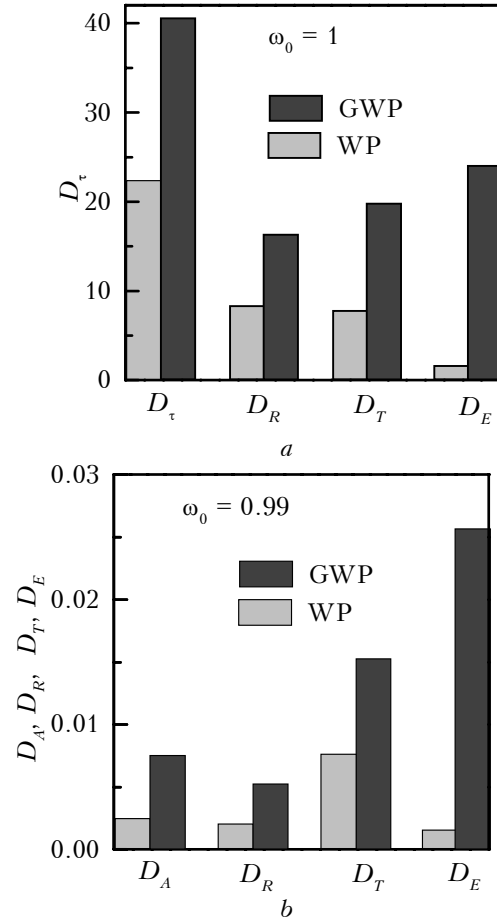


FIG. 3. Variances of the optical thickness  $D_\tau$  (a), albedo  $R$  (a, b), transmission  $D_T$  (a, b), absorption  $D_A$  (b), and horizontal transport  $D_E$  (a, b) for two cloud models at  $\xi_0 = 60^\circ$ ,  $A_s = 0$  (ocean), and  $\omega_0 = 1$ , (a), and  $\omega_0 = 0.99$  (b).

For definiteness, we assume that in front of the first and behind the third pixels, optically thin fragments of realization  $\tau_{WP}$  are located. The second optically thick pixel screens the region located behind from incident solar radiation. It is named optical shadow. The radiant energy entering the left side of the third pixel is decreased because of screening. By virtue of the strong elongation of the scattering phase function in the forward direction and small optical thickness of the region located behind, a significant fraction of radiation incident at the top of the third pixel, leaks through its right side. For above-indicated reasons, the third pixel will lose a larger portion of radiant energy than it receives ( $E > 0$ ).

The opposite situation occurs for the first pixel located in front of the optically thick pixel. A portion

of radiation entering the second pixel after multiple scattering leaks into the first optically thin pixel. Owing to this additional contribution, the first pixel will receive more radiant energy than it loses ( $E < 0$ ). Let us examine these three pixels for GWP model. From the algorithm of this model, it follows that the values of the extinction coefficients of these pixels remained unchanged and only cloud top altitudes changed. Let us assume that the geometrical thicknesses of the first and third pixels are decreased and  $H$  of the second pixel is increased (see Fig. 4). We also assume that in front of the first and behind of the third pixels, optically thin fragments of realization  $\tau_{\text{GWP}}$  are located. The radiation entering the top and left sides of the second pixel is attenuated before it

reaches the level  $z = \Delta H$ ; therefore, the radiant energy entering the optical shadow zone decreases. Optical shadow becomes darker (the first reason). Further, as far as the height of the second pixel exceeds  $\Delta H$ , it produces not only optical but also *geometrical* shadow (the second reason). At the level  $\Delta H = z$ , the extent of the geometrical shadow is  $(H - \Delta H) \tan(\xi_0)$ . By the above-indicated two reasons, for the irregular cloud top geometry the third pixel receives a smaller amount of radiation; hence, for this pixel the inequality  $E_{\text{GWP}} > E_{\text{WP}}$  is true. Let us examine how the irregular cloud top affects the horizontal transport in the first pixel. Its height is smaller than that of the second pixel; therefore, the *unscattered* radiation enters the unshaded fragment of the pixel.

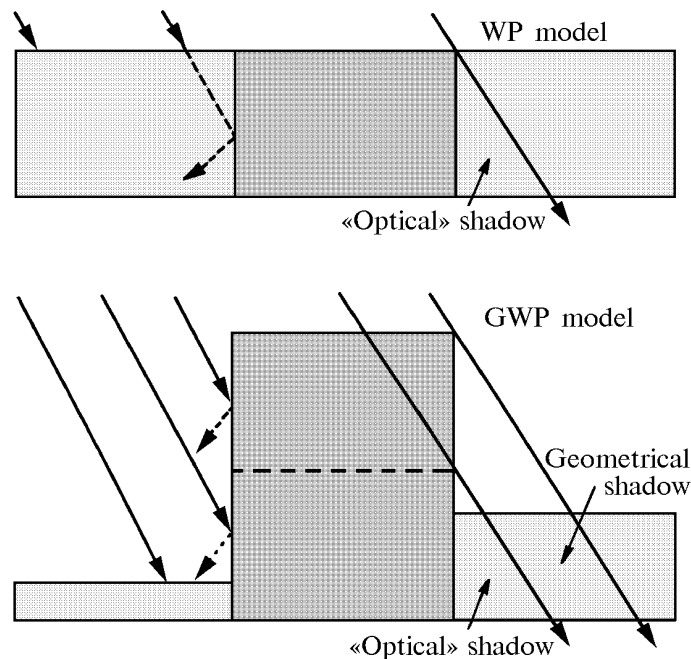


FIG. 4. Scheme illustrating the influence of the cloud top stochastic geometry on the horizontal radiative transport.

This is the main reason why the left side of the second pixel receives and hence reflects a larger amount of radiant energy for the stochastic cloud top geometry. A considerable portion of energy enters the first pixel and that is why the inequality  $|E_{\text{GWP}}| > |E_{\text{WP}}|$  is fulfilled.

The stochastic geometry increases the fluctuation amplitude of horizontal transport and its contribution to the variance  $D_E$  is dominant. Because with the increase of the fluctuation amplitude  $E(x)$  the connection between the true and reconstructed cloud absorption deteriorates,<sup>7</sup> we may expect that irregular cloud top geometry will have considerable influence on the accuracy of retrieval of the absorption by stratocumulus.

#### 4. HORIZONTAL TRANSPORT AND ABSORPTION IN STRATOCUMULUS

Recall the essence of the problem on determining the absorption in inhomogeneous clouds. For simplicity

of presentation we assume that the clouds are located above the nonreflecting underlying surface ( $A_s = 0$ ) and their optical characteristics depend solely on horizontal coordinate  $x$ . With consideration for the above assumptions, the radiant energy conservation law for inhomogeneous clouds has the form<sup>7-9</sup>

$$R(x) + T(x) + A(x) = 1 - E(x). \quad (2)$$

Equation (2) contains four functions: albedo  $R(x)$ , transmission  $T(x)$ , absorption  $A(x)$ , and horizontal transport  $E(x)$ . To find the absorption  $A(x)$  from this equation, it is necessary to know  $R(x)$ ,  $T(x)$ , and  $E(x)$ . In practice, only albedo and transmission are commonly measured. In this case, it is impossible to determine  $A(x)$  from Eq. (2). Instead of the true absorption  $A(x)$ , we can determine only the reconstructed one  $A'(x)$

$$A'(x) = A(x) + E(x) = 1 - R(x) - T(x). \quad (3)$$

From Eq. (3) it follows that when the horizontal transport is comparable with  $A(x)$  by the order of magnitude, the reconstructed absorption  $A'(x)$  and the true absorption  $A(x)$  will differ strongly.

In Fig. 5 the dependence  $A(x_i)$  on  $A'(x_i)$ ,  $i = 1, \dots, 4096$ , is shown. It can be seen that for the stochastic geometry the range of variations of the true absorption is increased more than twice and the limits of variations of the reconstructed absorption  $A'_{GWP}(x)$  are increased more than *three* times in comparison with  $A_{WP}(x)$ .

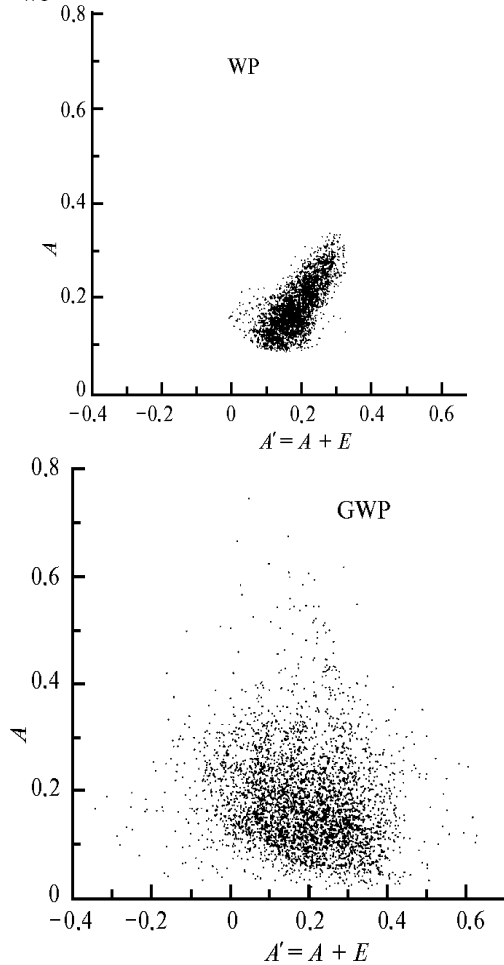


FIG. 5. Absorption  $A$  as a function of the reconstructed absorption  $A' = A + E$  at  $\xi_0 = 60^\circ$ ,  $A_s = 0$  (ocean), and  $\omega_0 = 0.99$ .

On the basis of two approaches suggested in Refs. 7 and 9 we examine the effect of the cloud top altitude fluctuations on the accuracy of determination of the cloud absorption.

The first approach involves the *spatial* averaging of the radiative characteristics over  $2^{nx}$  pixels,  $nx = 0, \dots, 12$

$$F_j(nx) = \frac{1}{2^{nx}} \sum_{i=k}^m F(x_i), \quad k = (j - 1) 2^{nx} + 1,$$

$$m = j 2^{nx}, \quad j = 1, \dots, 2^{12-nx}, \quad (4)$$

where  $F_j(nx)$  denote the radiative characteristics  $R_j(nx)$ ,  $T_j(nx)$ ,  $A_j(nx)$ , and  $E_j(nx)$ . The length of the spatial averaging is  $l(nx) = 2^{nx} \Delta x$  and the number of nonoverlapping intervals into which the realization of length  $L = \Delta x 2^{12}$  is divided, is  $2^{12-nx}$ . If after averaging over the space (over  $2^{2nx}$  pixels) the quantity  $E_j(nx^*) \approx 0$  for the given value  $nx^*$ , formula (2) can be used to estimate the true absorption for the  $j$ th interval of the realization. We note that in Refs. 8 and 9 the value  $l(nx^*)$  was calculated by formula (5) only for a *single* value  $j = 1$  and  $nx = 0, \dots, 12$ .

From the homogeneity of boundary conditions and random processes  $\tau_{WP}(x)$  and  $\tau_{GWP}(x)$  it follows that  $E_{WP}(x)$  and  $E_{GWP}(x)$  are also homogeneous random processes. From the theory of homogeneous random processes it is known that for sufficiently long averaging interval  $l(nx)$ , the variance of the *sample* mean  $E(nx)$  is approximately equal to  $D_E \rho_E / l(nx)$  (see, for example, Ref. 17, p. 258), where  $D_E$  and  $\rho_E$  are the variance and the correlation radius of the homogeneous horizontal transport, respectively. Hence, the *sample* mean  $E(nx)$  is close to the true mean  $\langle E(nx) \rangle = 0$  only when the averaging interval  $l(nx) \gg D_E \rho_E$ . Further, the correlation radius  $E$  is equal approximately to 0.25 km and depends weakly on the choice of the cloud model (Fig. 6).

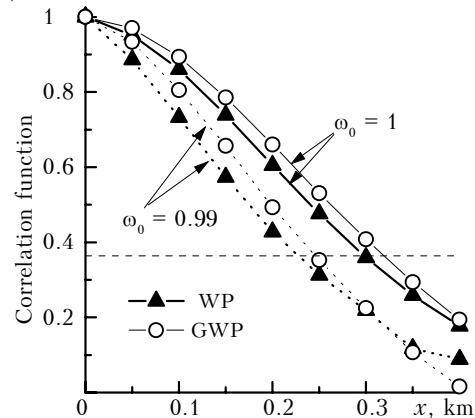


FIG. 6. Correlation functions of the horizontal transport for two cloud models at  $\xi_0 = 60^\circ$ ,  $A_s = 0$  (ocean); the dashed curves are for  $\omega_0 = 0.99$  and the solid curves are for  $\omega_0 = 1$ .

The variance of  $E_{GWP}$  exceeds approximately by an order of magnitude the variance of  $E_{WP}$ ; therefore, we may expect that  $l_{GWP}(nx)$  will be several times greater than  $l_{WP}(nx^*)$ . The results of our calculations support this assumption (Fig. 7).

Inequalities  $|E_{WP}(nx^*)| \leq 0.01$  and  $|E_{GWP}(nx^*)| \leq 0.01$  are satisfied for averaging scales approximately equal to 6 and 30 km, respectively. Thus, to obtain a reliable estimate of the solar radiation absorption by stratocumulus with inhomogeneous internal structure and stochastic cloud top geometry, the length of the spatial averaging of the net fluxes measured at the cloud layer top and bottom, should be approximately 30 km. Fluctuations of the Sc cloud top altitude degrade the maximum spatial resolution approximately five times.

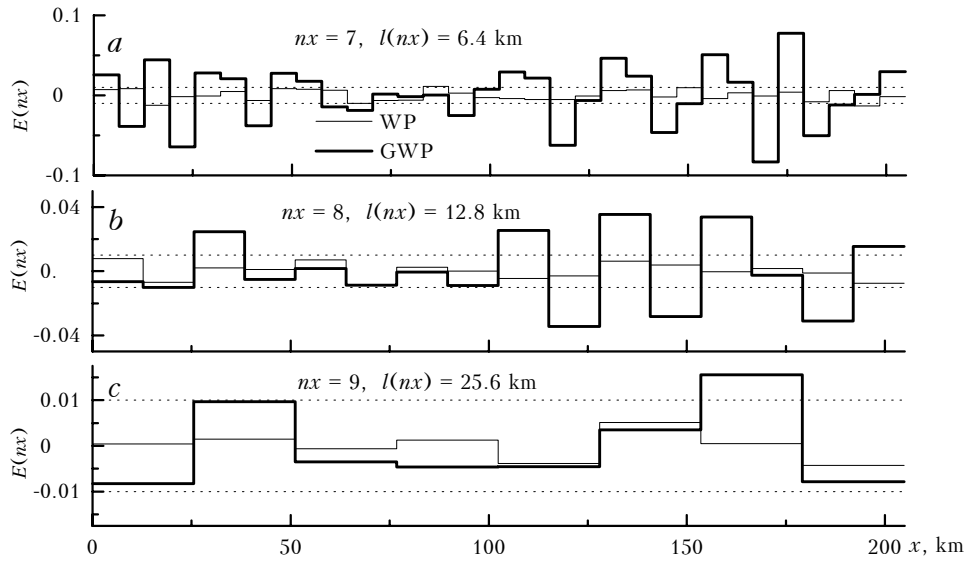


FIG. 7. Dependence of the horizontal transport  $E(nx)$  for two cloud models on the length of averaging interval  $l(nx)$  at  $\xi_0 = 60^\circ$ ,  $A_s = 0$  (ocean), and  $\omega_0 = 0.99$ ;  $l(nx) = 6.4$  (a),  $12.8$  (b), and  $25.6$  km (c).

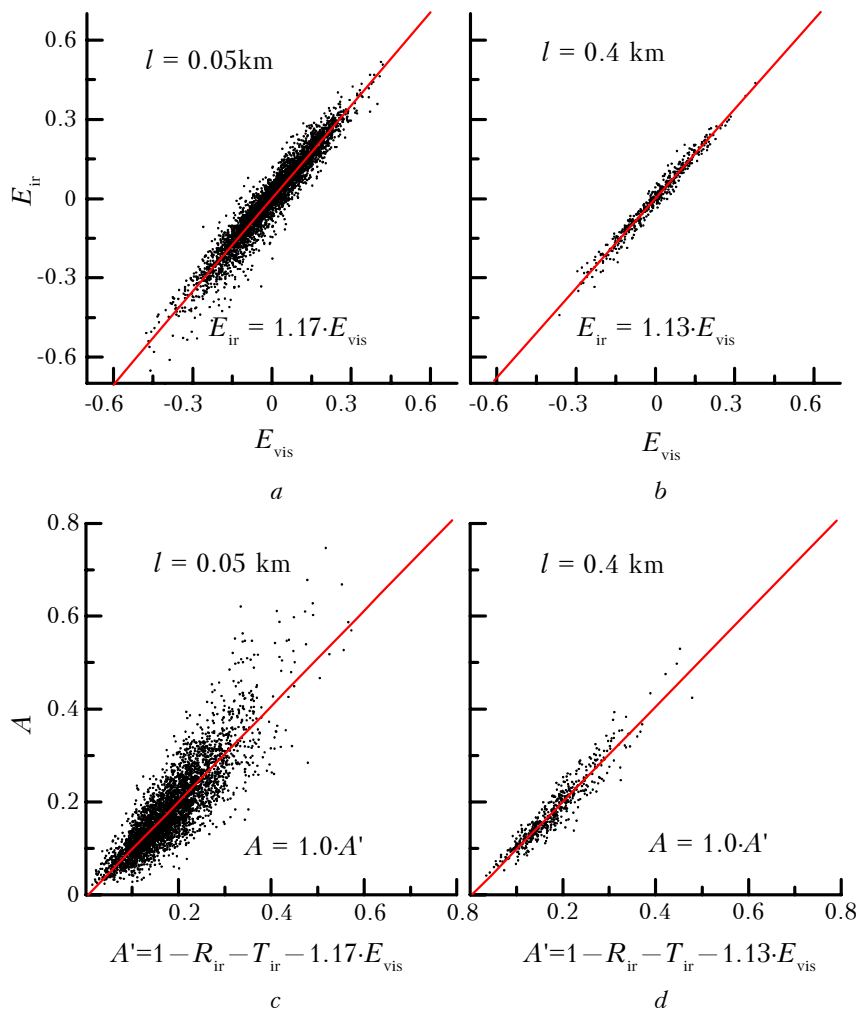


FIG. 8. Linear regression between  $E_{ir}$  and  $E_{vis}$  (a, b) and absorption  $A$  as a function of the refined estimate of absorption  $A'$  (c, d) for different spatial resolution  $l = 0.05$  (a, c) and  $0.4$  km (b, d) at  $\xi_0 = 60^\circ$ ,  $A_s = 0$  (ocean), and  $\omega_0 = 0.99$ .

The second approach that permits one to study small-scale (~0.1 km) variations of absorption is based on<sup>8-9</sup>:

- simultaneous measurements of the albedo and transmission in the visible (vis) and nearly-IR (ir) spectral ranges. Because  $A_{\text{vis}}(x) = 0$ ,  $E_{\text{vis}}(x) = 1 - R_{\text{vis}}(x) - T_{\text{vis}}(x)$ ;

- linear regression between  $E_{\text{vis}}(x)$  and  $E_{\text{ir}}(x)$ , that is,  $E_{\text{ir}}(x) = b E_{\text{vis}}(x)$ . To calculate the coefficient  $b$ , mathematical modeling is used.

Absorption by the clouds is calculated from the formula

$$A'_{\text{ir}}(x) = 1 - R_{\text{ir}}(x) - T_{\text{ir}}(x) - E_{\text{ir}}(x) \approx \\ \approx 1 - R_{\text{ir}}(x) - T_{\text{ir}}(x) - b E_{\text{vis}}(x). \quad (5)$$

We can assume that if for fixed  $F_{\text{vis}}(x)$  the spread of  $E_{\text{ir}}(x)$  about the regression straight line  $E_{\text{ir}}(x) = b E_{\text{vis}}(x)$  is insignificant, the reliable estimate of the absorption  $A_{\text{ir}}(x)$  can be calculated with the help of Eq. (5). Results shown in Fig. 8 confirm this assumption. At  $l(2^0) = 0.05$  km, a large spread of  $E_{\text{ir}}$  is observed about the regression straight line (Fig. 8a), therefore, the true absorption  $A_{\text{ir}}(x)$  and the reconstructed absorption  $A'_{\text{ir}}(x)$  differ strongly (Fig. 8c).

After averaging of  $E_{\text{vis}}(x)$ ,  $E_{\text{ir}}(x)$ , and radiative characteristics entering formula (5), the reliable estimate of the absorption can be obtained for the fragments of the realization of length  $l(2^3) = 0.4$  km (Fig. 8d). This is explained mainly by the fact that after averaging the variances of  $E_{\text{vis}}$  and  $E_{\text{ir}}$  decrease and hence the spread of  $E_{\text{ir}}(x)$  about the regression straight line (Fig. 8b) decreases.

Thus, the reliable estimate of the cloud absorption  $A_{\text{GWP}}$  can be obtained with a maximum spatial resolution of ~0.4 km from simultaneous measurements of the albedo and transmission in the visible and near-IR spectral ranges. Recall that for the WP model this approach allows one to study smaller-scale (~0.05 km) fluctuations of absorption.<sup>7-9</sup> Hence, the stochastic cloud top geometry decreases by approximately an order of magnitude the maximum spatial resolution with which we can obtain the cloud absorption.

We note that the conclusions drawn in this and preceding sections correspond to the *maximum* influence of the stochastic geometry of inhomogeneous stratocumulus.

## 5. CONCLUSION

Comparative analysis of the sensitivity of the statistical characteristics of albedo, transmission, absorption, and horizontal transport of marine stratocumulus to the fluctuations of the liquid water content and cloud top altitude is made. It is shown that the contributions of the stochastic geometry and inhomogeneous internal structure to the mean albedo and transmission are *comparable*. The mean absorption of stratocumulus depends *weakly* on the stochastic

cloud top geometry. Fluctuations of the liquid water content and cloud top altitude affect approximately *identically* the variance of the albedo, transmission, and absorption. The variance of the horizontal transport is caused primarily by the fluctuations of the cloud top altitude. The correlation radius of the horizontal transport is several hundreds of meters and depends weakly on the choice of the cloud model.

The influence of the stochastic cloud top geometry of stratocumulus with inhomogeneous internal structure on the reconstruction accuracy of the cloud absorption is studied. The net fluxes measured in one spectral interval at the cloud layer top and bottom can be used to study only large-scale (~30 km) variations of absorption. Irregular cloud top geometry deteriorates the maximum spatial resolution of low-frequency fluctuations of absorption approximately *five* times. Synchronous measurements of fluxes in the visible and near-IR spectral ranges can be used to study the small-scale (~0.4 km) variations of absorption. The stochastic geometry deteriorates approximately *by an order of magnitude* the accuracy of reconstruction of the high-frequency fluctuations of cloud absorption.

Thus, the stochastic cloud top geometry of stratocumulus may affect significantly the mean and the variance of the albedo and transmission as well as the absorption variance. Therefore, in parametrization of the radiative characteristics of these clouds for the models of global circulation of the atmosphere, analysis, and interpretation of field measurements not only the inhomogeneous internal structure, but also the stochastic top should be considered.

The work was supported by the DOE's ARM Program (contract No. 350114-A-Q1) and Russian Foundation for Basic Researches (grant No. 96-05-64275).

## REFERENCES

1. R.F. Cahalan and J.B. Snider, Remote Sens. Environ. **28**, 95-107 (1989).
2. A. Marshak, A. Davis, R.F. Cahalan, and W.J. Wiscombe, Phys. Rev. **E49**, 55-79 (1994).
3. S.M. Prigarin and G.A. Titov, Atmos. Oceanic Opt. **9**, No. 7, 629-635 (1996).
4. R.F. Cahalan, W. Ridgway, W.J. Wiscombe, et al., J. Atmos. Sci. **51**, No. 16, 2434-2455 (1994).
5. R.F. Cahalan, W. Ridgway, W.J. Wiscombe, et al., J. Atmos. Sci. **51**, 3776-3790 (1994).
6. R.F. Cahalan, Nonlin. Proc. Geophys. **1**, 156-167 (1994).
7. G.A. Titov, Atmos. Oceanic Opt. **9**, No. 10, 825-832 (1996).
8. G.A. Titov, *ibid.* **9**, No. 10, 833-838 (1996).
9. G.A. Titov, J. Atmos. Sci. (1996) (in press).
10. S.Yu. Popov and G.A. Titov, Atmos. Oceanic Opt. **7**, No. 3, 154-157 (1994).
11. G.A. Titov and E.I. Kas'yanov, Atmos. Oceanic Opt. **8**, No. 12, 1028-1034 (1995).



12. K.Ya. Kondrat'ev, ed., *Albedo and Angular Characteristics of Reflection from the Underlying Surface and Clouds* (Gidrometeoizdat, Leningrad, 1981), 232 pp.
13. G.I. Marchuk, G.A. Mikhailov, N.A. Nazarialiev, et al., *Monte-Carlo Method in Atmospheric Optics* (Nauka, Novosibirsk, 1976), 280 pp.
14. D. Deirmendjian, *Electromagnetic Scattering on Spherical Polydispersions* (Elsevier, Amsterdam; American Elsevier, New York, 1969).
15. R.G. Timanovskaya and E.M. Feigel'son, *Meteorol. Hidrol.*, No. 11, 44–51 (1970).
16. G.A. Titov, *Atmos. Oceanic Opt.* **9**, No. 1, 1–6 (1996).
17. J.S. Bendat and A.G. Piersol, *Random Data: Analysis and Measurement Procedures* (Wiley, New York, 1971).