Influence of the sea foam coverage on the power of lidar return at sensing the sea surface along slant paths

M.L. Belov

R&D Institute of Radioelectronics and Laser Technologies at N.E. Bauman Moscow State Technical University, Moscow

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Formula has been derived for description of the effect of foam on the lidar return signal from sea surface sounded along slant paths. Results calculated by this formula are compared with the results calculated numerically using different models of the sea surface coverage with foam. It is shown that the foam covering the sea surface strongly affects the power of lidar returns. The effect of foam essentially depends on the sounding angles and models of the foam used.

The influence of the sea foam on the power of the laser signal received by a lidar at a continuous irradiation of the sea surface from an aircraft (when shading of some surface elements by others can be neglected) was studied in Refs. 1 and 2.

Below we consider slant (almost horizontal) paths characteristic of the coastal sea surface sensing. Formula has been derived which describes the influence of the sea foam on the power of the lidar return signal under conditions of strong shading of some sea surface elements by others. Results calculated by this formula are compared with the results calculated numerically using different models of the sea surface coverage with foam.

The mean power P received by lidar in sensing the sea surface partially covered with foam can be presented as (Ref. 1)

$$P = (1 - C_{\rm f})P_{\rm s} + C_{\rm f}P_{\rm f}, \tag{1}$$

where $P_{\rm s}$, $P_{\rm f}$ are the mean powers of the signals returned from the sea surface free of foam and totally covered with the foam; $C_{\rm f}$ is the fraction of the sea surface covered with foam.

Assume that an IR sounding radiation is used, which is strongly absorbed by water and thus most of the return power is generated due to mirror reflection from the air–water interface, while the portion of light diffusely reflected by water column can be neglected. The model of sea roughness is represented as a Gaussian random process (Gaussian distribution of sea surface slopes is close to that observed experimentally[3]) with local surface elements producing specular returns. The foam spots are considered as Lambert reflectors (see, for example, Refs. 4–5) located on the wave slopes and the distribution of the slopes of foam spots is considered to be the same as of the wave slopes[5] (i. e., the model of the sea surface totally covered with foam is represented as a Gaussian random process with a local Lambert reflection of surface elements).

The integral formulas for $P_{\rm s}$ and $P_{\rm f}$ under conditions of strong shading of some sea surface elements by other ones have been obtained in Ref. 6 (it is assumed that source and receiver and their optical axes are in the same XOZ-plane):

$$P_{\rm s} = V^2 \frac{q^4}{4q_z^4} \int_{-\infty}^{\infty} W_a(\zeta;\theta_{\rm s},\theta_{\rm r})\,{\rm d}\zeta \int_{S_0} {\rm d}\mathbf{R}_0 E_{\rm s}^n(\mathbf{R}'_{0\zeta}) E_{\rm r}^n(\mathbf{R}''_{0\zeta}) \times$$

$$\times W\left(\gamma_x = -\frac{q_x}{q_z} - \frac{R_{0x}}{q_z}T;\ \gamma_y = -\frac{R_{0y}}{q_z}s\right), \tag{2}$$

$$P_{\rm f} \cong \frac{A}{\pi} \int_{-\infty}^{\infty} {\rm d}\zeta \int_{-\infty}^{\infty} W_a(\zeta,\gamma;\theta_{\rm s},\theta_{\rm r})\,{\rm d}\gamma \times$$

$$\times \int_{S_0} \frac{{\rm d}\mathbf{R}_0}{n_z} E_{\rm s}(\mathbf{R}'_{0\zeta}) E_{\rm r}(\mathbf{R}''_{0\zeta}), \tag{3}$$

where

$$s = \frac{1}{L_{\rm s}} + \frac{1}{L_{\rm r}};\quad T = \frac{\cos^2\theta_{\rm s}}{L_{\rm s}} + \frac{\cos^2\theta_{\rm r}}{L_{\rm r}};$$

$$E_{\rm s}(\mathbf{R}'_{0\zeta}) = E_{\rm s}^n(\mathbf{R}'_{0\zeta})(\mathbf{n}\mathbf{m}_{\rm s});\quad E_{\rm r}(\mathbf{R}''_{0\zeta}) = E_{\rm r}^n(\mathbf{R}''_{0\zeta})(\mathbf{n}\mathbf{m}_{\rm r});$$

$$q_x = \sin\theta_{\rm s} + \sin\theta_{\rm r};\quad q_z = -(\cos\theta_{\rm s} + \cos\theta_{\rm r});$$

$$q^2 = q_x^2 + q_z^2;$$

$$\mathbf{R}'_{0\zeta} = \left\{[R_{0x}\cot\theta_{\rm s} - \zeta]\sin\theta_{\rm s}, R_{0y}\right\};$$

$$\mathbf{R}''_{0\zeta} = \left\{[R_{0x}\cot\theta_{\rm r} - \zeta]\sin\theta_{\rm r}, R_{0y}\right\};$$

if source and receiver are located on one and the same side from the normal to the surface S_0, then

$$W_a(\zeta,\gamma;\theta_{\rm s},\theta_{\rm r}) \cong W_a(\zeta;\theta_{\rm s},\theta_{\rm r})\Theta(\cot\theta - \gamma_x)W(\gamma),$$

$$W_a(\zeta;\theta_{\rm s},\theta_{\rm r}) \cong W(\zeta)\exp\left\{-\Lambda(a)\int_{\zeta}^{\infty} W(\zeta'){\rm d}\zeta'\right\};$$

$$a = \frac{\cot\theta}{(\overline{\gamma_x^2})^{1/2}};$$

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$$\Lambda\left(\frac{\cot\theta}{(\overline{\gamma_x^2})^{1/2}}\right) = \tan\theta \int\limits_{\cot\theta}^{\infty} (\gamma'_x - \cot\theta) W(\gamma'_x) d\gamma'_x;$$

$$\theta = \max(\theta_s, \theta_r);$$

$\Theta(x)$ is a step function determined as follows:

$$\Theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases};$$

Λ is the parameter characterizing the degree of shading. Under strong shading ($\Lambda \gg 1$)

$$\Lambda \approx (\overline{\gamma_x^2})^{1/2} / \sqrt{2\pi}\cot\theta,$$

where ζ, $\boldsymbol{\gamma} = (\gamma_x, \gamma_y)$, $\mathbf{n} = (n_x, n_y, n_z)$ are the random height, slope vector, and the unit vector normal to the sea surface; $E^n_{s,r}(\mathbf{R})$ are the values of illumination in the beam cross section from the actual and an apparent (with parameters of the receiver) sources; $\mathbf{m}_{s,r}$ are the unit vectors defining the emission and reception directions; $W(\zeta)$, $W(\boldsymbol{\gamma}) = (\gamma_x, \gamma_y)$ are the distribution functions for surface heights and slopes; V^2 is the Fresnel reflection coefficient of foam-free sea surface; A is the albedo of foam-covered surface region; $L_{s,r}$ are the slant distances from the source and receiver to the surface; $\theta_{s,r}$ are the angles between normal to the plane $z = 0$ and optical axes of the source and receiver, respectively; $W(\gamma_x)$, $\overline{\gamma_x^2}$ are the distribution function and the slope variance of rough sea surface along the X direction.

In the integral expressions (2) and (3), the integration is done over the surface S_0 (the projection of a randomly rough sea surface onto the plane $z = 0$).

It is possible to evaluate the integrals in Eqs. (2) and (3) with some approximations and derive formulas for the power returned from the sea surface free of foam (randomly rough locally specular reflecting surface) and from the surface completely covered with foam (randomly rough locally Lambert surface) under conditions of strong shading ($\Lambda \gg 1$).

In a monostatic optical arrangement of sensing ($\theta_s = \theta_r = \theta$, $L_s = L_r = L$) the analytical formulas for the characteristics P_s, P_f look as follows (in the case when the height of the sensing laser beam over the sea surface is much greater than the root-mean-square wave height; as usual, it is satisfied that the root-mean-square wave slope be much larger than the source divergence angle and the receiver field of view):

$$P_s \cong \frac{q^4 V^2 a_s a_r}{q_z^4 \tilde{L}^4 (\tilde{C}_s + \tilde{C}_r)} W\left(\gamma_x = -\frac{q_x}{q_z}, \gamma_y = 0\right) \times$$

$$\times \frac{[1 - \exp(-\Lambda(a)]}{\Lambda(a)}, \qquad (4)$$

$$P_f \cong \frac{A a_s a_r \omega}{\tilde{L}^4 (\tilde{C}_s + \tilde{C}_r)} \frac{[1 - \exp(-\Lambda(a)]}{\Lambda(a)}, \qquad (5)$$

where

$$\tilde{L} = L - \mu\sin\theta; \quad \mu = \zeta_m \tan\theta; \quad \zeta_m = \frac{\Lambda\sigma F(\alpha)}{\sqrt{2\pi}}; \quad \alpha = \frac{\Lambda^2}{4\pi};$$

$$F(\alpha) \approx \left\{\frac{1}{2\alpha}\left[\ln\alpha - \ln\ln 2\alpha - \ln\left(1 - \frac{\ln\ln\alpha}{\ln\alpha}\right)\right]\right\}^{1/2}.$$

For a clear atmosphere:

$$\tilde{C}_{s,r} = (\alpha_{s,r}\tilde{L})^{-2}; \quad a_r = \pi r_r^2, \quad a_s = \frac{P_0}{\pi\alpha_s^2};$$

σ is the root-mean-square wave height; $\alpha_{s,r}$ are the source radiation divergence angles and the receiver field of view, respectively; P_0 is the source emission power; r_r is effective radius of the receiving aperture.

In the approximation of the isotropic sea roughness (slope variances of the rough sea surface along the X and Y axes are $\overline{\gamma_x^2} \cong \overline{\gamma_y^2} = \overline{\gamma^2}$), the approximation for the parameter ω for conditions of strong shading ($\cot\theta \ll (\overline{\gamma^2})^{1/2}$) has the form:

$$\omega \cong \exp\left(\frac{1}{4\overline{\gamma^2}}\right)\left[0.5\cos^2\theta\left(\frac{1}{2\overline{\gamma^2}}\right)^{1/4} W_{-1/4,-1/4}\left(\frac{1}{2\overline{\gamma^2}}\right) + \right.$$

$$+\sin\theta\cos\theta\frac{1}{\sqrt{\pi}} W_{-1/2,-1/2}\left(\frac{1}{2\overline{\gamma^2}}\right) +$$

$$\left. +\sin^2\theta(\overline{\gamma^2})^{1/2} 2^{-7/4} W_{-3/4,-3/4}\left(\frac{1}{2\overline{\gamma^2}}\right)\right], \qquad (6)$$

where $W_{m,n}(x)$ is the Whittaker function.

For the general case of anisotropic sea roughness, the expression for ω is more cumbersome with a power series of the parameter characterizing the roughness anisotropy:

$$\omega \cong \frac{1}{4\sqrt{\pi}(\overline{\gamma_x^2}\,\overline{\gamma_y^2})^{1/2}} \exp\left(\frac{1}{2\overline{\gamma_y^2}}\right)\left\{\cos^2\theta \sum_{k=0}^{\infty} (-1)^k \frac{\delta^k}{k!}\left(\frac{1}{4\overline{\gamma_y^2}}\right)^k \times\right.$$

$$\times\Gamma(k + 0.5) G^{20}_{12}\left(\frac{1}{2\overline{\gamma_y^2}}\middle| \begin{matrix} 0.5 \\ -k - 0.5; 0 \end{matrix}\right) + 2\cos\theta\sin\theta \times$$

$$\times\sum_{k=0}^{\infty} (-1)^k \frac{\delta^k}{k!}\left(\frac{1}{4\overline{\gamma_y^2}}\right)^k \Gamma(k+1) G^{30}_{23}\left(\frac{1}{2\overline{\gamma_y^2}}\middle| \begin{matrix} 0; 0.5 \\ -k - 1; 0; 0 \end{matrix}\right) + \sin^2\theta \times$$

$$\left.\times\sum_{k=0}^{\infty} (-1)^k \frac{\delta^k}{k!}\left(\frac{1}{4\overline{\gamma_y^2}}\right)^k \Gamma(k + 1.5) G^{20}_{12}\left(\frac{1}{2\overline{\gamma_y^2}}\middle| \begin{matrix} 0.5 \\ -k - 1.5; 0 \end{matrix}\right)\right\}, \qquad (7)$$

where $\delta = 2\left(\frac{\overline{\gamma_y^2}}{\overline{\gamma_x^2}} - 1\right)$; $\Gamma(k)$ is the gamma function; $G^{m,n}_{p,q}\left(z\middle| \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix}\right)$ is the Meijer function.

For calculating P_s and P_f the results were used, as in Ref. 2, of statistical processing of the observation data (in different climatic zones of the world ocean) on the relative areas of sea foam coverage C_f. As a result of statistical data processing, empirical relations were derived that strongly depend on geography and sea surface temperature T_w. These relations in terms of three model dependences of C_f on the near-water wind speed U are presented in Table 1.[7,8]

The parameter U_w in the third line of the table is a certain value of near-water wind speed at which the foam starts to form. The parameter T_w is the sea surface temperature governing U_w value in accordance with the empirical formula presented in Table.

The foam reflection was measured in Ref. 9, where it was shown that the foam albedo A is ≈ 0.5 in the wavelength region from 0.5 to 1 μm.

Figure 1 shows the variations of returned power $P = (1 - C_f)P_s + C_f P_f$ for different wind speeds U. The calculations were made for the case of monostatic sensing and the foam models presented in Table 1, assuming the following model parameters: $V^2 = 0.02$; $A = 0.5$; $P_0 = 1$ W; $\alpha_s = 0.5$ mrad; $\alpha_r = 1$ mrad.

For the range of the near-water wind speeds, for which the calculations were made (6–18 m/s), the root-mean-square wave slope varies from 0.14 to 0.24 (Ref. 3), which at $\theta = 89°$ corresponds to the values of Λ (the parameter that characterises shading) ~ from 2.5 to 5, and at $\theta = 89.5°$ to the values from 5 to 10.

In the calculation by the analytical equation (5), sea surface was assumed smoothly rough ($(\overline{\gamma^2})^{1/2} \ll 1$), and the parameter ω was approximated by the following expression (based on the asymptotic series for the Whittaker functions ($W_{m,n}(x)$):

$$\omega \approx 0.5\cos^2\theta + 2\sqrt{\frac{\overline{\gamma^2}}{2\pi}}\sin\theta\cos\theta + 0.5\overline{\gamma^2}\sin^2\theta.$$

Variances of the surface slopes $\overline{\gamma^2_{x,y}}$ were calculated by the Cox and Munk formulas, and the root-mean-square height was calculated by the formula (Ref. 10): $\sigma \cong 0.016U^2$.

The results on P calculated by the formulas (1), (4), and (5) are sown by solid curves. Dots stand for numerical calculations by the formulas (1)–(3). Dashed curves are the results on P calculated ignoring the shading effect (using the results from Refs. 1 and 2).

It is seen from the Fig. 1 that the foam on the sea surface strongly influences the power of lidar return signals at almost horizontal sounding paths. The return power is noticeably different from zero only starting from a certain (characteristic of a foam model) value of the near-water wind speed, at which foam forms on the sea surface. With the increase of the near-water wind speed, the foam-covered area grows as well. But the degree of influence of the near-surface wind speed on the return signal essentially depends on the foam model (i.e., on the sea surface temperature, geographical location of the experiment, etc.).

Neglect of the shading effect in the case of sounding the sea surface along almost horizontal paths results in a strong (an order of magnitude and more) overestimation of the values of return signal power (this is clearly seen from comparison of the dashed and solid curves in the Fig. 1).

If all the conditions, for which the formulas (4) and (5) have been derived, are fulfilled, results on P calculated by the analytical formulas (solid curves) well agree with the numerical simulations (dots),

Dependence of C_f on the near-water wind speed

Number of the model	T_w, °C	U, m/s	$C_r \cdot 10^2$
1	6–22	9–23	$C_f = 0.009U^3 - 0.3296U^2 + 4.54U - 21.33$
2	3	9–16	$C_f = 0.189U - 1.285$
3	> 14	$U > U_w$	$C_f = 2.95 \cdot 10^{-4}U^{3.52}$; $U_w = 3.36 \cdot 10^{-0.00309T_w}$

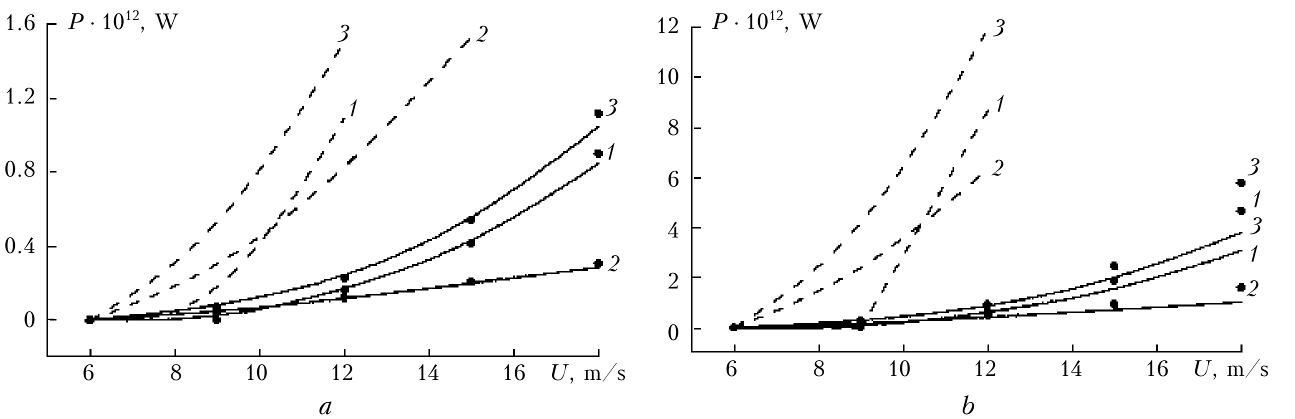

Fig. 1. Dependence of P on the near-surface wind speed: $\theta = 89°$, $L = 10$ km (a); $\theta = 89.5°$, $L = 5$ km (b).

(see Fig. 1*a*). If some conditions are not fulfilled, then agreement between calculations by analytical formulas with the numerical calculations is still satisfactory, see Fig. 1*b* (for Fig. 1*b*, for most part of the sensing path, the root-mean-square wave height is comparable with the laser beam height above the sea surface).

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