THEORETICAL MODEL OF RADIONUCLIDES SPREAD AFTER POSSIBLE INDUSTRIAL EMERGENCY

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The trajectory model of pollution transport in the atmospheric boundary layer is described, which takes into account both horizontal and vertical turbulent diffusion of a pollutant as well as dry and humid sedimentation onto the Earth's surface. The model can be applied to calculation of spread of any pollutant from a point source as well as radionuclides after possible emergency emissions from repositories of radioactive waste and nuclear power plants. The calculational results are presented and analyzed.

Nowdays state of the art of ecological environmental studies is characterized by a greater attention paid to the problem of optimal evaluation of spatial spread of toxic pollutants emitted into the atmosphere as a result of industrial emergencies. In this case the emissions of radioactive decay products, or radionuclides (isotopes of plutonium, cesium, ruthenium, zirconium, niobium, etc.[1]), which are, as a rule, the results of emergencies at nuclear power plants, are the most serious hazard. An important feature of such emergencies is possible emission in the form of volley, as took place, for example, in emergency at Chernobyl nuclear power plant on April 26, 1986 (see Refs. 2 and 3) and Siberian Chemical Plant, Tomsk, on April 6, 1993 (see Ref. 4).

Since the existing network of ecological stations is of much lower density than it is required for ecological monitoring of atmospheric pollution, for ecological safety it is necessary to be able to calculate the pollution spread from a point emission source and concentration of pollutants at any point of the space. To this end, various theoretical models of pollution transport being the basis for a computer software (see, for example, Refs. 5 and 6) are usually used. The present paper is devoted to consideration of one of such models, namely, the trajectory model of pollution transport.

The model equation for the atmospheric boundary layer is derived by integrating the equation of pollution transport[5–7] over altitude from $z = 0$ to $z = H$ (boundary layer altitude)

$$\frac{\partial s}{\partial t} = u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z} + \frac{\partial w_a s}{\partial z} =$$

$$= \frac{\partial}{\partial x} k_1 \frac{\partial s}{\partial x} + \frac{\partial}{\partial y} k_1 \frac{\partial s}{\partial y} + \frac{\partial}{\partial z} k \frac{\partial s}{\partial z} + \varepsilon_a \qquad (1)$$

with the following boundary conditions:

$$\text{for } z = 0\ (z = z_0)\quad k\frac{\partial s}{\partial z} - w_a s = \beta s - f_0\,,$$
$$\text{for } z = H(H_1)\quad w = w_H\,,\quad k = k_H\,, \qquad (2)$$

where s is the exchange concentration of a pollutant; w_a is the vertical speed of a given pollutant a; β is the rate of dry absorption of a pollutant by the Earth's surface; k_1 is the coefficient of turbulence in a horizontal motion; k is the coefficient of turbulence in a vertical motion; k_H and w_a are the coefficient of turbulence and air vertical speed at the altitude $z = H$; ε_a is the rate of creation or annihilation of the admixture at which a pollutant either enters or leaves the boundary layer due to emission from sources in the layer, sedimentation with the atmospheric precipitation, and chemical transformations; f_0 is the rate of pollutant emission from a ground–based source.

As a result of integrating the equation of pollution transport (1) with regard to boundary conditions (2) after introducing the values averaged over the boundary layer following the expressions

$$\bar{s} = \frac{1}{H}\int_0^H s(z)\,\mathrm{d}z\,,\quad \bar{u} = \frac{1}{H}\int_0^H u(z)\,\mathrm{d}\varepsilon\,,$$

$$\varepsilon_a = \frac{1}{H}\int_0^H \varepsilon_a(z)\,\mathrm{d}z\,,\ \text{etc.},$$

we derive the following model equation for the boundary layer (omitting the bar atop the symbols, which marks the layer–averaged values):

$$\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} = -\sigma s + \frac{\partial}{\partial x} k\frac{\partial s}{\partial x} + \frac{\partial}{\partial y} k\frac{\partial s}{\partial y} + \varphi\,, \qquad (3)$$

where

$$\varepsilon_a = F - \sigma_2 s - \sigma_3 s\,;$$

$$\varphi = \varphi(x, y, t) = F(x, y, t) + \frac{1}{H} f_0(x, y, t)\,;$$

$$\sigma = \sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 + \sigma_5 + \sigma_6\,;$$

F is the rate of a pollutant emission from high–altitude sources; $\sigma_1 = \alpha_0\beta/H$; $\sigma_2 = \alpha^* I$; I is the precipitation intensity; α^* is the coefficient; σ_3 is the coefficient taking into account the pollutant chemical transformations;

$$\sigma_4 = (\alpha_0 - \alpha_H)\,k_H / H^2\,;\ \sigma_5 = \alpha_H w_H / H\,;\ \sigma_6 = \alpha_H w_a / H\,;$$

$\alpha_0 = s_0/s$; $\alpha_H = s_H/s$, where s_0 and s_H are the values of s at $z = 0$ and $z = H$; σ_4, σ_5, and σ_6 are semiempirical coefficients found from the data on vertical profile of pollution.

Depending on power of a pollution source as well as on meteorological conditions, equation (3) is integrated over

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internal (near–ground) boundary layer (IBL) of height $H = H_1 = 0.1\ k_{1m}/l$ for a low power emission or over the planetary boundary layer (PBL) of height H = HML (HML is the height of mixing layer) for a high power emission. The height in the latter case is determined from the relationship $H = 0.2V_*/l$ (see Ref. 8) or is taken to be equal to HML = k_{1m}/l. Here $l = 2\omega \sin\varphi$ is the Coriolis parameter, k_{1m} is the turbulence coefficient at the altitude of 1 m above the Earth's surface, V_* is the dynamic speed.

Having obtained the data on wind speed at an altitude z_1 (the level of a wind vane of 10–12 m), the parameters of IBL are determined, according to the theory, by the following relations (for nearly neutral stratification):

$$V_* = V(z_1)\,\kappa/[\ln(z_1+z_0)/z_0];\ V(z) = (V_*/\kappa)\ln[(z+z_0)/z_0]\ ,$$

$$k = k(z) = k_2 + a\,z\ ,\ k_2 = \kappa\,V_*\,z_0\ ,\ \overline{V} = \frac{V_\kappa}{k}\ln\left(\frac{H_1}{z_0} - 1\right), \quad (4)$$

where κ is the Karman constant and z_0 is the parameter of roughness. In so doing the wind direction in IBL is considered to be constant. The pollution spread is computed for short (about 100 km) distances (local model).

For high power sources, PBL serves as a boundary layer. The wind velocity and direction changes in this layer are determined according to the theory of planetary boundary layer and using the value of geostrophic wind.[9] In this case the pollution spread is computed for long (about 1000 km) distances (regional model) with simultaneous determination of the particle trajectories from sources.[6]

In both cases the source power is taken to be constant during the period of integration.

EXAMPLES OF POLLUTION TRANSPORT COMPUTATION FOR SHORT DISTANCES

The computer program based on a model of pollution transport in IBL was created in order to estimate the degree of air pollution at short distances (up to 100 km) from an isolated ground–based source of pollution. The emissions from ground–based repositories of nuclear waste having constant power can be considered as such sources. In computations the influence of different model parameters on pollutant concentration was also estimated. The model can be applied to any pollutant including radioactive ones.

When integrating Eq. (3) the process is considered to be stationary ($\partial s/\partial t = 0$, f_0 = const); wind velocity is taken at the level of wind vane. In this case, if x axis is directed along the vector, Eq. (3) takes the form:

$$u\,\frac{\partial s}{\partial x} = -\sigma\,s + \frac{\partial}{\partial y}\,k_1\,\frac{\partial s}{\partial y} + \varphi\ . \quad (5)$$

A solution to this equation, similar to that proposed by Berlyand in Ref. 3, can be presented as follows:

$$s(x,\ y) = P(x,\ y)\ s'(x)\ . \quad (6)$$

For variables P and s' we derive the equations:

$$\frac{\partial P}{\partial x} = \frac{k_1}{u}\,\frac{\partial^2 P}{\partial y^2}\ , \qquad u\,\frac{\partial s'}{\partial x} = -\sigma\,s' + \frac{\varphi}{P}\ . \quad (7)$$

One can see that the solution to the former equation with $k = a^2ux$ and under the condition that $P \to 0$ at $y \to \pm\infty$ has the form

$$P(x,\ y) = \frac{1}{\sqrt{2\pi}\,\sigma_y}\exp(-y^2/2\sigma_y^2) \quad (8)$$

for $\sigma_y = a\,x$ ($a \approx 10^{-1} - 10^{-2}$), and the solution to the latter equation is as follows:

$$s'(x) = \exp(-\sigma\,x/u)\left(s'^0 + \int_0^x \exp(-\sigma\,x'/u)\,\frac{\varphi\,x'}{u\,P}\,\mathrm{d}\,x'\right), \quad (9)$$

where s'^0 is the value of s' at $x = 0$, $s'^0 = s^0/P(0)$, and s^0 is the value of s at $x = 0$.

TABLE I. Pollutant concentration (μg/m³) per unit air column in internal boundary layer of H_1 in height on the plume axis at different distances x from a point ground–based source with the intensity of upward emission $f_0 = 0.2\cdot10^{-3}$ g/(m²s) for different values of model parameters.

Parameters							x, km				
V_1, m/s	z_0, m	V_*, m/s	H_1, m	$\overline{V}$, m/s	s^0, μg/m³	σ_y	0.1	1	10	50	100
5	0.1	0.40	14	4.2	3420	0.01 x	1340	116	2.5	0	0
						0.1 x	134	12	0.1	0	0
	0.5	0.61	167	7.1	169	0.01 x	68	7	0.6	0	0
						0.1 x	7	0.7	0.1	0	0
10	0.1	0.81	29	9.8	702	0.01 x	279	27	2	0.1	0
						0.1 x	28	2.7	0.2	0	0
	0.5	1.21	234	16.4	52	0.01 x	21	2.1	0.2	0	0
						0.1 x	2.1	0.2	0	0	0

Tabulated in Table I are the computational results for a pollutant concentration at a distances up to 100 km from a ground–based point source with the power $f_0 = 0.2\cdot10^{-3}$ g/(m²·s) for different values of model parameters $V(z_1)$, z_0, and $s^0 = f_0 d/\overline{V}H_1$, where $d = \Delta y$

and $\sigma_y = 0.01\ x$ and $0.1\ x$. The dependence of a pollutant concentration on these parameters is clearly seen from the data tabulated.

The model for computation of pollution transport for short distance (local model) was applied to estimate the spread of air polluted with radionuclides after an emergency that took place at the Siberian Chemical Plant, Tomsk–7 (now Seversk), on April 6, 1993 (see Ref. 4). In this computations we took that radionuclides emitted into the air spread in the same way as any other "weightless" gaseous pollutant.

As follows from the meteorological data, at the emergency the velocity of south–east wind in Tomsk region was 5 m/s at the level of wind vane; it was lightly sleeting. In our calculations we assumed $z_0 = 0.5$ cm (that corresponds to rugged terrain with trees and buildings) and $\beta = 1$ cm/s. Calculated height of the IBL amounted to 109 m. The process was assumed to be stationary during a day.

Table II presents the data on γ–radiation dose rate[1] (in μR/h) per unit air column in IBL of 109 m height on the plume axis for different distances from the emergency point (the town of Seversk) in the nort–east direction with the dose rate in the emergency zone $J_0 = 1100$ μR/h. The data were calculated using the model with regard for turbulence, scattering, surface absorption, and sedimentation with precipitation

TABLE II. Gamma–radiation dose rate.

x, km	0	0.5	1	5	10	20	50	100
J, μR/h	1100	730	360	69	32	14.3	3.8	1.0

It follows from the data tabulated that the air radioactivity rapidly decreases with the distance from the emergency point. So at 1 km distance from the source the radioactivity it was 360 μR/h, or 33% of the initial one, and at a distance of 20 km from the source it was 14.3 μR/h, or 1.3%, at a distance of 100 km only traces of radioactivity (1 μR/h, or 0.1% of the initial value) occurred.

The radioactivity values above 12 μR/h (this value corresponds to natural background of the Earth's surface) were marked from our calculations at a distances up to 22 km from the emergency point. Radioactivity rapidly decreases along the normal to the plume axis. For example, for $x = 10$ km at a distance of 0.2 km from the plume axis the radioactivity amounted to about 10% of that on the plume axis.

The values of γ–radiation dose rate computated using our model do not contradict those obtained from air survey.[4]

EXAMPLE OF POLLUTION TRANSPORT COMPUTATION FOR LONG DISTANCES

The model of a distant transport of pollution from a point source is the trajectory one,[6,10] therefore trajectories of particles from sources are first calculated and then the equation of pollution transport is integrated within a time period up to 24 h with the time step of 2 h.

The mean wind velocities in PBL are calculated according to the theory of PBL[8] with regard to the wind change with altitude. Trajectories are calculated for the air volumes moving in the PBL. The transport equation is integrated along the trajectory. At any point of the trajectory, x axis is directed along the trajectory, whereas y axis is directed normally to it. The equation of pollution transport is written in the form

$$\frac{d_r s}{d t} = -\sigma s + k_0 u \frac{\partial^2 s}{\partial y^2} + \varphi , \qquad (10)$$

where $\frac{\partial_r s}{\partial t} = \frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x}$ is the individual derivative taken when moving along the trajectory r, and $k_0 = k_1/u$ is a constant.

Solution is sought in the form

$$s(x, y, t) = P(x, y)\, s'(x, t) .$$

Thus we derive the equations for the functions P and s'

$$\frac{\partial P}{\partial x} = k_0 \frac{\partial^2 P}{\partial y^2} , \quad \frac{d_r s'}{d t} = -\sigma s + \frac{\varphi}{P} .$$

The solution to the equation for P has the form[3]:

$$P(x, y) = \frac{1}{\sqrt{2 \pi k_0 x}} \exp(-y^2 / 4 k_0 x) . \qquad (11)$$

The equation for s' is solved numerically by substituting the integration over time by the integration over trajectory. Let us approximate this equation as follows:

$$s'^{(n)} = s'^{(n-1)} - \Delta t\, \sigma s'^{(n, n-1)} + \Delta t \frac{1}{P} \varphi^{(n, n-1)} , \qquad (12)$$

where n is the current number of the time step ($n = 1, 2, \ldots$), $s'^{(n-1)}$ is the value of s' at $(n-1)$ time moment, and $s'^{(n)}$ is that at the subsequent moment n. These moments correspond to the initial and final points of the trajectory during the time step Δt.

The functions $\sigma s'^{(n, n-1)}$ and $(1/P)\, \varphi^{(n, n-1)}$ are the averages of $\sigma s'$ and $(1/P)\, \varphi$ on the trajectory for a given time step. These averages may be presented following two schemes: an explicit scheme,

$$\overline{\sigma s'^{(n, n-1)}} = \sigma s'^{(n-1)} , \quad \overline{\frac{1}{P} \varphi^{(n, n-1)}} = \frac{1}{P} \varphi^{(n, n-1)}) ,$$

and an implicit one,

$$\overline{\sigma s'^{(n, n-1)}} = \frac{1}{2} \left[\sigma s'^{(n)} + \sigma s'^{(n-1)} \right] ,$$

$$\overline{\frac{1}{P} \varphi^{(n, n-1)}} = \frac{1}{2} \left[\frac{1}{P} \varphi^{n} + \frac{1}{P} \varphi^{n-1} \right] .$$

Then the value of s'^n at the end of a step is, according to the explicit scheme,

$$s'^{(n)} = (1 - \sigma \Delta t)\, s'^{(n-1)} + \Delta t \frac{1}{P} \varphi^{n-1} ,$$

and, according to the implicit one,

$$s'^{(n)} = \frac{1 - \sigma \Delta t/2}{1 + \sigma \Delta t/2} s'^{(n-1)} + \frac{\Delta t/2}{1 + \sigma \Delta t/2} \left[\frac{\varphi^n}{P} + \frac{\varphi^{n-1}}{P} \right] .$$

The expressions derived give the solution to the problem under the condition that at every step the trajectory is known. We have tested both integration schemes and the tests show that the implicit scheme offered advantages over the explicit one. Let us also note that the function $P(x, y)$ describes the distribution of a pollutant (in m^{-1}) at distances x and y from a point source along the corresponding axes, whereas function $s'(x, y)$ describes the distribution of the pollutant (in $kg \cdot m^2$) from an infinite linear source placed on the axis.

Let us now consider an example of pollution transport calculation. Table III presents the values of concentration of a conventional pollutant in PBL along the trajectory given at two–hour time interval (up to 24 h) on the plume axis and at a distance of 1 km from the axis. The value of conventional pollutant concentration s^0 at the emission point was taken to be 1000 $\mu g/m^3$.

TABLE III. Pollutant concentration ($\mu g/m^3$) per unit air column in the PBL of 980 m *in height for different distances from a point snap–action source for different model parameters (s^0 = 1000 $\mu g/m$).*

Parameter		t, hours				
		2	6	12	18	24
β,	w_H,	x, km				
cm/s	cm/s	70	201	434	690	997
		on the plume axis				
1	0	1.42	0.74	0.42	0.28	0.20
0	0	1.51	0.89	0.61	0.48	0.40
1	+ 0.5	1.41	0.73	0.41	0.27	0.18
		at a distance of 1 km from the plume axis				
1	0	0.04	0.21	0.24	0.19	0.15

The calculations were performed for no vertical motion and vertical speed at the upper boundary of the PBL w_H = 0.5 cm/s. It is clearly seen from Table III that the pollutant concentration rapidly decreases with distance from the emission point. So within 2 h after the start of particle motion at a distance about 70 km from the emission point, the pollutant concentration is 1.42 $\mu g/m^2$ that amounts to 0.14% of the initial concentration. Upward motions at the upper boundary of the PBL reduce the concentration in the PBL, since they transport a part of pollution into the upper atmosphere.

Presented here are the results obtained using the trajectory model in the problem of numerical evaluation of the pollution transport for long distances.

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