PROBLEM ON RECONSTRUCTION OF THE PARAMETERS OF A CHARGED PARTICLE TRACK

A.S. Burundukov

Pacific Institute of Oceanography,
Far–Eastern Branch of the Russian Academy of Sciences, Vladivostok
Received March 19, 1993

The problem on reconstruction of the geometric parameters of a charged particle track in the sea water as well as particle charge, mass, and energy from the Čerenkov optical radiation is considered. Algorithms for primary signal filtration by modular units of a detector array are proposed.

The Project DUMAND (on deep–water detection of muons and neutrinos) proposed in the middle 70s, together with the Program ATENA (of the experiment with the atmospheric neutrinos of high energy) and the Program UNICORN (on underwater detection of interstellar cosmic neutrinos)[1] is as before the only accessible method to penetrate far beyond the teraelectronvolt horizon of physics of elementary particles, because mankind will apparently be forced to accept monopoly of the universe on the production of particles with 10^7–10^8 TeV energy. The short fluorishing state of experimental physics of elementary particles is near the completion, and it will be compelled, in near future, to turn from experimental science into an observational one like astronomy. So after a short spell of oblivion, a renaissance of the Project DUMAND and a renewal of interest in problems of detection of high–energy particles, nuclear–electromagnetic cascades, etc. are inevitable as a result of the tremendours progress of experimental physics of elementary particles in the early 80s. Because the optical methods of recording have higher level of information content as compared with the acoustic ones (in spite of the latter have relatively low cost), one can assert with a high degree of probability that the project will be realized in an optical variant. Hence the need arises of more detailed experimental and theoretical study of the propagation and absorption of electromagnetic waves of the optical range in the see water as well as the study of the Čerenkov radiation process, the detection and filtration of the optical signal against the bioluminescence background, etc. In this paper the problem is briefly considered on the reconstruction of the geometric parameters of a charged particle track as well as particle charge, mass, and energy from the recorded Čerenkov radiation, the algorithms for the primary signal filtration by modular units of the detector array, and feasibility of recording the exotic processes as part of the Program DEGRE (on detection of the (super)gravitation effects[2]).

1. THE ČERENKOV RADIATION AS A SOURCE OF INFORMATION ABOUT THE PARAMETERS OF A CHARGED PARTICLE TRACK

Here we consider the case of motion of a single charged particle in water. The Čerenkov radiation process has been well studied theoretically both on the classical and quantum levels, which allows us to construct the convenient analytical algorithms for reconstruction of the track geometric parameters, the physical characteristics, and the parameters of the medium. In the most general form the problem on the reconstruction of the parameters can be defined more accurately using the following diagram:

$$\begin{array}{ccccccc}
 & & E^3\times E^3 & & & & \\
 & & \downarrow c & & & & \\
D(R^{\kappa}) & \xrightarrow{p} & J(R^4) & \xrightarrow{g} & Y(R^{MN}) & \xrightarrow{a} & U(R^{MN}) \\
 & & & & & & \downarrow G_1 \\
 & & & & r & & U^*(R^{3M}) \\
 & & & & & & \downarrow G_2 \\
 & & & & & & H(R^{\kappa})\times L(R^{3M-\kappa}).
\end{array}$$

Here $D(R^k)$ is the parametric space of the model, k is the dimensionality of this model space, $J(R^4)$ is the track space or the space of the physical realization of the model, $E^3\times E^3$ is the 6–D space of straight lines in the Euclidean space R^3, $Y(R^{MN})$ is the space of the geometric realization of the model, M is the number of units in a setup, $2N$ is the number of photomultipliers in one modular unit ($N \geq 6$), $U(R^{MN})$ is the space of amplitudes, $U^*(R^{3M})$ is the reduced amplitude space, $H(R^{\kappa})$ is the κ–dimentional manifold of a noiseless signal, $L(^{3M-\kappa})$ is the space of the separable noise, p is the mapping of the parameter space into the space of the physical realization, c is the canonical projection, g is the mapping from the space of physical realization of the model into the space of geometric realization which reduces to the selection of the geometry of the setup and the geometry of arrangement of photomultipliers in the unit of modules, a is the mapping of the instrumental realization connected in particular with sensitivity of the photomultiplier, G_1 is the primary (intramodular) geometric filtration of the signal, G_2 is the secondary (intermodular) filtration, and r is the isomorphic mapping of $H(R^{\kappa})$ into $D(R^{\kappa})$.

The minimum dimensionality of the parameter space $\dim D_{\min}(R^{\kappa}) = 4$, considering that it corresponds to the number of the track geometric characteristics. They can be identified with the three Euler angles and one parameter of length which is determined by the distance covered by the light from the particle trajectory to the origin of the coordinate system which we can affix to the geometric center of the setup. The maximum dimensionality $\dim D_{\max}(R^{\kappa}) = 9$, considering that we can determine, in addition to the geometric parameters, some physical ones and the reference parameters of the medium. Thus in the most general case the detection of the charged particle

0235-6880/93/07 481-06 $02.00

corresponds to the point of the 9–D space of the parameters $(\alpha, \beta, \gamma, L, q, m, \varepsilon, \mu, n)$, where α, β, and γ are the Euler angles; L is the track length; q is the charge, m is the mass, and ε is the energy of the particle; and, μ and n are the optical coefficients of absorption and refraction, respectively. In principle it is possible to construct $\sum_{i=0}^{5} c_5^i$ models in which, in addition to the geometric parameters, the arbitrary combinations of the rest of five parameters are used. For the models with the number of the parameters $\kappa \le 6$ at least two modular units must come into operation, while their number is three for $6 < \kappa \le 9$.

The space of linear trajectories in R^3 in the parametric form is given as

$$\mathbf{r} = \mathbf{a} + \mathbf{b}t ,$$

i.e., a straight line in R^3 is represented by a point in $E^3 \times E^3$. The canonical projection c: $E^3 \times E^3 \to C^2 \times S^2 = T^3 \times R^1$, where S^2 is a two–dimensional sphere of unit radius, C^2 is a two–dimensional cone, T^3 is a three–dimensional tore, is defined in the form

$$\begin{cases} \mathbf{a} \to \mathbf{a} = \mathbf{a} - \dfrac{\mathbf{b}}{\|\mathbf{b}\|^2}\left[(\mathbf{a},\mathbf{b}) + \sqrt{\|\mathbf{a}\|^2\|\mathbf{b}\|^2 - (\mathbf{a},\mathbf{b})^2}\right]\cot\hat{c} \overset{\mathrm{def}}{=} \mathbf{L} = L\mathbf{I}, \\ \mathbf{b} \to \dfrac{\mathbf{b}}{\|\mathbf{b}\|^2} \overset{\mathrm{def}}{=} \mathbf{n}. \end{cases}$$

where $\mathbf{L}$ and $\mathbf{n}$ are the canonical coordinates of the trajectory, $\mathbf{I}$ is a unit vector in the direction $\mathbf{L}$, and $\hat{c}$ is a Čerenkov angle ($\hat{c} \simeq 41°$). Here $(\mathbf{l}, \mathbf{n}) = -\cos\hat{c}$, i.e., $\mathbf{L}$ is directed from the origin of coordinates to the point of the trajectory from which the Čerenkov radiation comes. The kernel of the canonical projection is the 2–D manifold invariant under translation group in the direction $\mathbf{n}$

$$\begin{cases} \mathbf{a} = \mathbf{a} + \lambda\mathbf{b} , \\ \mathbf{b}' = \sigma\mathbf{b} . \end{cases}$$

The mapping $p(D(R^\kappa) \to J(R^4))$ can be defined more accurately by choosing a couple of vectors $\mathbf{I}_0$ and $\mathbf{n}_0$ in the form, for example,

$$\begin{pmatrix} \mathbf{I}_0 \\ \mathbf{n}_0 \end{pmatrix} = \begin{pmatrix} \sin\hat{c} \\ 0 \\ \cos\hat{c} \\ 0 \\ 0 \\ -1 \end{pmatrix} ,$$

and by using the representation of $SO(3)$ group as a group of the orthogonal transformations R^3

$$P: (\alpha, \beta, \gamma, L) \to J(R^4) ,$$

$$L = L(A_3(\alpha)\, A_2(\beta)\, A_3(\gamma)\, \mathbf{I}_0) ,$$

$$\mathbf{n} = A_3(\alpha)\, A_2(\beta)\, A_3(\gamma)\, \mathbf{n}_0 ,$$

where

$$A_3(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad A_2(\beta) = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} .$$

Let $\mathbf{M}_\nu$ be a radius–vector of the νth module of the setup. The mapping g: $J(R^4) \to Y(R^{\mu\nu})$ is a product of two transformations: the translation $T_{\mu\nu}$ of the vector $\mathbf{L}$ at the vector $\mathbf{M}_\nu$

$$\mathbf{L}'_\nu = T_{\mu\nu} = \mathbf{L} - \mathbf{M}_\nu$$

and subsequent canonical projection

$$g(\mathbf{L}) = c \circ T_{\mu\nu}(\mathbf{L}) = c \circ (\mathbf{L} - \mathbf{M}_\nu) .$$

As a result of these operations, we derive for every $\mathbf{M}_\nu$ the vector $\mathbf{L}_\nu$ pointing to the point of the trajectory from which the signal came to the point $\mathbf{M}_\nu$. Let us define the commutator

$$[c, T_{\mu\nu}]\mathbf{L} = \mathbf{R} = \pm R\mathbf{n} ,$$

where $\mathbf{R}$ is a vector joining two points of the trajectory; moreover, the light from one point arrives at the origin of the coordinates and from the other enters the point $\mathbf{M}_\nu$. Then we construct the object $G_{\sigma\nu}$ as a scalar product of N_σ by L_ν

$$G_{\sigma\nu} = (N_\sigma\, L_\nu) .$$

Here ($\sigma = 1, 2, ..., N$, $\nu = 1, 2, ..., M$), where N_σ is a unit vector specifying the direction in R^3. It is realized in the physical space by two photomultipliers directed oppositely. The magnitude of the vector projection on this direction equals the difference between the signals of two photomultipliers and the totality of all N_σ forms the geometry of modular unit of the setup. Thus we can say that $Y(\mu)$ is the intermodular geometry of the setup while $Y(\nu)$ is its intramodular geometry.

And finally we consider the mapping a: $Y(R^{\mu\nu}) \to U(R^{\mu\nu})$ that we determine by the formula

$$A_{\sigma\nu} = \frac{q^2\kappa}{||L_\nu||^2}\, G_{\sigma\nu} \exp(-\mu(\lambda)\, L_\nu) ,$$

where $A_{\sigma\nu}$ is an average amplitude in photoelectrons, q is a dimensionless charge of a particle in units multiple to the electron charge, k is a constant of a photomultiplier (according to the data of the scientists of the Baikal group, it makes 8.57±0.43 meters per one photoelectron), $\mu(\lambda)$ is a sea water absorption coefficient, G – filtration consists in division of the amplitude space U into the submanifold of the filtered–out signal and that of the removable noise. The intramodular filtration is the simplest operation as in this case the submanifold of the filtered–out signal is a linear 3–D subspace. To solve the problem, one needs only one transformation. We consider this procedure in detail in Sec. 2. The intermodular filtration G_2 essentially depends on the overall geometry of the setup. For this reason, no consideration has been given here. We only limit ourselves to the example of reconstruction of the track geometric parameters with simultaneous detection of the Čerenkov radiation by two units of the setup.

Let $\mathbf{M}_1$ and $\mathbf{M}_2$ be the radius–vectors of two units of the setup, and A_1 and A_2 be the filtered signals. From the equality

$$\mathbf{L} = c \circ (\mathbf{L}_1 + \mathbf{M}_1) = c \circ (\mathbf{L}_2 - \mathbf{M}_2) , \quad (1)$$

by multiplying it by $\mathbf{n}$, we derive

$$(\sqrt{(\mathbf{L}_1+\mathbf{M}_1)^2-(\mathbf{L}_1+\mathbf{M}_1,\mathbf{n})^2}=\sqrt{(\mathbf{L}_2+\mathbf{M}_2)^2-(\mathbf{L}_2+\mathbf{M}_2,\mathbf{n})^2}\,. \quad (2)$$

By multiplying Eq. (1) by $\mathbf{l}_1$ and $\mathbf{l}_2$ and making addition and subtraction of the obtained equations as well as by using Eq. (2), we find the solution of the system of equations (3)

$$(\mathbf{L}_1+\mathbf{L}_2)=\left(\mathbf{M}_2-\mathbf{M}_1,\frac{\mathbf{l}_1-\mathbf{l}_2}{1-(\mathbf{l}_1\,\mathbf{l}_2)}\right),$$

$$(\mathbf{L}_1-\mathbf{L}_2)=\left(\mathbf{M}_2-\mathbf{M}_1,\frac{\mathbf{l}_1+\mathbf{l}_2-2\cos\hat{c}\,\mathbf{n}}{1+(\mathbf{l}_1\,\mathbf{l}_2)-2\cos^2\hat{c}}\right). \quad (3)$$

To solve the problem completely, one needs to find the direction of movement of a particle specified by the vector $\mathbf{n}$. It can be done by using the ratio $(\mathbf{l},\mathbf{n})=-\cos\hat{c}$.

Then

$$\mathbf{n}=-\frac{\cos\hat{c}}{1+(\mathbf{l}_1\,\mathbf{l}_2)}(\mathbf{l}_1+\mathbf{l}_2)\pm\sqrt{\frac{1+(\mathbf{l}_1\,\mathbf{l}_2)-2\cos^2\hat{c}}{1-(\mathbf{l}_1\,\mathbf{l}_2)^2}}\;\mathbf{l}_1\times\mathbf{l}_2\,, \quad (4)$$

from which we find the expressions for L_1 and L_2

$$L_1=$$

$$=\left(\mathbf{M}_2-\mathbf{M}_1,\frac{\mathbf{l}_1-(\mathbf{l}_1\,\mathbf{l}_2)\mathbf{l}_2}{1-(\mathbf{l}_1\,\mathbf{l}_2)^2}\pm\frac{\mathbf{l}_1\times\mathbf{l}_2\cos\hat{c}}{\sqrt{((\mathbf{l}_1\,\mathbf{l}_2)-2\cos\hat{c})(1-(\mathbf{l}_1\,\mathbf{l}_2)^2)}}\right),$$

$$L_2=$$

$$=-\left(\mathbf{M}_2-\mathbf{M}_1,\frac{\mathbf{l}_2-\mathbf{l}_1(\mathbf{l}_1\,\mathbf{l}_2)}{1-(\mathbf{l}_1\,\mathbf{l}_2)^2}\mp\frac{\mathbf{l}_1\times\mathbf{l}_2\cos\hat{c}}{\sqrt{((\mathbf{l}_1\,\mathbf{l}_2)-\cos 2\hat{c})(1-(\mathbf{l}_1\,\mathbf{l}_2)^2)}}\right), \quad (5)$$

Because the distance L is inversely proportional to the signal amplitude, the sign in Eqs. (4) and (5) is taken depending on the ratio of the amplitudes A_1 and A_2.

2. PLATONIC SOLIDS AND FILTRATION OF THE OPTICAL SIGNAL

Now we consider the G–filtration when the photomultipliers of the modules form the regular polyhedrons (they are arranged quasispherically in the space). We limit ourselves to Platonic solids such as tetrahedron, cube, octahedron, dodecahedron, icosahedron, and two solids of mixed octahedron–cube symmetry.

The arbitrary vector $\mathbf{x}$ in E^3, can be represented on the orthogonal Descartes' basis $\mathbf{e}_i$: $\mathbf{x}=x^i\mathbf{e}_i$ $(i=1,2,3)$. Let us introduce N unit vectors $\mathbf{N}_\sigma$ $(\sigma>3)$ into E^3 and take the scalar product $(\mathbf{x},\mathbf{N}_\sigma)=A_\sigma$, where $A_\sigma\in U^{\mu\nu}$ and μ is fixed. Thus the vector $\mathbf{N}_r$ can be represented on the initial basis $\mathbf{N}_\sigma=N_\sigma^i\mathbf{e}_i$. In this case the scalar product $(\mathbf{x},\mathbf{N}_\sigma)$ can be rewritten in the form

$$(\mathbf{x},\mathbf{N}_\sigma)=x^i N_\sigma^j(\mathbf{e}_i\,\mathbf{e}_j)=x^i N_\sigma^i\delta_{ij}=x^i N_{\sigma i}=A_\sigma\,, \quad (6)$$

where δ_{ij} is Kronecker's delta symbol.

The problem of $\mathbf{x}$ reconstruction from the values of A_r can be solved by two methods.

1. Choose C_N^3 of triplets of vectors in E^3 in the form of a basis set, and find for every independent triplet N_τ $(\tau=1,2,3)$ the corresponding inverse matrix N_j^τ such that $N_{\tau i}N_j^\tau=\delta_{ij}$. Then the components of the vector x^i in Descartes' basis are found by multiplication of Eq. (6) by N_j^τ:

$$x^i N_{\tau j}N_j^\tau=x^i\delta_{ij}=A_s N_j^\tau=x^j\,.$$

As a result, Descartes' coordinates x^i can be expressed in terms of the amplitudes of the signal by the following formula

$$x^i=\frac{1}{C_N^3}\sum A_\tau N^{\tau i}\,,$$

where, in addition to the summation over all dummy indices, the sum is taken over all linear–independent triplets of vectors. The method is obviously cumbersome due to the fact that one needs to calculate C_N^3 inverse matrices. For dodecahedron $(N=5)$ $C_5^3=10$, while for icosahedron $C_{10}^3=120$.

2. It is convenient to consider the matrix N_τ^i as the triplet of vectors in the N–space of U^N rather than a set of N unit vectors in E^3. The idea has been conceived to complete this triplet to the total orthogonal basis of N vectors in U^N and by orthogonal rotation and normalization to obtain such a coordinate system in U^N in which the first three coordinates correspond to the components x^i and the rest form the subspace of the separable noise. In this case the problem reduces merely to the Gram–Schmidt orthogonalization. In the matrix representation this procedure can be performed in the following way. The mapping $x^i\to A_\tau$ is written in the form

$$\left\|\begin{matrix}A_1\\A_1\\.\\.\\.\\.\\A_N\end{matrix}\right\|=\left\|\begin{matrix}N_{11}&N_{12}&N_{13}\\N_{21}&N_{22}&N_{23}\\.&.&.\\.&.&.\\.&.&.\\.&.&.\\N_{N1}&N_{N2}&N_{N3}\end{matrix}\right\|\begin{pmatrix}x^1\\x^2\\x^3\end{pmatrix}.$$

We must find the matrix of the orthogonal transformation $M_{\tau\rho}$ such that

$$\left\|\begin{matrix}M_{11}&M_{12}&...&M_{1N}\\.&.&...&.\\.&.&...&.\\.&.&..&.\\.&.&...&.\\M_{N1}&M_{N2}&...&M_{NN}\end{matrix}\right\|\left\|\begin{matrix}N_{11}&N_{12}&N_{13}\\.&.&.\\.&.&.\\.&.&.\\.&.&.\\N_{N1}&N_{N2}&N_{N3}\end{matrix}\right\|=\left\|\begin{matrix}1&0&0\\0&1&0\\0&0&1\\&&\\0&0&0\\0&0&0\end{matrix}\right\|.$$

The first three rows of the matrix $M_{\tau\rho}$ are merely the columns $N_{11}\ldots N_{N1}$; $\ldots N_{12}\ldots N_{N2}$; $N_{13}\ldots N_{N3}$ divided by the square of their absolute value. The rest of the vectors of the same length are found by the standard Gram–Schmidt procedure.

Let us consider some examples.

(a) T e t r a h e d r o n. To reconstruct unambiguously the amplitude and direction of propagation of the signal from all R^3, the number of the tetrahedron faces $2N = 4$ is insufficient.

(b) Cube. Cube has $2N = 6$ faces. Choosing the vectors in the directions of Descartes' basis $\mathbf{e}_i$, we derive

$$N = \left\|\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right\| , \text{ i.e., } \begin{matrix} x^1 = A_1 \\ x^2 = A_2 \\ x^3 = A_3 \end{matrix} .$$

(c) Octahedron. Octahedron has $2N = 8$, i.e., abundance of the basis makes up $4 - 3 = 1$. Take the basis in the form

$$N = \left\|\begin{matrix} \sqrt{\frac{2}{3}} & 0 & \frac{1}{\sqrt{3}} \\ 0 & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ -\sqrt{\frac{2}{3}} & 0 & \frac{1}{\sqrt{3}} \\ 0 & -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \end{matrix}\right\| .$$

The solution of the problem on reconstruction of the components x_i from the abundant data is written down in the form

$$x_1 = \frac{\sqrt{3}}{2\sqrt{2}} (A_1 - A_3) ,$$

$$x_2 = \frac{\sqrt{3}}{2\sqrt{2}} (A_2 - A_4) ,$$

$$x_3 = \frac{\sqrt{3}}{4} (A_1 + A_2 + A_3 + A_4) ,$$

$$d_1 = \frac{\sqrt{3}}{4} (A_1 - A_2 + A_3 - A_4) .$$

Here d_1 is the component of the separable noise.

Rotating octahedron at $\pi/4$ about the axis z, we derive a more symmetric matrix

$$N = \frac{1}{\sqrt{3}} \left\|\begin{matrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \\ 1 & -1 & 1 \end{matrix}\right\|$$

and more symmetric solution

$$x_1 = \frac{\sqrt{3}}{4} (A_1 - A_2 - A_3 + A_4) ,$$

$$x_2 = \frac{\sqrt{3}}{4} (A_1 + A_2 - A_3 - A_4) ,$$

$$x_3 = \frac{\sqrt{3}}{4} (A_1 + A_2 + A_3 + A_4) ,$$

$$d_1 = \frac{\sqrt{3}}{4} (A_1 - A_2 + A_3 - A_4) .$$

(d) Dodecahedron. It has $2N = 12$ faces, i.e. the dimensionality of the space of the separable noise equals 3. Choose the basis in E^3 in the following way:

$$N = \left\|\begin{matrix} 0 & 0 & 1 \\ 2/\sqrt{5} & 0 & 1/\sqrt{5} \\ (\sqrt{5}-1)/2\sqrt{5} & \sqrt{(\sqrt{5}+1)/2\sqrt{5}} & 1/\sqrt{5} \\ -(\sqrt{5}+1)/2\sqrt{5} & -\sqrt{(\sqrt{5}-1)/2\sqrt{5}} & 1/\sqrt{5} \\ -(\sqrt{5}+1)/2\sqrt{5} & -\sqrt{(\sqrt{5}-1)/2\sqrt{5}} & 1/\sqrt{5} \\ (\sqrt{5}-1)/2\sqrt{5} & -\sqrt{(\sqrt{5}+1)/2\sqrt{5}} & 1/\sqrt{5} \end{matrix}\right\| .$$

In this case the solution is written down in the following way:

$$x_1 = \frac{1}{\sqrt{5}} A_2 + \frac{\sqrt{5}-1}{4\sqrt{5}} (A_3 + A_6) - \frac{\sqrt{5}+1}{4\sqrt{5}} (A_4 + A_5) ,$$

$$x_2 = \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2\sqrt{5}}} (A_3 - A_6) + \frac{1}{2}\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{5}}} (A_4 - A_5) ,$$

$$x_3 = \frac{1}{2} A_1 + \frac{1}{2\sqrt{5}} (A_2 + A_3 + A_4 + A_5 + A_6) ,$$

$$d_1 = -\frac{1}{2}\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{5}}} (A_1 + A_2) + \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2\sqrt{5}}} (A_3 + A_6) ,$$

$$d_2 = -\frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2\sqrt{5}}} (A_1 + A_2) + \frac{1}{2}\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{5}}} (A_4 + A_5) ,$$

$$d_3 = \frac{1}{2}\sqrt{\frac{1}{2} - \frac{1}{2\sqrt{5}}} (A_3 - A_6) - \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2\sqrt{5}}} (A_4 - A_5) .$$

(e) The octahedron–cubic (OC) symmetry. The number of faces for polyhedron of the octahedron–cubic symmetry equals 14, i.e., abundance of geometry of the module equals 4. Two modifications of the OC symmetry are possible. We define the first one with the matrix

$$N = \frac{1}{\sqrt{3}} \left\|\begin{matrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \\ 1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \\ 1 & -1 & 1 \end{matrix}\right\| .$$

The solution of the problem will be

$$x_1 = \frac{3}{7} A_1 + \frac{\sqrt{3}}{7} (A_4 - A_5 - A_6 + A_7) ,$$

$$x_2 = \frac{3}{7} A_2 + \frac{\sqrt{3}}{7} (A_4 + A_5 - A_6 - A_7) ,$$

$$x_3 = \frac{3}{7} A_3 + \frac{\sqrt{3}}{7} (A_4 + A_5 + A_6 + A_7) ,$$

$$d_1 = \frac{1}{2}\sqrt{\frac{3}{7}} (A_4 - A_5 + A_6 - A_7) ,$$

$$d_2 = \frac{2\sqrt{3}}{7} A_1 - \frac{3}{14} (A_4 - A_5 - A_6 + A_7) ,$$

$$d_3 = \frac{2\sqrt{3}}{7} A_2 - \frac{3}{14}(A_4 + A_5 - A_6 - A_7) ,$$

$$d_4 = \frac{2\sqrt{3}}{7} A_3 - \frac{3}{14}(A_4 + A_5 + A_6 + A_7) .$$

The second modification can be defined in the following way:

$$N = \left\| \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \sqrt{2/3} & 0 & 1/\sqrt{3} \\ 0 & \sqrt{2/3} & 1/\sqrt{3} \\ -\sqrt{2/3} & 0 & 1/\sqrt{3} \\ 0 & -\sqrt{2/3} & 1/\sqrt{3} \end{array} \right\| .$$

Then the solution is written down in the form

$$x_1 = \frac{3}{7} A_1 + \frac{\sqrt{6}}{7}(A_4 - A_6) ,$$

$$x_2 = \frac{3}{7} A_2 + \frac{\sqrt{6}}{7}(A_5 - A_6) ,$$

$$x_3 = \frac{3}{7} A_3 + \frac{\sqrt{3}}{7}(A_4 + A_5 + A_6 + A_7) ,$$

$$d_1 = \frac{1}{2}\sqrt{\frac{3}{7}}(A_4 - A_5 + A_6 - A_7) ,$$

$$d_2 = \frac{2\sqrt{3}}{7} A_1 - \frac{3}{7\sqrt{2}}(A_4 - A_6) ,$$

$$d_3 = \frac{2\sqrt{3}}{7} A_2 - \frac{3}{7\sqrt{2}}(A_5 - A_7) ,$$

$$d_4 = \frac{2\sqrt{3}}{7} A_3 - \frac{3}{14}(A_4 + A_5 + A_6 + A_7) .$$

It should be noted that the concrete form of the components d_i makes no physical sence, for the meaning has only

$$d = \sqrt{\sum_{1}^{N=4} d_i^2} .$$

(f) I c o s a h e d r o n . Finally we complete our examples with the case of icosahedron symmetry. The number of icosahedron faces equals 20, and dimensionality of the separable noise equals $2N = 7$. The appropriate choice of the basis ensures the brevity of the algorithms for reconstruction.

Let us choose the basis in the form

$$N = \left\| \begin{array}{ccc} 0 & 0 & 1 \\ 2/3 & 0 & \sqrt{5}/3 \\ -1/3 & 1/\sqrt{3} & \sqrt{5}/3 \\ -1/3 & -1/\sqrt{3} & \sqrt{5}/3 \\ \sqrt{5}/3 & 1/\sqrt{3} & 1/3 \\ \sqrt{5}/3 & -1/\sqrt{3} & 1/3 \\ (3-\sqrt{5})/6 & (\sqrt{5}+1)/2\sqrt{3} & 1/3 \\ -(3+\sqrt{5})/6 & (\sqrt{5}-1)/2\sqrt{3} & 1/3 \\ -(3+\sqrt{5})/6 & -(\sqrt{5}-1)/2\sqrt{3} & 1/3 \\ (3-\sqrt{5})/6 & -(\sqrt{5}+1)/2\sqrt{3} & 1/3 \end{array} \right\| .$$

Then the solution of the problem will be

$$x_1 = \frac{1}{5} A_2 - \frac{1}{10}(A_3 + A_4) + \frac{1}{2\sqrt{5}}(A_5 + A_6) +$$

$$+ \frac{3-\sqrt{5}}{20}(A_7 + A_{10}) - \frac{3+\sqrt{5}}{20}(A_8 + A_9) ,$$

$$x_2 = \frac{\sqrt{3}}{10}(A_3 - A_4 + A_5 - A_6) + \frac{\sqrt{3}}{20}(\sqrt{5} + 1)(A_7 - A_{10}) +$$

$$+ \frac{\sqrt{3}}{20}(\sqrt{5} - 1)(A_8 - A_9) ,$$

$$x_3 = \frac{1}{10}(3A_1 + A_5 + A_6 + A_7 + A_8 + A_9 + A_{10}) +$$

$$+ \frac{1}{2\sqrt{5}}(A_2 + A_3 + A_4) ,$$

$$d_1 = -\frac{\sqrt{3}}{4\sqrt{5}}(\sqrt{5}(A_1 - A_2 - A_3 - A_4) ,$$

$$d_2 = -\frac{\sqrt{3}}{4\sqrt{55}}(A_1 + 3\sqrt{5}A_2 - \sqrt{5}(A_3 + A_4 + A_5 + A_6)) ,$$

$$d_3 = \sqrt{\frac{3}{40}}(A_3 - A_4 - A_5 + A_6) ,$$

$$d_4 = -\frac{\sqrt{5}-1}{4\sqrt{5}}(A_7 - A_{10}) + \frac{\sqrt{5}+1}{4\sqrt{5}}(A_8 + A_9) ,$$

$$d_5 = \frac{3}{10\sqrt{2}}(A_3 - A_4 + A_5 - A_6) + \frac{\sqrt{5}+1}{10\sqrt{2}}(A_7 - A_{10}) -$$

$$- \frac{\sqrt{5}-1}{10\sqrt{2}}(A_8 - A_9) ,$$

$$d_6 = -\frac{\sqrt{3}}{2\sqrt{385}} A_1 + \frac{1}{10}\sqrt{\frac{3}{11}} A_2 - \frac{3}{5}\sqrt{\frac{3}{77}}(A_3 + A_4) +$$

$$+ 2\sqrt{\frac{3}{385}}(A_5 + A_6) - \frac{1}{10}\sqrt{\frac{33}{7}}(A_7 - A_8 - A_9 + A_{10}) ,$$

$$d_7 = \frac{1}{6}\sqrt{\frac{3}{7}} A_1 + \frac{1}{2}\sqrt{\frac{33}{35}}(A_3 + A_4) - \frac{\sqrt{3}}{10\sqrt{35}}(A_5 + A_6) -$$

$$- \frac{\sqrt{3}}{20\sqrt{7}}(7 + \sqrt{5})(A_7 + A_{10}) - \frac{\sqrt{3}}{20\sqrt{7}}(7 - \sqrt{5})(A_8 + A_9) .$$

3. PROGRAM DEGRE

Here we briefly dwell on the prospects of detection of gravitons and other exotic particles such as photino, gravitino, gluino, etc. Modern theories of quatum gravitation do not satisfy us for many reasons. The main of them are violation of unitarity with increase of energy and the lack of renormalization. The Fermi theory of weak interactions had the same drawbacks but nevertheless it gave good approximation up to the energies comparable with the mass of the W boson. So we hope that more fundamental theories having changed our notion of the gravitational interaction on the Plank scales nevertheless will not change the behavior of the scattering cross section for energy $\varepsilon \ll 10^{19}$ GeV.

In the first order of the perturbation theory for the gravity constant the following five processes are possible:

1) The graviton decay in two photons (gluons) in dielectric (barion).[3] The probabilities of two–photon W_γ and two–gluon W_{gl} decays equal, respectively

$$W_\gamma \simeq \kappa^2 (n_\gamma^2 - 1)^2 \varepsilon^3 \simeq 5{,}1\cdot 10^{-38} (n^2 - 1)^2 \varepsilon^3 [\text{eV}] ,$$

$$W_{gl} \simeq \kappa^2 \varepsilon^3 \simeq 6{,}3\cdot 10^{-13} \varepsilon^3 [\text{GeV}] .$$

2) The graviton resonance on barions. The corresponding probability equals

$$W \simeq (\kappa^2/32\pi)\, m^3 .$$

Here $m \simeq \varepsilon$ it is the mass of the barion resonance.

3) The graviton ionization.[4] The cross section σ of this process equals

$$\sigma \simeq \frac{k^2 Z^2 e^{10}}{8\pi} \frac{m}{\varepsilon} \simeq 6{,}2\cdot 10^{-62} \varepsilon^{-1} [\text{GeV}]\ \text{cm}^2 .$$

4) The graviton–photon inversion on nuclei:

$$\sigma \simeq \frac{k^2 Z^2 e^2}{128\pi} \simeq 1{,}6\cdot 10^{-63}\ \text{см}^2 .$$

5) The graviton production of higgs as well as W and Z bosons and X and Y leptonquarks

$$\sigma \simeq \frac{k^2 G_F m^2 x}{8\sqrt{2}\pi} \simeq 4{,}2\cdot 10^{-66}\ \text{cm}^2 .$$

The first and second processes have the maximum probability of detection. The frequency of the quantum graviton processes in the setup with the volume of 1 km^3 $R \simeq 6.1\cdot 10^{-31} \varepsilon^3 [\text{GeV}]\, F\, [\text{graviton}/\text{km}^2\cdot \text{s}]$.

The minimum density of the graviton flux for its detection must be

$$F \geq 5\cdot 10^{24} \varepsilon^{-2} [\text{GeV}] .$$

The theory of supergravitation predicts the existence of the new particles being super analogs for the usual ones such as photino, gluino, gravitino, etc. According to estimates of the scattering cross–section reported in Ref. 5, we have

$$\sigma \simeq 10^{-38} \varepsilon [\text{GeV}]\ \text{cm}^2 .$$

Thus the frequency of occuring of the events R in the setup is approximately equal to $6\cdot 10^{10} \varepsilon [\text{GeV}]\, F\, [\text{particles}/\text{km}^2\cdot\text{s}]$, and the minimum density of the flux is $F \geq 5\cdot 10\ \varepsilon^{-1} [\text{GeV}]$.

REFERENCES

1. P. Kotzer, ed., *Proceedings of 1975 Summer Workshop on DUMAND* (West. State College, Wash, 1976), 176 pp.
2. A.S. Burundukov and U.H. Kopvillem, in: *Abstract of Reports at the Tenth Intern. Conf. on General Relativity and Gravitation*, Padova (1983), p. 901.
3. A.S. Burundukov, in: *Abstracts of Reports at the Sixth Soviet Gravitation Conference,* Moscow (1984), p. 172.
4. A.S. Burundukov, Gen. Relativ. Gravit. **17**, No. 4, 311–318 (1985).
5. P. Fayet, Phys. Lett. **B86**, 272–275 (1979).