MONOSTATIC DOUBLE SENSING OF EDDY INHOMOGENEITY FLUXES WITH A MOVING REFLECTOR

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The autocorrelation function and energy spectrum of the phase and frequency fluctuations are considered for the case of monostatic double sensing of a flux of eddy inhomogeneities in the microwave and optical ranges employing a moving reflector. The possibility of retrieving the transport velocity and spatial intensity distribution for atmospheric turbulence is demonstrated. The problem is generalized to the case of a moving radiation source, and an example demonstrating practical applicability of the suggested technique to such tasks is presented.

INTRODUCTION

Lately many reliable techniques have been suggested for the remote sensing of atmospheric turbulence which are capable of yielding its transport velocity, intensity, its spatial inhomogeneity spectrum, etc., from variations in the parameters of electromagnetic and acoustic waves.[1,2] The existing techniques for sensing such inhomogeneities can be tentatively divided into two groups. The first has to do with detecting radiation that has been diffusely scattered by the medium inhomogeneities.[2] Such techniques have been implemented in lidar systems for sensing air parcels of atmospheric aerosols and in incoherent scattering stations. The second group includes techniques for recording radiation which has passed once through the studied inhomogeneities. In the case of systems of double sensing the radiation passes through the medium twice, e.g., it is reflected by mirrors at the ends of the path.[3] Techniques for this second group are energetically more effective, and may be employed for long-range sensing. However they need specially organized beam paths with emitters, detectors, and reflectors positioned at opposite ends of the path.[2] The use of spatially separated paths makes it possible to measure transverse flux velocities and the spatial structure of the eddy inhomogeneities of the medium by such techniques. To that end bistatic lidar and radar measurements can be performed with various reflectors. Wind drift of the eddies results in lateral displacements of pockets of excessive backscatter,[2,3] and in shifts of the range of effective focusing of the inverse wave field.[4–6] By adequate spacing of the emitter and the detector the described shift can be traced out and the eddy transport velocity retrieved. In a monostatic sensing scheme such a result can be obtained by displacing either the locating system itself[6] or the reflector. The aim of this paper is to analyze the temporal fluctuations of the phase and frequency in a monostatic sensing scheme with a moving reflector (which so far has not received proper attention in the literature and to within the turbulent layer at the same rate as the design a technique on that basis for retrieving beam line itself, i.e., the parameters of the eddy flux.

SPATIAL-TEMPORAL RELATIONSHIPS FOR DOUBLE SENSING

Let us consider the scheme of double sensing presented in Fig. 1. The signal is emitted at point A, is reflected at point B from a reflecting object which moves at a velocity V, and is then detected at point A. The angle between the direction of reflector movement and the path AB is denoted by α. We assume that the eddy inhomogeneities in the medium which produce the phase fluctuations in the signal are transported at a velocity V, parallel to reflector movement but in the opposite direction, and are concentrated in a layer whose boundaries are shown by the dashed lines in Fig. 1.

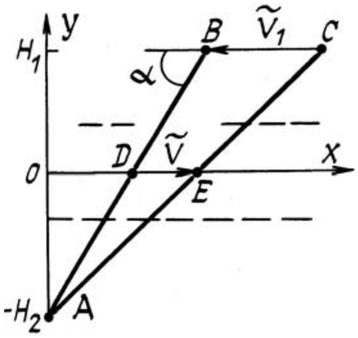

FIG. 1. Sensing scheme.

During its propagation from the emitter to the reflector and back the wave interacts with the medium twice: first on its way to, and second — on its way

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back from the reflector. This would mean that any particular inhomogeneity affects the wave phase twice. The reflector moves as well, so the beam line is constantly moving against the eddy flux. It is therefore convenient to transform to a coordinate system that rotates within the turbulent layer at the same rate as the beam line itself, i.e.,

$$u = V_1 H_2/H, \qquad H = H_1 + H_2 \tag{1}$$

Here H_1 and H_2 determine the position of reflector and the radiation source with respect to the central line of the turbulent layer, shown by the line DE in Fig. 1. After transforming to the new coordinate system the reflector velocity V_1 is replaced by $\tilde{V}_1 = V_1 - u$, and the eddy velocity V by $\tilde{V} = V + u$. In this coordinate system the indicated inhomogeneity produces its first phase distortion in the vicinity of point D as the wave propagates from the emitter to the reflector. After reflection of the wave at point B that distortion will be received at point A after a time $t_1 = t_0 + \Delta t_2$ where $t_0 = 2\Delta t_1$ and $\Delta t_{1,2} = H_{1,2}/c\sin\alpha$, where c is the speed of light. The second distortion of the phase by the same inhomogeneity will take place along the beam AC in the vicinity of point E as the wave propagates back from the reflector, which at some previous instant had been at point C. The resulting secondary phase distortion will be detected at point A at the time $t_2 = \Delta t + \Delta t_2$ where $\Delta t = d/\tilde{V}$ is the time it takes for the inhomogeneity to travel from point D to point E. Obviously, this second phase distortion will be received at our observation point earlier than the first one by the time

$$\tau_0 = t_1 - t_2 = t_0 - d/\tilde{V}. \tag{2}$$

The length of the line segment $DE = d$ can be found if we take into account the time it takes the beam to rotate from position AC to position $AB = 2H/c\ \sin\alpha$. During that time interval the reflector passes through the distance $BC = V_1 2H/c\ \sin\alpha$. From similarity we can write $d = BC \cdot H_2/H = ut_0$. Substituting this value into Eq. (2) we have on account of Eq. (1)

$$\tau_0 = t_0\left[1 - \frac{u}{\tilde{V}}\right] = t_0 V/\left[V + V_1 H_2/H\right]. \tag{3}$$

The value of τ_0 significantly affects the correlation properties of the phase and frequency fluctuations in the chosen sensing scheme.

THE CORRELATION FUNCTION AND THE ENERGY SPECTRUM OF THE PHASE AND FREQUENCY FLUCTUATIONS FOR THE DOUBLE SENSING SCHEME

We represent the fluctuational component $s(t)$ of the total phase as a sum of two terms:

$$s(t) = s_1(t) + s_2(t), \tag{4}$$

where $s_1$and s_2 are the values of the phase fluctuations, accumulated during the propagation of the wave to reflector and back. Within the geometric optics approximation the terms s_1 and s_2 can be expressed as (see Fig. 1)

$$s_1(t) = k\int\limits_{-H_2}^{H_1}\frac{dy'}{\sin\alpha}\times$$

$$\times N\left[x' = \frac{H_2 + y'}{\mathrm{tg}\alpha},\ y', z' = 0,\ t' = t - \frac{H + H_2 - y'}{c\ \sin\alpha}\right],$$

$$s_2(t) = k\int\limits_{-H_2}^{H_1}\frac{dy''}{\sin\alpha}\times$$

$$\times N\left[x'' = \frac{H_2 + y''}{\mathrm{tg}\alpha} + d,\ y'',\ z'' = 0,\ t'' = t - \frac{H_2 + y''}{c\ \sin\alpha}\right], \tag{5}$$

where $N(x, y, z, t)$ is the fluctuational part of the atmospheric refractive index and k is the wave number. It has been recalled here that the beams AB and AC can be considered to be practically parallel to each other in the inhomogeneity layer. Assuming the inhomogeneities to be "frozen in place" and the eddies to be statistically locally homogeneous, we can write

$$\langle N\left[x',\ y',\ z,\ t'\right] N\left[x'',\ y'',\ z,\ t''\right]\rangle =$$

$$= \sigma^2\left[\frac{y' + y''}{2}\right]\iint\limits_{-\infty}^{+\infty} d\kappa_x d\kappa_y \Phi\left(\kappa_x,\ \kappa_y\right)\times$$

$$\times\exp\left[i\kappa_x\left(x' - x'' - \tilde{V}(t' - t'')\right) + i\kappa_y(y' - y'')\right]. \tag{6}$$

Here $\sigma^2(y)$ is the variance, which depends on the coordinate y, and $\Phi\left[\kappa_x,\ \kappa_y\right] = \Phi\left[\sqrt{\kappa_x^2 + \kappa_y^2}\right]$ is the normalized spatial spectrum of the refractive index fluctuations N.

After averaging over the ensemble of realizations we can write for the autocorrelation function of the phase fluctuations (4)

$$B_s(\tau) = \langle s(t)s(t + \tau)\rangle =$$

$$= 2B_0(\tau) + B_1\left(\tau - \tau_0\right) + B_1\left(\tau + \tau_0\right). \tag{7}$$

Here the terms

$$B_0(\tau) = \langle s_1(t)\ s_1(t + \tau)\rangle = \langle s_2(t)\ s_2(t + \tau)\rangle$$

and

$$B_1(\tau) = \langle s_1(t)\ s_2(t + \tau)\rangle$$

represent respectively the autocorrelation function for the fluctuations in the case of single sensing and the correlation function of the phase fluctuations in the case in which the wave propagates to the reflector and back. The Wiener-Khinchine formula relates these functions to the respective energy spectra $W_0(\omega)$ and $W_1(\omega)$, which, on account of Eqs. (5) and (6) can be written as

$$W_0(\omega) = \frac{2\pi k^2}{\sin^2\alpha \tilde{V}} \Phi\left[\frac{\omega}{\tilde{V}\sin\alpha}\right] q(0);$$

$$W_1(\omega) = W_0(\omega)\, Q\left[\kappa = \frac{2\omega}{c\sin\alpha}\right] e^{i\omega\tau_0}, \quad (8)$$

where

$$q(\kappa) = \int_{-H_2}^{H_1} e^{-i\kappa y}\sigma^2(y)dy,$$

$$Q(\kappa) = q(\kappa)/q(0), \quad (9)$$

and the value of time lag τ_0 is given by Eq. (3). In accordance with Eqs. (7) and (8) the energy spectrum of the fluctuations of the total phase is written as

$$W_s(\omega) = \int_{-\infty}^{+\infty} e^{i\omega\tau} B_s(\tau)d\tau =$$

$$= 2W_0(\omega)\left[1 + Q\left[\frac{2\omega}{c\sin\alpha}\right]\cos\omega\tau_0\right]. \quad (10)$$

Note at once that although this expression is obtained here within the geometric optics approximation, it holds for a wider class of situations. One only has to replace the function $W_0(\omega)$ in Eq. (10) by the respective expression for the phase fluctuation spectrum obtained by the "smooth perturbations" technique[1-3] for single mode sensing.

It follows from Eq. (7) that the phase autocorrelation function $B_s(\tau)$ has one central maximum at $\tau = 0$ and two symmetric side maxima at $\tau = \pm\tau_0$. The sign of τ_0 may vary, which is of no fundamental importance, because of the symmetry of expression (7). For this reason in what follows we will take τ_q to be positive. According to Eq. (3) the value of τ_0 depends uniquely on the transport velocity of the inhomogeneities V, and on the speed at which the beam line moves through the turbulent layer u. As can be seen from Fig. 2, where the dependence of τ_0 on V is presented in normalized form, one may place the range of velocities V between $-4u$ and $2u$, where the turbulent flux velocities may be retrieved from the measured time lag τ_0 with satisfactory accuracy. Note that when the vector of the inhomogeneities transport velocity and that of the beam line movement approximately coincide $(V - u)$, there is a singularity in the profile $(\tau_0 - \infty]$, so to determine the velocity V in that case one needs quite a long record of phase fluctuations.

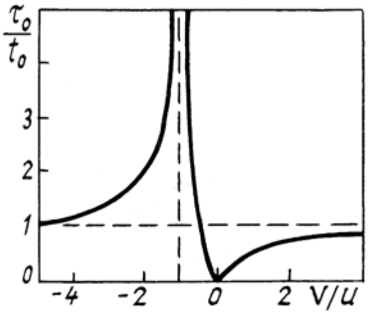

FIG. 2. Correlation lag time as a function of inhomogeneities transport velocity.

The value of τ_0 and the transport velocity of the inhomogeneities V can also be found from the energy spectrum of the phase fluctuations (10), where τ_0 determines the characteristic period of the interference oscillations $F_0 = 1/\tau_0$. The amplitudes of the maxima and minima in the energy spectrum $W_s(\omega)$, whose positions are determined by the conditions $\omega\tau_0 = \pi(2n - 1)$ and $\omega\tau_0 = 2\pi n$, $n = 1, 2, 3, \ldots$, gradually decrease as the factor $Q(2\omega/c\sin\alpha)$ increases. The thicker the turbulence layer, the faster this factor decreases at higher frequencies ω. In that sense the factor Q describes the averaging effect of the finite layer thickness upon the phase fluctuation spectrum in the case of double sensing. According to Eq. (9) the function $Q(\kappa)$ is the normalized spatial spectrum of $\sigma^2(y)$. Measurements of $Q(\kappa)$ provide, after inverting the integral transformation Eq. (9), information on the spatial variations of the variance of the refractive index fluctuations $\sigma^2(y)$.

All the above relates to phase fluctuations but is easily generalized to describe the frequency fluctuations $F = (2\pi)^{-1}ds/dt$. To obtain the autocorrelation function $B_F(\tau)$ and the energy spectrum $W_F(\omega)$ of the frequency fluctuations, it suffices to recall that

$$B_F(\tau) = \frac{-1}{(2\pi)^2} \cdot \frac{d^2}{d\tau^2} B_s(\tau),$$

$$W_F(\omega) = \left[\frac{\omega}{2\pi}\right]^2 W_s(\omega).$$

An analysis of the results obtained from these relationships demonstrates that all the conclusions formulated above for double sensing are also valid for frequency fluctuations.

DOUBLE SENSING WITH A MOVING RADIATION SOURCE

The obtained solution can be generalized to a source moving at a velocity V_A. It then suffices to transform to a coordinate system moving along with the source, and to apply the relations obtained above, in which the variable V_1 is replaced by the variable $V_1' = V_1 + V_A$, and V — by $V' = V - V_A$. We then have

$$\tilde{V} = V + V_1 \frac{H_2}{H} - V_A \frac{H_1}{H} ; \qquad (11)$$

$$\tau_0 = t_0 \left[V - V_A \right] / \left[V + V_1 \frac{H_2}{H} - V_A \frac{H_1}{H} \right]. \qquad (12)$$

Another important case follows from relations (11) and (12), which are rather general in their form anyhow. Indeed, with the source at rest ($V_1 = 0$) we can write

$$\tilde{V} = V - V_A \frac{H_1}{H},$$

$$\tau_0 = t_0 \left[V - V_A \right] / \left[V - V_A \frac{H_1}{H} \right].$$

Expressions for the correlation and energy spectra of the phase and frequency retain their original form. This case of sensing may be realized if a microwave or optical locating system is mounted on a moving ground-based or airborne platform, e.g., an aircraft.

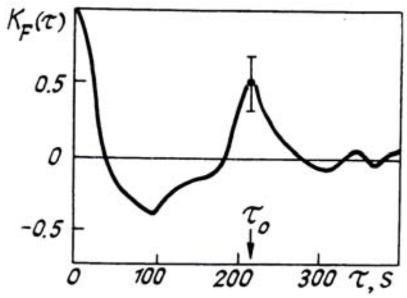

FIG. 3. Experimental autocorrelation coefficient for the frequency fluctuations

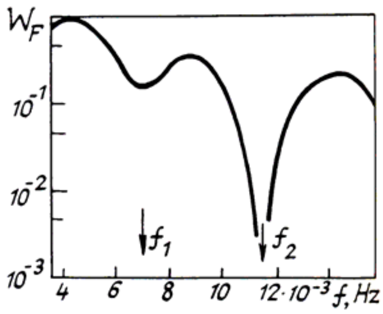

FIG. 4. The normalized energy spectrum for the frequency fluctuations

In conclusion let us consider an example which demonstrates the practical applicability of the suggested technique that has to do with double microwave sensing of eddy inhomogeneities in the solar wind.[7] A monochromatic signal in the decimeter range was emitted from the Earth, received on board the "Venera-15" spacecraft and retranslated back to the Earth, preserving its state of phase coherence. Figure 3 presents the results of measurements of K_F — the autocorrelation coefficient for the frequency fluctuations retrieved from its 30-second running averages, when the sensing beam passed close to the Sun at a sighting distance of 4.9 solar radii. Together with the central maximum at $\tau = 0$ a distinct side maximum at $\tau = \tau_0 = 220$ s is observed. The vertical bar shows the 90% significance interval for $K_F(\tau)$ in the vicinity of $\tau = \tau_0$. According to Eq. (12) the found value of τ_0 corresponds to a solar wind velocity of $V = 46$ km/s. Such a value agrees quite well with the known values of V for this part of solar space. Figure 4 presents a fragment of the normalized energy spectrum for the frequency fluctuations, which has been Fourier-transformed and corresponds to the correlation function plotted in Fig. 3.

The abscissas give the cyclic spectral frequency $f = \omega/2\pi$. It can be seen that sharp minima are present at $f_1 = 7 \cdot 10^{-3}$ Hz and $f_2 = 11.5 \cdot 10^{-3}$ Hz. Their position (corresponding to $f_1 = 3/2\tau_0$ and $f_2 = 5/2\tau_0$) and their repetition period ($F_0 = f_2 - f_1 = 1/\tau_0$) testify to the accuracy of our results ($T_0 = 220$ s, $V = 46$ km/s). The possible error in estimating τ_0 (± 30 s) should only result in an error of ± 3 km/s in V. The envelope of the spectrum can be employed to find $Q(\kappa)$ and thereby retrieve the spatial dependence of the refractive index fluctuations intensity.

CONCLUSION

It has been shown as a result of our analysis that monostatic double sensing of eddy fluxes using a moving reflector is equivalent to spatially distributed sensing and that the former technique can be applied to measuring the velocity and spatial distribution of atmospheric inhomogeneities. The applicability of the suggested technique to sensing extended inhomogeneous media has been demonstrated by experiment. Such a technique can be applied to sensing the lower and upper terrestrial atmosphere, and interplanetary and circumsolar space in both the microwave and the optical ranges.

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