EFFICIENCY OF ADAPTIVE CORRECTION OF RANDOM WAVEFRONT TILTS OF LASER BEAMS PROPAGATING THROUGH THE TURBULENT ATMOSPHERE

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Efficiency of phase-conjugated correction of random wavefront tilts is studied to shape laser beams propagating through the turbulent atmosphere. Amplification of the average intensity of radiation and degree of suppression of the intensity fluctuations on the beam axis in the receiving plane chosen as perfomance figures of the correction efficiency are analyzed. Assuming that the correction for random tilts is practically identical to control of the random shifts of the beam's energy center of gravity we have avaluated the relative contribution of the latter to the diminishment of the average intensity of collimated beams with the Fresnel number of the order of unity.

Laser beam propagation through the turbulent atmosphere is always accompanied by beam broadening as well as by radiation intensity fluctuations at a point of reception.[1] Such phenomena have a negative effect on the operation of atmospheric laser systems, in particular, lines of optical communication, thereby deteriorating, e.g., noiseproof characteristics and reliability of information transmission. To compensate for the effect of the atmospheric turbulence, the adaptive optical systems[2,3] can be used.

The present paper concerns with investigation of the efficiency of a phase–conjugated system intended for correcting the direction of laser beam propagation. It is assumed here that the correction is based on the measurements of random wavefront tilts of the wave field being analyzed within the transmitting aperture and formed by a point reference source. Such a correction is the simplest from the viewpoint of technical realization. Moreover, it is the large–scale phase fluctuations (within the aperture associated with random wavefront tilts) for which the principle of additivity can be expected to be hold[2] even under conditions of radiation propagating along the extended paths. The wavefront tilts are less sensitive than small–scale phase fluctuations to the effect of the other factors degrading the correction efficiency.[2]

The fact that the correction under study is practically identical to the control of the random shifts of the laser beam's energy center of gravity in the receiving plane evokes additional interest.[3] In close connection with this is the problem of the effect of the beam shifts on relative variance of intensity fluctuations β^2, i.e., the difference between β^2 calculated (or experimentally measured) in a stationary coordinate system and the one obtained in the coordinate system affixed to the laser beam's center of gravity[4,5] as well as the problem of the β^2 dependence on a type of radiation diffraction at the transmitting aperture.[6,7]

To obtain the numerical results, we make use of the phase approximation of the Huygens–Kirchhoff method[1] generalized to randomly inhomogeneous media according to which the corrected field u_c at a point $\mathbf{R}$ of the receiving plane $x = x_0 + L$ is written down in the form

$$u_c(L,\mathbf{R}) = \frac{k}{2\pi iL}\int d^2\rho_0 u_0(\rho_0)\times$$

$$\times\exp\left[\frac{ik}{2L}(\mathbf{R}-\rho_0)^2 + iS(\rho_0) - iS_c(\rho_0)\right], \quad (1)$$

where $u_0(\rho_0)$ is the initial distribution of the laser beam field in the transmitting plane $x = x_0$, $S(\rho_0)$ is the random phase of an elementary spherical wave propagating from the point ρ_0 of the plane x_0 to the point R of the plane $x_0 + L$, $S_c(\rho_0)$ is the wave phase of the point reference source employed in correcting and located in the receiving plane, L is the path length, and $k = 2\pi/\lambda$ is the wave number.

If the correction of the wavefront tilts alone is accomplished

$$S_c(\rho_0) = \alpha\rho_0, \quad (2)$$

then a random vector α for the axisymmetric amplitude function $\Omega_0(\rho_0)$ of the transmitting aperture is determined in the following way:[8,9]

$$\alpha = \int d^2\rho_0\rho_0 W_0(\rho_0)S_r(\rho_0)/[\pi\int\limits_0^\infty d\rho_0\rho W_0(\rho_0)], \quad (3)$$

where $S_r(\rho_0)$ is the wave phase of the reference source.

The initial distribution of the beam field in the transmitting plane is assumed to be Gaussian

$$u_0(\rho_0) = u_0 W_0(\rho_0)\exp\left(-i\frac{k\rho_0^2}{2F}\right),$$

$$W_0(\rho_0) = \exp\left(-\frac{\rho_0^2}{2a^2}\right), \quad (4)$$

where F is the curvature radius of the phase front at the center of the emitting aperture and $2a = d$ is the effective diameter of the beam.

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The value $\theta(\rho_0) = S(\rho_0) - S_c(\rho_0)$ entering into relation (1) is the corrected phase of the elementary spherical wave. Under the assumption of normal distribution of θ the second $\Gamma_{c2}(L, \mathbf{R})$ and the fourth $\Gamma_{c4}(L, \mathbf{R})$ statistical moments of the field $u_c(L, \mathbf{R})$ are expressed in terms of the structure function

$$D_\theta(\rho_{01}, \rho_{02}) = < [\theta(\rho_{01}) - \theta(\rho_{02})]^2 > , \quad (5)$$

where the angular brackets dentote averaging over an ensemble of realizations of the propagation medium. Let Eq. (5) be represented in the form

$$D_\theta(\rho_{01}, \rho_{02}) = D_s(\rho_{01}, \rho_{02}) - D_c(\rho_{01}, \rho_{02}) , \quad (6)$$

where the components D_s and D_c are determined from the expression following the relations

$$D_s(\rho_{01}, \rho_{02}) = < [S(\rho_{01}) - S(\rho_{02})]^2 > ; \quad (7)$$

$$D_c(\rho_{01}, \rho_{02}) = 2 < [S_c(\rho_{01}) - S_c(\rho_{02})][S(\rho_{02}) - S(\rho_{01})] > ; \quad (8)$$

$$- < [S_c(\rho_{01}) - S_c(\rho_{02})]^2 > .$$

The first component is the structure function of the phase fluctuations of spherical waves without correction. For a locally homogeneous and isotropic turbulent medium with the Kolmogorov spectrum of fluctuations in the dielectric constant, the function D_s depends only on the absolute value of the vector $\rho_0 = \rho_{01} - \rho_{02}$ and is determined from the relations[1]

$$D_s(t) = D_0(d)\varphi(t; l_a) , \ \varphi(t; z) = (t^2 + z^2)^{5/6} - z^{5/3} , \quad (9)$$

where $D_0(z) = 0.27C_\varepsilon^2 k^2 L z^{5/3}$, $l_a = (2a\kappa_e)^{-1}$; C_ε^2 is the structure characteristics of fluctuations in the dielectric constant; $\kappa_e = 1.60/l_0$ is the wave number corresponding to the inner scale l_0 of turbulent inhomogeneities, and $t = \rho_0/d$ is the normalized spacing.

The second component D_c was calculated elsewhere.[3,10,11] Using the results obtained in Ref. 11 after some transformations, we have

$$D_c(\rho_{01}, \rho_{02}) = 1.09D_0(d)t^2 1 - 0.15[t^2 + T^2(1 + 2\cos(\mathbf{t}, \hat{\mathbf{T}}))]\}, \quad (10)$$

where $\mathbf{T} = (\rho_{01} + \rho_{02})/d$. Relation (10) indicates that the phase fluctuations determined by the structure function D_c are inhomogeneous since they depend not only on the distance between the points ρ_{01} and ρ_{02} but also on the choice of the position of the point (vector $\mathbf{T}$). However, the degree of this inhomogeneity is not very high, therefore the expression in the square brackets of formula (10) may be averaged over the aperture, i.e., taking into account the amplitude coefficient $W_0(\rho_0)$. Thus we finally arrive at the following approximation:

$$D_c(t) = 0.76\, D_0(d)t^2 , \quad (11)$$

which differs from the corresponding relation derived in Ref. 10 on the assumption that the random wavefront tilts α and small–scale phase fluctuations $[S(\rho_{0i}) - \alpha\rho_{0i}]$ are independent only in the numerical coefficient 0.76 in place of 0.88.

Approximation (11) for the structure function D_c can be used, when writing down the field moments u_c, to reduce the multiplicity of integration and finally to obtain

$$\Gamma_{c2}(L, \mathbf{R}) = \frac{\Omega^2}{\pi} u_0^2 \int d^2t \cos\left(2\Omega \frac{\mathbf{R}}{a} \mathbf{t} \right) \times$$

$$\times \exp\left[- g^2t^2 - \frac{1}{2} D_0(d)\varphi_c(t; l_a) \right] , \quad (12)$$

$$\Gamma_{c4}(L, \mathbf{R}) =$$

$$= \left(\frac{2}{\pi} \right)^3 \Omega^4 u_0^4 \int d^6t_{1,2,3} \times \cos\left[4\Omega(1 - L/F)\mathbf{t}_1\mathbf{t}_2 - 4\Omega \frac{\mathbf{R}}{a} \mathbf{t}_3 \right] \times$$

$$\times \exp\left\{ - 2[t_1^2 + t_2^2 + g^2t_3^2] - \frac{1}{2} D_0(d) \times \right.$$

$$\times \left[\sum_{n=1}^{2} \varphi_c(|\mathbf{t}_3 + (-1)^n\mathbf{t}_2|; l_a) + \right.$$

$$\left. \left. + \sum_{j=1}^{2} \sum_{m=2}^{3} (-1)^{m-1}\varphi_c(|\mathbf{t}_1 + (-1)^j \mathbf{t}_m|; l_a) \right] \right\} , \quad (13)$$

where $\Omega = a^2/\rho_F^2$ is the Fresnel number of the transmitting aperture, $\rho_F = \sqrt{L/k}$, $g^2 = 1 + \Omega^2(1 - L/F)^2$, and $\varphi_c(t; z) = \varphi_c(t; z) - 0.76\, t^2$.

It should be noted that the relations for the moments of the laser beam field without correction can be derived from Eqs. (12) and (13) by replacing φ_c by φ. Recall also that the calculation of Γ_{c2} yields the average intensity of the corrected field $<I_c>$ and the fourth moment Γ_{c4} determines the relative variance of the intensity fluctuations

$$\beta_c^2 = \Gamma_{c4}(L, \mathbf{R}) / \Gamma(L, \mathbf{R}) - 1 . \quad (14)$$

Equations (12) and (13) were integrated out numerically using the Monte Carlo method.[12] The relative error estimated from the standard deviations obtained in the calculations from the sequence of random values did not exceed 2% for Γ_{c2} and 10% for Γ_{c4} (except for one regime of propagation described below).

To compare the results of calculation from formulas (12)–(14) with experimental and theoretical studies presented, e.g., in Refs. 13–17, we express the structure function $D_0(d)$ in terms of the transverse coherence radius $r_0 = (0.37C_\varepsilon^2k^2L)^{-3/5}$ of a plane wave

$$D_0(d) = 0.74\, (d/r_0)^{5/3} . \quad (15)$$

In this case, the ratio d/r_0 is also related to the relative variance of the intensity fluctuations of a plane wave $\beta_0^2 = 0.31C_\varepsilon^2k^{7/6}L^{11/6}$ calculated in the first approximation of the smooth perturbation method and characterizing the turbulence intensity along the propagation path. The aforementioned relationship is given by the formula

$$\beta_0^2 = 0.26\Omega^{-5/6}(d/r_0)^{5/3} . \quad (16)$$

Let us consider the characteristics of the corrected field on the axis ($R = 0$) of the focused ($L/F = 1$) and narrow collimated ($L/F = 0$, and $\Omega = 1$) laser beams. Given in the table are the results of calculation of the ratio $\eta_0 = \langle I_c\rangle/\langle I\rangle$ (in decibels) as functions of the parameter (d/r_0). Presented here too are the corresponding values of β_0 calculated for the beam with the Fresnel number $\Omega = 1$ as well.

The maximum $\eta_0 \simeq 5.3$ dB for a focused beam is in a good agreement with the result derived in Ref. 11 using exact relation (10). The calculated results for a focused beam are also plotted in Fig. 1 (solid line) along with the experimental data (dots) borrowed from Ref. 14 ($\lambda = 0.63$ μm, $L = 1.6$ km, and $d \sim 15$ cm). It is difficult to compare these results with the experimental ones due to the difference between the field distributions over the really employed transmitting apertures and that of unlimited Gaussian beam as well as due to the discrepancy in the really controlled wavefront tilts and the values of α given by relation (5). However, as shown in Refs. 3 and 15, these discrepancies are not so large. Therefore the experimental data of Ref. 14 are plotted in Fig. 1 at the same scale but are displaced along the abscissa at the fixed distance which provides their best agreement with the calculated results. Taking into account a large spread of points and possible effect of the above–indicated factors, it is possible to ascertain quite satisfactory agreement between the calculated and experimental data. That is also an indirect verification of applicability of approximation (11) for the description of the structure function D_c.

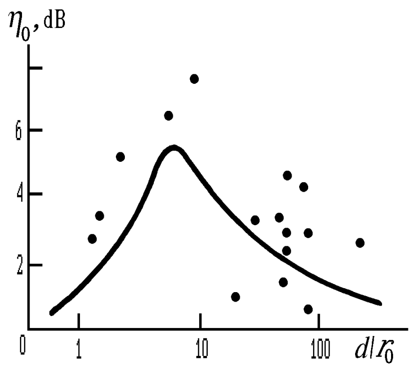

FIG. 1.

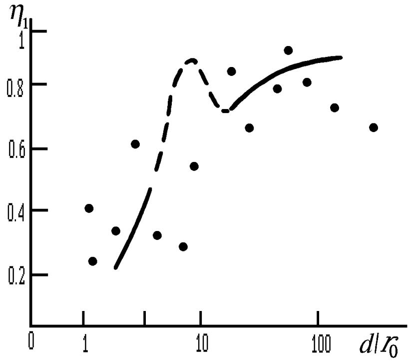

FIG. 2.

As follows from examination of the tabulated values, for a narrow collimated beam a gain in η_0 in correcting is somewhat smaller than that for a focused beam and the maximum in η_0 is shifted toward increased turbulent intensity on the path. The increase of the inner scale of turbulence results in the η_0 increase, particularly for strong turbulence as was noted previously in Ref. 16. In all instances the marked effect caused by correction of the tilts holds even for the very large ratios (d/r_0).

As has already been noted above, the correction of the beam propagation direction based on the analysis of random tilts of the radiation wavefront of a point reference source in the transmitting plane is essentially equivalent to processing of random displacements of the beam energy center of gravity in the receiving plane.[3] In this connection it is interesting to compare the results of more or less rigorous calculations by formulas (12) and (13) with those made using a phenomenological model[4,5,17] following which the beam radius (at e^{-1}) of maximum without correction w (in a stationary system of coordinates), is determined by the beam radius with correction w_c and by the variance of the shift of its center of gravity σ^2_{sh}:

$$w^2 = w_c^2 + \sigma^2_{sh} \,. \qquad (17)$$

This formula is based on the assumption of the Gaussian (on the average) beam profile as well as on the Gaussian shifts of its center of gravity. If relation (17) is valid the relative amplification of the average intensity on the beam axis, when introducing the correction of the shifts $\eta_0 = w^2/w_c^2$, can be estimated as

$$\eta_0 = [1 - \sigma^2_{sh}/w^2]^{-1} \,. \qquad (18)$$

The relation for the variance of the random shifts obtained in Markovian approximation has the form[1]

$$\sigma^2_{sh} = \frac{\pi}{2P_0^2}\int\limits_0^L dx(L-x)^2\int d^2\mathrm{k}k^2\,\Phi_\varepsilon(x,\mathbf{k}) \times$$

$$\times \int d^4R_{1,2}\exp[i\mathbf{k}(\mathbf{R}_1-\mathbf{R}_2)]\,\langle I(x,\mathbf{R}_1)I(x,\mathbf{R}_2)\rangle \,, \qquad (19)$$

where $P_0 = u_0^2\int d^2\rho_0 W_0^2(\rho_0)$ is the total radiation power, $\Phi_\varepsilon(x, \kappa)$ is the three–dimensional spectral density of the dielectric constant fluctuations in the medium. Following Ref. 18, we write down the second moment of the intensity in terms of the correlation coefficient K_I

$$\langle I(x,\mathbf{R}_1)I(x,\mathbf{R}_2)\rangle = \langle I(x,\mathbf{R}_1)\rangle\langle I(x,\mathbf{R}_2)\rangle\times$$

$$\times\,[1 + K_I(x,\mathbf{R}_1,\mathbf{R}_2)\beta(x,\mathbf{R}_1)\beta(x,\mathbf{R}_2)] \,. \qquad (20)$$

It is evident that for weak turbulence ($\beta \lesssim 1$) the second term in the square brackets of Eq. (20) can be neglected which is the essence of the so–called average–intensity approximation.[1] Calculations from formula (19) for a locally homogeneous turbulent medium with the Kolmogorov spectrum $\Phi_\varepsilon(x, \kappa)$ and a beam with the initial field distribution in the form of Eq. (4) enable us to go over to the following representation

$$\sigma^2_{sh} = 0.34\ (1 + \alpha)\ f_{sh}\ C^2_\varepsilon\ L^3\ a^{-1/3}\ , \qquad (21)$$

where

$$f_{sh} = 3\int\limits_0^1 d\xi(1-\xi)^2 q^{-1/6}(\xi)\ ,\ q(\xi) = (1-\xi L/F)^2 + \Omega^{-2}\xi^2[1 + $$

$$+\ 0.44 D_0^{6/5}\ (d)\xi^{6/5}]\ .$$

Taking into account that the beam radius in the stationary coordinate system is estimated as $w = aq^{1/2}(1)$, the effect of introduction of the correction is given by the simple relation:

$$\eta_0 = [1 - 0.38\ (1 + \alpha)\ f_{sh}\ q^{-1}\ (1)\ \Omega^{-2} D_0(d)]^{-1}\ . \qquad (22)$$

The coefficient α introduced into relation (21) is the correction for the average–intensity approximation (for which $\alpha = 0$). The physical meaning of this coefficient consists of the consideration of the effect of the effective beam radius fluctuations along the propagation path on the shifts of its center of gravity in the receiving plane. Unfortunately, it is difficult to estimate the value of α (the corresponding asymptotic expressions were given in Ref. 1, however, in the region of the parameters of interest for us the residual terms have the same order as the principal terms of the expansion have). Therefore, as the first approximation, the values of η_0 were calculated using Eq. (22) for $\alpha = 0$. The results for a narrow collimated beam are listed in Table I. It is noteworthy that for weak turbulence which the conditions of applicability of the average–intensity approximation hold for, the results of η_0 calulations from formulas (12) and (22) agree. From stronger turbulence the values of η_0 determined from Eq. (22) turn out to be underestimated (for $\alpha = 0$), though the general form of the $\eta_0(d/r_0)$ dependence remains the same. If we assume that for strong turbulence the values of η_0 obtained by different methods must agree, then it is easy to estimate the correspondingly required values of α (see Table I for $L/F = 0$ and $\Omega = 1$). Analogous results can be obtained for a focused ($L/F = 1$) beam. The beam defocusing ($L/F \neq 1$) leads to the $q(1)$ increase and, since f_{sh} is a slowly varying function of L/F, to the degradation of the correction efficiency (estimated by the value of η_0) due to the decrease of the relative contribution of random shifts to the total beam broadening.

TABLE I.

d/r_0	η_0 ($L/F = 1$, $l_a = 0$)	β_0	η_0 ($L/F = 0$, $\Omega = 1$) $l_a = 0$	$l_a = 0.2$	from Eq. (25) for $\alpha = 0$	α
0.97	1.02	0.5	0.54	0.54	0.55	−0.01
2.23	2,99	1.0	1.73	1.74	1.59	0.07
3.63	4,53	1.5	2.86	2.92	2.22	0.21
5.13	5.27	2.0	3.60	3.72	2.38	0.33
6.70	5.26	2.5	3.93	4.18	2.33	0.43
8.34	4.89	3.0	3.99	4.34	2.21	0.51
15.40	3.48	5.0	3.33	4.07	1.73	0.63
23.06	2.84	7.0	2.77	3.79	1.41	0.70
35.38	2.31	10.0	2.29	3.57	1.12	0.81
57.55	1.85	15.0	1.89	3.46	0.85	0.99

Let us proceed now to the estimate of the efficiency of correction of the wavefront tilts for suppression of the intensity fluctuations. Depicted in Fig. 2*a* are the results of calculation of the ratio $\eta_1 = \beta_c/\beta$ for a focused beam (the designations are the same as in Fig. 1). From this figure it can be seen that with a general monotonous behavior of the $\eta_1(d/r_0)$ dependence, in the region of the so–called strong random focusing of radiation ($D_0(d) \sim 25$), the fluctuation suppression for a focused beam deteriorates significantly (the calculated values of η_1 belonging to this region are shown in Fig. 2 with a dashed line). However, it should be noted that the relative error of calculation of β_c for this regimes sharply increased (up to ~ 30% for the selected number of statistical tests). This requires careful analysis of the corresponding data, though, generally speaking, such a result could be expected. Virtually in this region the inhomogeneities which cause random focusing of radiation (for a focused beam we are most likely dealing with random defocusing) make greater contribution and, hence, the relative contribution of the beam diviation from the initial direction of propagation to β^2 becomes smaller.

For a narrow collimated beam a monotonous behavior of $\eta_1(d/r_0)$ in fact is not violated. For strong turbulence, the η_1 values for the collimated (η_1^{col}) and focused (η_1^f) beams coincide. For weak turbulence $\eta_1^{col} > \eta_1^f$, e.g., $\eta_1^{col}(2) = 0.58$ and $\eta_1^f(2) = 0.24$. The increase in l_a, when $\beta_0^2 \gg 1$, results in the decrease of η_1 (e.g., when $l_a = 0.2$ and $\beta_0 = 15$, we have $\eta_1^{col} = 0.73$ in place of $\eta_1^{col} = 0.89$ corresponding to the zeroth inner scale).

In defocusing, taking into account practically constant absolute value of σ^2_{sh}, the values of η_1^{-1}, along with η_0, would decrease, at least, for relatively weak turbulence. However, such a conclusion is unreasonable since the degree of internal inhomogeneity of the beam speckle structure can vary. Analogous variations can also be observed when the size of the aperture Ω changes for both focused and collimated beams. Therefore, in contrast to the mean values for fluctuation characteristics of spatially limited laser beams, the simplified phenomenological approach assuming a sort of additivity of fluctuations caused by different scales of turbulent inhomogeneities in the propagation medium cannot be considered adequate to the physics of the phenomenon. Nevertheless, it is possible to assume that the increase of the variance of the intensity fluctuation for collimated beams, when $\Omega \sim 1$, for a relatively strong turbulence[6,7] is associated with the increased contribution of random shifts of the beam's center of gravity to β^2.

By comparing the obtained dependences of the performance figures of the correction efficiency η_0 and η_1 on the parameter (d/r_0) it is not difficult to establish that the optimal suppression of fluctuations (η_1), on the one hand, and the optimal amplification of the radiation intensity on the beam axis (η_0), on the other hand, are provided for different regimes of turbulence. This is particularly true for a focused beam for which a substantial supression of fluctuations is observed at a much lower level of turbulence compared to the value corresponding to the maximum value of η_0.

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