Approximate methods for simulation of diffuse radiation

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The Eddington model approximation for calculation of diffuse ultraviolet radiation is considered. All equations needed for modeling are derived, and a computer code is developed. The two-stream approximation is considered as well. The model data are compared with the results of the Monte Carlo method and experimental data, satisfactory agreement is demonstrated, and the Eddington model approximation is shown to be applicable to solution of practical problems of atmospheric optics in the ultraviolet spectral region.

Introduction

Ultraviolet (UV) radiation makes up less than 5% in the net flux of the solar radiation,[1] but its impact on the human environment is quite significant. This is due to strong influence of the UV radiation on many biological and chemical processes. For example, the ecologically hazardous phenomenon of smog formation in the atmosphere is, to a great degree, due to its stimulating effect on the rate of photochemical reactions.

Spectral composition of the UV radiation in the atmosphere is determined by absorption by ozone in the Hartley and Higgins bands and oxygen (in a short UV range of 180–220 nm). From the viewpoint of efficiency of the biological effect, three wavelength ranges are usually separated[1]: A ($315 < \lambda < 400$ nm), B ($280 < \lambda < 315$ nm), and C ($\lambda < 280$ nm). In this paper we consider the A and B ranges. Atmospheric aerosol and, to a great degree, clouds also affect propagation of the UV radiation.

The net UV radiation near the Earth's surface is mainly determined by the Sun elevation, cloudiness, total column ozone (TCO) in the atmosphere, surface albedo, and the aerosol composition of the atmosphere. Continuous UV monitoring in different regions of the globe becomes increasingly urgent in recent years in connection with the observed climate and ecology changes, especially, in the ozone layer. However, direct measurements of UV fluxes are rather difficult, therefore, calculation of UV levels based on the data on the atmospheric conditions at some particular points of the globe is now an inevitable part of the UV monitoring. This approach to organization of the UV monitoring network requires a rather fast algorithm for high-accuracy estimation of the UV radiation intensity.

1. Problem formulation

Estimation of characteristics of the UV radiation field in the corresponding spectral range presents significant difficulties associated with multiple scattering of UV radiation in the atmosphere. Algorithms based on the Monte Carlo method are widely used in the recent years for description of this process.[2,3] Without dwelling on advantages and disadvantages of the Monte Carlo method, note that this method, being highly computationally expensive, is of little use for engineering calculations. Besides, the initial data (for example, optical properties of clouds) are usually known only approximately and, therefore, the main advantages of this method (possibility of detailed consideration of all the factors determining the radiative transfer process in a medium and high accuracy of calculation) loss their significance. That is why engineering calculations widely involve approximate methods for calculation of diffuse radiation (see, for example, the Lowtran-7 software,[4] Refs. 5–10). This was favored, in particular, by the work of the International Commission on Radiation, which initiated the program of investigations on intercomparison of different methods for calculation of diffuse radiation fluxes in the non-absorbing atmosphere. It was shown in Ref. 7, in particular, that the approximate Eddington method provides for a good accuracy (within 10%).[6,7] In this paper, a similar comparison (for the Eddington method) is performed for the absorbing atmosphere in the presence of a reflecting surface.

Distinctive features of the UV region are much stronger effect of Rayleigh scattering and the presence of a strong ozone absorption band. This fact calls for some changes to be introduced into the numerical scheme of calculation of the scattered radiation both

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in the Monte Carlo method when modeling trajectories of photon random walks and in approximate methods.

The principles of the method are considered by Sobolev in Ref. 6, and it is outlined in Refs. 7 and 11. Reference 6 presents the equations for the transmission and reflection coefficients, albedo, and diffuse transmission of the atmosphere for the case of the "black" surface (zero surface albedo). For the case of the reflecting surface, Ref. 6 gives the equations for the characteristics mentioned above only for pure scattering. The equations for albedo and atmospheric transmittance in the case of the black surface are presented in Ref. 7. Here we also use the Sobolev's approach when deriving the equations of the Eddington method, which was chosen because it likely is the only approximate method that allows calculation of not only fluxes, but also intensities. Further we will calculate just the diffuse radiation intensity. Below we give the main prerequisites of the approximate Eddington method, transfer equations, and boundary conditions. The final equations are too cumbersome and therefore they are presented in Appendix. Then the Eddington method is used for simulation of the UV radiation (A and B ranges). The two-stream approximation is considered as well. The results are compared with those of the Monte Carlo method (considered as a reference one) and experimental data. The investigations are performed for the following formulation of the problem.

Let a parallel flux of solar radiation with the power πS be incident on the top boundary of the plane-parallel atmosphere at the angle of ν_0. Write the radiative transfer equation for the plane-parallel atmosphere[6]:

$$\mu\frac{dI(\tau,\mu,\eta,\varphi)}{d\tau} = -I(\tau,\mu,\eta,\varphi) + \frac{\omega_0}{4\pi}\int_0^{2\pi}d\varphi'\int_{-1}^{1}I(\tau,\mu',\eta,\varphi')p(\gamma')d\mu' + \frac{\omega_0}{4}Sp(\gamma)e^{-\tau/\eta}, \quad (1)$$

where

$$\eta = \cos\nu_0;\ \mu = \cos\nu;$$

$$\cos\gamma' = \mu\mu' + \sqrt{(1-\mu^2)(1-\mu'^2)}\cos(\varphi-\varphi');$$

$$\cos\gamma = \mu\eta + \sqrt{(1-\mu^2)(1-\eta^2)}\cos\varphi.$$

Here ν_0 is the zenith angle of solar ray incidence; ν is the zenith angle of observation; φ is the azimuth angle of observation; γ is the angle between the directions of the incident and scattered radiation; ω_0 is the single scattering albedo ($\omega_0 = \sigma/\alpha$, where σ is the scattering coefficient; α is the extinction coefficient; $\alpha = \sigma + \alpha_a + \alpha_g$, where α_a, α_g are the aerosol and gas absorption coefficients); πS is the illumination at the top boundary of the atmosphere (solar constant); $\tau = \int_h^{\infty}\alpha(h)dh$ is the optical depth; $\tau_0 = \int_0^{\infty}\alpha(h)dh$ is the optical thickness of the atmosphere; $p(\gamma)$ is the scattering phase function satisfying the normalization condition:

$$\frac{1}{2}\int_0^{\pi}p(\gamma)\sin\gamma\, d\gamma = 1.$$

2. Eddington approximation

Expand the scattering phase function $p(\gamma)$ into a series in terms of the Legendre polynomials up to the linear term inclusive:

$$p(\gamma) = 1 + p_1\cos\gamma. \quad (2)$$

Then, according to Ref. 6, the radiation intensity can be presented as a sum of two components I^0 and I^1:

$$I(\tau,\mu,\eta,\varphi) = I^0(\tau,\mu,\eta) + I^1(\tau,\mu,\eta)\cos\varphi. \quad (3)$$

For determination of each of these terms, we have individual integro-differential equations of the following form:

$$\mu\frac{dI^m(\tau,\mu,\eta)}{d\tau} = -I^m(\tau,\mu,\eta) + B^m(\tau,\mu,\eta),\quad m = 0,1, \quad (4)$$

where

$$B^m(\tau,\mu,\eta) = \frac{\omega_0}{4\pi}\int_0^{2\pi}d\varphi'\int_{-1}^{1}I^m(\tau,\mu',\eta)p(\gamma')d\mu' + \frac{\omega_0}{4}Sp(\gamma)e^{-\tau/\eta}.$$

These equations are solved under the following boundary conditions: (1) there is no incident diffuse radiation at the top boundary; (2) the diffuse radiation at the bottom boundary is formed from the bottom by the Lambert reflecting surface with the albedo A_0.

The component I^0 is the azimuthally averaged radiation intensity. The second term in Eq. (3) is the azimuth-dependent addition to the radiation intensity.

The main idea in derivation of the equations for the diffuse radiation intensity is the following: substitute Eq. (2) into Eq. (4), then for I^0 we can write

$$\mu\frac{dI(\tau,\mu,\eta)}{d\tau} = -I(\tau,\mu,\eta) + \omega_0\bar{I}(\tau,\eta) + \omega_0\beta_1\bar{H}(\tau,\eta)\mu + \frac{\omega_0}{4}S(1+\beta_1\mu\eta)\exp\left(-\frac{\tau}{\eta}\right). \quad (5)$$

In Eq. (5) the function I^0 is denoted as I for simplicity and

$$\bar{I}(\tau,\eta) = \frac{1}{2}\int_{-1}^{1}I(\tau,\mu,\eta)d\mu,\ \bar{H}(\tau,\eta) = \frac{1}{2}\int_{-1}^{1}I(\tau,\mu,\eta)\mu d\mu.$$

From Eq. (5) we can get the system of approximate equations for the functions $\bar{I}$ and $\bar{H}$. Integrating Eq. (5) over μ from –1 to 1, we obtain

$$\frac{d\bar{H}(\tau,\eta)}{d\tau} = -(1-\omega_0)\bar{I}(\tau,\eta) + \frac{\omega_0}{4} S \exp\left(-\frac{\tau}{\eta}\right). \quad (6)$$

Multiplying Eq. (5) by μ and integrating it over μ from –1 to 1, as well as using the approximation

$$\frac{1}{2}\int_{-1}^{1} \bar{I}(\tau,\mu,\eta)\mu^2 d\mu = \frac{1}{3}\bar{I}(\tau,\eta),$$

we can write

$$\frac{d\bar{I}(\tau,\eta)}{d\tau} = -(3-\omega_0\beta_1)\bar{H}(\tau,\eta) + \beta_1 \frac{\omega_0}{4} S \exp\left(-\frac{\tau}{\eta}\right)\eta. \quad (7)$$

From Eqs. (6) and (7) we have the equation for $\bar{I}(\tau,\eta)$:

$$\frac{d^2\bar{I}(\tau,\eta)}{d\tau^2} = k^2\bar{I}(\tau,\eta) - \left[3 + (1-\omega_0)\beta_1\right]\frac{1}{\omega_0} S \exp\left(-\frac{\tau}{\eta}\right),$$

where

$$k^2 = (1-\omega_0)(3-\omega_0\beta_1).$$

The solution of this equation has the form

$$\bar{I}(\tau,\eta) = C_1\exp(-k\tau) + C_2\exp(k\tau) + D\exp\left(-\frac{\tau}{\eta}\right). \quad (8)$$

Substituting Eq. (8) into Eq. (7) we obtain

$$\bar{H}(\tau,\eta) = \left[C_1\exp(-k\tau) - C_2\exp(k\tau)\right]\frac{k}{3-\omega_0\beta_1} -$$

$$-\frac{\omega_0}{4} S \exp\left(-\frac{\tau}{\eta}\right)\frac{\eta}{1-k^2\eta^2}\left(1 + (1-\omega_0)\beta_1\eta^2\right). \quad (9)$$

To find the coefficients C_1, C_2 entering into Eqs. (8) and (9), we have to apply the boundary conditions. At $\tau = 0$ (on the top boundary) there is no diffuse radiation incident from the above, therefore we can write:

$$\bar{H}(0,\eta) = \frac{1}{2}\int_{-1}^{0} I(0,\mu,\eta)\mu d\mu = -\frac{1}{4}\int_{-1}^{0} I(0,\mu,\eta) d\mu,$$

and, consequently,

$$2\bar{H}(0,\eta) = -\bar{I}(0,\eta).$$

At $\tau = \tau_0$ on the bottom boundary we have the diffuse radiation formed by the reflecting Lambert surface with the albedo A_0. Let $I_r(\tau_0,\eta)$ stand for the intensity of the reflected radiation and $I_s(\tau_0,\eta)$ stand for the mean intensity of the diffuse radiation incident on the surface. Hence we can write

$$I_r(\tau_0,\eta) = A_0\left[I_s(\tau_0,\eta) + S\eta\exp\left(-\frac{\tau_0}{\eta}\right)\right].$$

Using the approximations

$$\bar{I}(\tau_0,\eta) = \frac{1}{2}\left(I_s(\tau_0,\eta) + I_r(\tau_0,\eta)\right),$$

$$\bar{H}(\tau_0,\eta) = \frac{1}{4}\left(I_s(\tau_0,\eta) - I_r(\tau_0,\eta)\right)$$

we find the boundary condition at $\tau = \tau_0$:

$$\bar{I}(\tau_0,\eta) - 2\bar{H}(\tau_0,\eta) =$$

$$= A_0[\bar{I}(\tau_0,\eta) + 2\bar{H}(\tau_0,\eta) + S\eta\exp(-\tau_0/\eta)].$$

Having substituted the determined functions $\bar{I}(\tau,\eta)$ and $\bar{H}(\tau,\eta)$ into Eq. (4) and solved it, we get the equation for $I(\tau,\mu,\eta)$. The integration constant arising in this process can be determined from the following boundary conditions. For the intensity of the downward radiation we have a zero boundary condition $I^{\downarrow}(\tau = 0,\eta,\mu) = 0$ (no diffuse radiation incident on the top boundary). For the intensity of the upward radiation, we take into account the diffuse radiation reflected from the surface

$$I^{\uparrow}(\tau_0,-\mu,\eta) = A_0[\bar{I}(\tau_0,\eta) + 2\bar{H}(\tau_0,\eta) + S\eta e^{-\tau_0/\eta}].$$

The solution for $I^1(\tau,\mu,\eta)$ can be found similarly.

Final equations for the intensity of the upward and downward radiation are given in Appendix.

The angle-integral atmospheric characteristics can be found directly from $\bar{H}(\tau,\eta)$ or $\bar{I}(\tau,\eta)$. For the atmospheric albedo we have

$$A(\eta) = -\frac{4}{S\eta}\bar{H}(0,\eta) = \frac{2}{S\eta}\bar{I}(0,\eta) = \frac{2}{S\eta}(C_1 + C_2 + D).$$

The atmospheric transmittance $V(\tau_0,\eta)$ with no reflection (black surface) is equal to the ratio of the surface illumination

$$4\pi\bar{H}(\tau_0,\eta) + \pi S\eta\exp(-\tau_0/\eta)$$

to illumination of the atmospheric top $\pi S\eta$ (Ref. 6). In the case of reflecting surface with the albedo A_0, the transmittance is determined from the condition that the incident flux is equal to the difference between the surface illumination $\pi S\eta V(\tau_0,\eta)$ and the energy reflected from the surface $A_0\pi S\eta V(\tau_0,\eta)$, that is,

$$4\pi\bar{H}(\tau_0,\eta) + \pi S\eta\exp[-\tau_0/\eta] = (1-A_0)\pi S\eta V(\tau_0,\eta).$$

Hence, we have

$$V(\tau_0,\eta) = \frac{1}{(1-A_0)}\left[\frac{4}{S\eta}\bar{H}(\tau_0,\eta) + \exp\left(-\frac{\tau_0}{\eta}\right)\right]. \quad (10)$$

3. Delta-Eddington approximation

For scattering phase functions with large forward peaks (large particles, clouds) the above method can lead to large errors, since it involves only two terms of expansion [see Eq. (2)]. To improve the accuracy, Ref. 11 uses the delta approximation of the scattering phase function. In this approximation, the forward peak is represented by the delta function

$$\tilde{p} = 2f\delta(1-\cos\gamma) + (1-f)(1+p_1\cos\gamma), \qquad (11)$$

where f is the parameter characterizing the fraction of the forward scattered radiation [at $f = 1$, $\tilde{p} = 2\delta(1-\cos\gamma)$].

Having introduced the scattering phase function (11) into the transfer equation, we obtain two integro-differential equations for I^0 and I^1 [see Eq. (4)] that are equivalent to those considered above, but with

$$\omega_0' = \omega_0(1-f)/(1-\omega_0 f); \qquad (12)$$

$$\tau' = (1-\omega_0 f)\tau \qquad (13)$$

in place of ω_0 and τ .

4. Two-stream approximation

Consider one more approximate method for calculation of diffuse radiation – the so-called two-stream approximation.[7]

In the general form, the equations for the diffuse upward and downward fluxes can be represented as

$$\frac{dF^+(\tau)}{d\tau} = \alpha_1 F^+(\tau) - \alpha_2 F^-(\tau) - \alpha_3\omega_0\pi F e^{-\tau/\eta},$$

$$\frac{dF^-(\tau)}{d\tau} = \alpha_2 F^+(\tau) - \alpha_1 F^-(\tau) + \alpha_4\omega_0\pi F e^{-\tau/\eta}.$$

These equations can be easily solved with the boundary conditions for $F^-(0)$ and $F^+(\tau_0)$, then the equation for the total transmittance of the radiation (direct and diffuse) for the case $\omega \neq 1$ can be written in the form

$$T = (1-G^2)e^{-k\tau_0}/(1-G^2e^{-2k\tau_0}); \qquad (14)$$

where

$$G = (r-s)(r+s), \; r = 1-\omega_0 f + \omega_0 b,$$

$$s = \left[(1-\omega_0 f)^2 - \omega_0^2 b^2\right]^{1/2}, \; k = s/\eta, \qquad (15)$$

and the coefficients b and f determine the fractions of the backward and forward scattered radiation, respectively.

Similarly to the Eddington approximation, it is possible to introduce the delta-two-stream approximation. Then in Eqs. (14) and (15), ω_0' and τ', calculated by Eqs. (12) and (13) are used in place of ω_0 and τ.

5. Statistical simulation of diffuse radiation in ozone absorption bands

The simulation of optical radiation propagation in disperse media by the Monte Carlo method is described in sufficient detail in Ref. 2. This paper considers the Monte Carlo algorithm based on the method of local estimates in the scheme of conjugate wandering.[2] The algorithm was selected based on recommendations to selection of modifications of the Monte Carlo method,[3] as well as from comparison of computer resources taken by Monte Carlo algorithms for estimation of UV fluxes at the ground level.[17]

The idea of Monte Carlo algorithms involving conjugate trajectories is based on consideration of the conjugate transfer equation and application of the optical reciprocity theorem.[2] In these algorithms, the initial point of every trajectory of photon random walks is chosen using the random number generator from the receiver area in the direct formulation of the problem. Then the photon mean free path is calculated, the probability of photon absorption by medium elements is selected using the random number generator (or calculated through the weighting coefficient), directional cosines of the new direction of photon motion are calculated, and so on. To estimate the sought flux, the probability of photon escape from the medium at a certain scattering point in the direction to the source (local flux estimate) is calculated by the equation:

$$\psi_n = \frac{e^{-\tau(r_n)}g(-\omega^{(c)}\omega_n)q(r_n)}{2\pi}|\Omega|, \qquad (16)$$

where n is the number of collision in the particle trajectory; $\tau(r_n)$ is the optical length from the point r_n to the atmospheric boundary in the direction $-\omega^{(c)}$ (to the sun); ω_n is the direction of the particle motion before collision at the point r_n; $q(r) = \sigma_s(r)/\sigma(r)$, $\sigma(r)$ and $\sigma_s(r)$ are volume extinction and scattering coefficients in the atmosphere, $g(\mu)$ is the normalized atmospheric scattering phase function; Ω is the solid angle of observation. The sought flux of the multiply scattered solar radiation is estimated as:

$$I = M\xi, \; \xi = \sum_{n=1}^{N}\psi_n,$$

where N is the number of collisions in the particle trajectory.

The method of local estimates in the scheme of conjugate trajectories (or method of conjugate wandering) has some advantages.[3] It can be relatively readily implemented on a computer, and the estimate (16) has a finite variance. Besides, this method is efficient when estimating the flux intensity in small areas or at a point of the phase space in the illuminated part of the atmosphere. The main its disadvantage is that it is incapable of calculating the contribution of every collision point for several arbitrary observation points. However, the symmetry of the problem allows calculating the contributions of every collision to trajectories simultaneously for several solar zenith angles under the condition of that the receiver's aperture angle and sighting direction are constant. The accuracy of flux estimation by this method increases with the decrease of $|\Omega|$, all other conditions being the same.

6. Simulation of diffuse radiation

As was already mentioned, the spectral composition of the UV radiation in the atmosphere in the A ($315 < \lambda < 400$ nm) and B ($280 < \lambda < 315$ nm) bands is determined by the ozone absorption in the Hartley and Higgins bands, as well as by scattering and absorption at clouds and aerosols.

The atmosphere in the simulation was represented by a homogenous medium, and its parameters (ω_0, $p(\cos\theta)$) were specified as height averages. As the initial aerosol model, we used the LOWTRAN-7 optical model,[4] whose data were height averaged and then used in practical calculations (Table 1). The surface was assumed reflecting with the albedo equal to 0.6. The input data were the optical thickness and the cloud type (Table 2), as well as the scattering phase function.[4] The Monte Carlo method was used as a reference one.

Table 1. Atmospheric characteristics[5]

ν, nm	τ	ω_0
0.28	$0.3737 \cdot 10^2$	0.1076
0.29	$0.1404 \cdot 10^2$	0.1930
0.30	4.861	0.3322
0.31	2.179	0.4970
0.32	1.479	0.6337
0.33	1.107	0.8673
0.34	0.9851	0.9297
0.35	0.8862	0.9816
0.36	0.8098	0.9933
0.37	0.7440	0.9959
0.38	0.6868	0.9957
0.39	0.6363	0.9954
0.40	0.5916	0.9952

Table 2. Cloud characteristics[13]

Cloud type	Bottom, km	Thickness, km	Aerosol scattering coefficient ε
Upper-level clouds			
Ci, Cc, Cs	6	0.2–2	2.5
Middle-level clouds			
Ac, As	3	0.2–2	10
St, Sc	0.5	0.2–0.8	40
Lower-level clouds			
Ns	0.5	1–2	20

Figure 1 depicts the atmospheric transmittance in the UV spectral region for the net diffusely scattered radiation as calculated by the different methods.

It is seen from Fig. 1*a* that the Eddington and two-stream methods are the closest to the reference result for the cloudless atmosphere. For the cloudy atmosphere from Figs. 1*b* and *c* we can see the discrepancy between the approximate methods and the reference one.

For better agreement, we have introduced the fitting parameters into the approximate methods. For this purpose, the diffuse transmittance was calculated using the effective values of the optical thickness and the single scattering albedo specified by the following equations:

$$\omega_0' = c_1\,\omega_0; \qquad \tau' = c_2\,\tau.$$

The coefficients c_1 and c_2 were declared the fitting parameters and determined from comparison of the calculated results with the data obtained by the Monte Carlo method. The Eddington method was used as the approximate one, because only this method allows calculating not only fluxes, but also intensities.

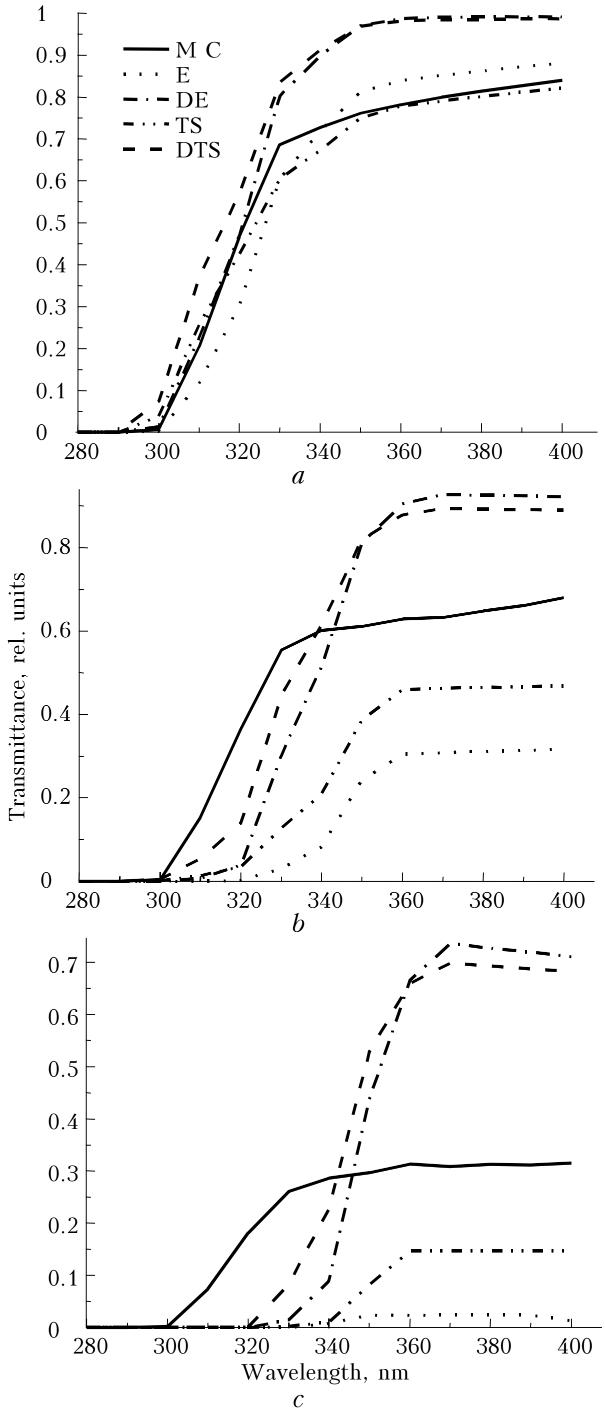

Fig. 1. Atmospheric transmittance calculated by the Monte Carlo method (M C), Eddington approximation (E), delta-Eddington approximation (DE), two-stream approximation (TS), and delta-two-stream approximation (DTS) for the case of the cloudless atmosphere (*a*), upper-level clouds 2 km thick (*b*), and middle-level clouds 2 km thick (*c*); the solar zenith angle of 40°.

Fitting was performed separately for different types of clouds. The transmittance calculated using the effective values of ω_0' are τ' is depicted in Fig. 2, wherefrom it is seen that the agreement is rather good for the 1 km thick clouds as well, though the coefficients were fitted for the limiting values of 0.2 and 2.0 km.

For the cloudless atmosphere the absolute agreement is achieved for all the considered wavelengths.

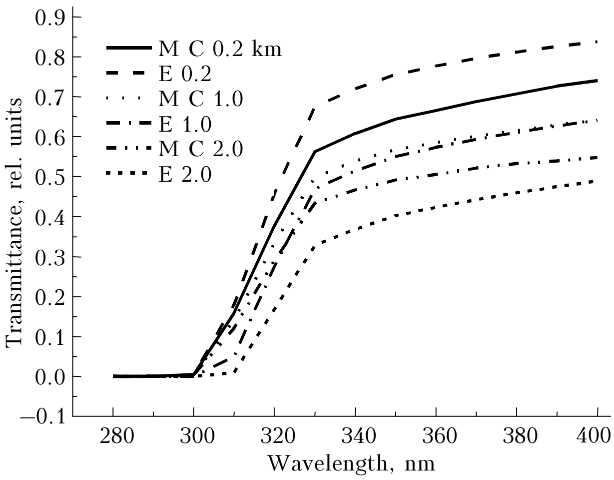

Fig. 2. Atmospheric transmittance as calculated by the Monte Carlo and Eddington methods using the effective values of the optical thickness and single scattering albedo for upper-level clouds 0.2, 1, and 2 km thick, the solar zenith angle of 40°.

Figure 3 depicts the curves of the fitting parameters c_1 and c_2 for upper-level clouds (Table 2). The analogous curves for the cloudless atmosphere are similar with the only difference that they are closer to unity. The coefficient of the optical thickness is less than unity. This is obviously caused by the fact that, despite the large optical thickness, absorption in clouds is low and scattering occurs largely in the forward direction. The accuracy of the Eddington method at significant asymmetry of the scattering phase function decreases.

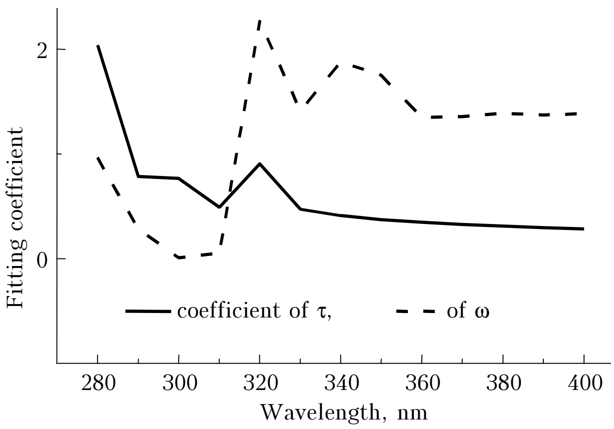

Fig. 3. Fitting coefficients for upper-level clouds.

7. Comparison of experimental data with simulated intensity of diffuse UV radiation for the cloudless atmosphere

In the early 1990s the Tomsk State University developed a spectrophotometer for measurement of the integral intensity of UV radiation.[14] This spectrophotometer has been used since 1994 for routine observations of UV fluxes at the ground level in Tomsk. The observations were conducted in two spectral regions using filters that had the pass bands at 353 (A region) and 281 nm (B region) and the halfwidths of 63 and 24 nm, respectively. The KU-2 quartz hemisphere with the convex side looking upward was employed as a receiving antenna. This antenna ensures accumulation of radiation in the solid angle with the radiation accumulation cone of 110°. The radiation was received by the FEU-170 PMT with a Te-Rb photocathode. The UV flux intensity was measured in relative units. To obtain calibrating curves, synchronous measurements of the solar UV radiation intensity by the spectrophotometer described above and the UBF ultraviolet biological photometer developed by the team headed by N.D. Lazarev[15] were used.

The observations of the UV fluxes were conducted in the following way. The spectrophotometer was installed on a horizontal surface, and the receiving antenna was directed at zenith. The observations were conducted for the sun elevation angles multiple of 5 (5, 10, 15°, an so on) in the A and B regions. For each spectral region, a series of five measurements with different PMT amplification coefficients was conducted. Every value obtained was converted into absolute value, in W/m^2, using the calibrating curves. The value averaged for the series of five measurements was taken as experimental finding.

To avoid the effect of the cloud cover and to smooth the annual TCO trend (study of the effect of these parameters on the UV radiation intensity is beyond the scope of this paper), for further studies we took the findings obtained in the summer (since May till October) and winter (since November till March) periods at the almost cloudless atmosphere (total cloudiness of 0–2, lower-level cloudiness of 0). The April data were excluded when obtaining average values, because this month is just transient from winter to summer, and observations may include both the presence and absence of the snow cover, which significantly affects the ground-level intensity of the diffuse UV radiation.[16]

Since starting from the sun elevation of 35° the spectrophotometer receives not only diffuse, but also direct radiation, the experimental data were taken for the sun elevation angles of 5–30°. The selected observations were averaged for each sun elevation. Thus, from the experimental observations we obtained the average values of the UV radiation intensity in the winter and summer periods for the sun elevation less than 35° under the conditions of the cloudless atmosphere.

The intensity of the diffuse UV radiation was calculated by the Monte Carlo method.[17] For computer realization of this method, we used the algorithm of local estimate in the scheme of conjugate wandering. In the calculations, 32-level atmospheric models generated by the LOWTRAN-7 (Ref. 4) and corresponding to the mid-latitude conditions with the continental aerosol and visibility range of 35 km in the atmospheric surface layer were employed. For the winter conditions, the surface albedo was equal to 0.6, and the reflection law was assumed Lambertian. The calculations involved the experimental geometry, optical characteristics of the spectrophotometer, and observation conditions.

The intensity of the diffuse UV radiation was calculated for the spherical model of the atmosphere. The intensity of UV fluxes was calculated in the spectral region of 290–395 nm with the step of 5 nm. The relative rms error of the results was within 5%.

The calculation by the Eddington method was performed for the same conditions (taking into account the surface albedo), but with the use of the plane-parallel model of the atmosphere. Just this fact can explain the increasing difference between the results for small sun elevation angles. The spectrophotometer geometry did not allow the flux to be determined directly from Eq. (15), therefore, the radiation intensity was calculated for different sun elevation angles with the following integration over the angle. The experimental findings are compared with the Monte Carlo calculations and calculations by the Eddington method in Fig. 4.

Conclusion

The simulated results obtained suggest the applicability of the approximate Eddington method to description of fluxes of atmospheric diffuse UV radiation for the cases of the cloudless and cloudy atmosphere. The discrepancies between the approximate and the reference (Monte Carlo) methods for transmittance are about 0.1 of the absolute value for the cloudless atmosphere and increase up to 0.3 for upper-level clouds. The Eddington approximation is shown to underestimate transmittance, but for many practical problems the accuracy ensured by this method is quite acceptable.

Comparison with the experimental data for the cloudless atmosphere confirms the applicability of the approximate method for description of the diffuse UV radiation. The result obtained can be confidently extrapolated to the cloudy atmosphere as well, since the approximate method was developed for the cloudy atmosphere in the first turn.

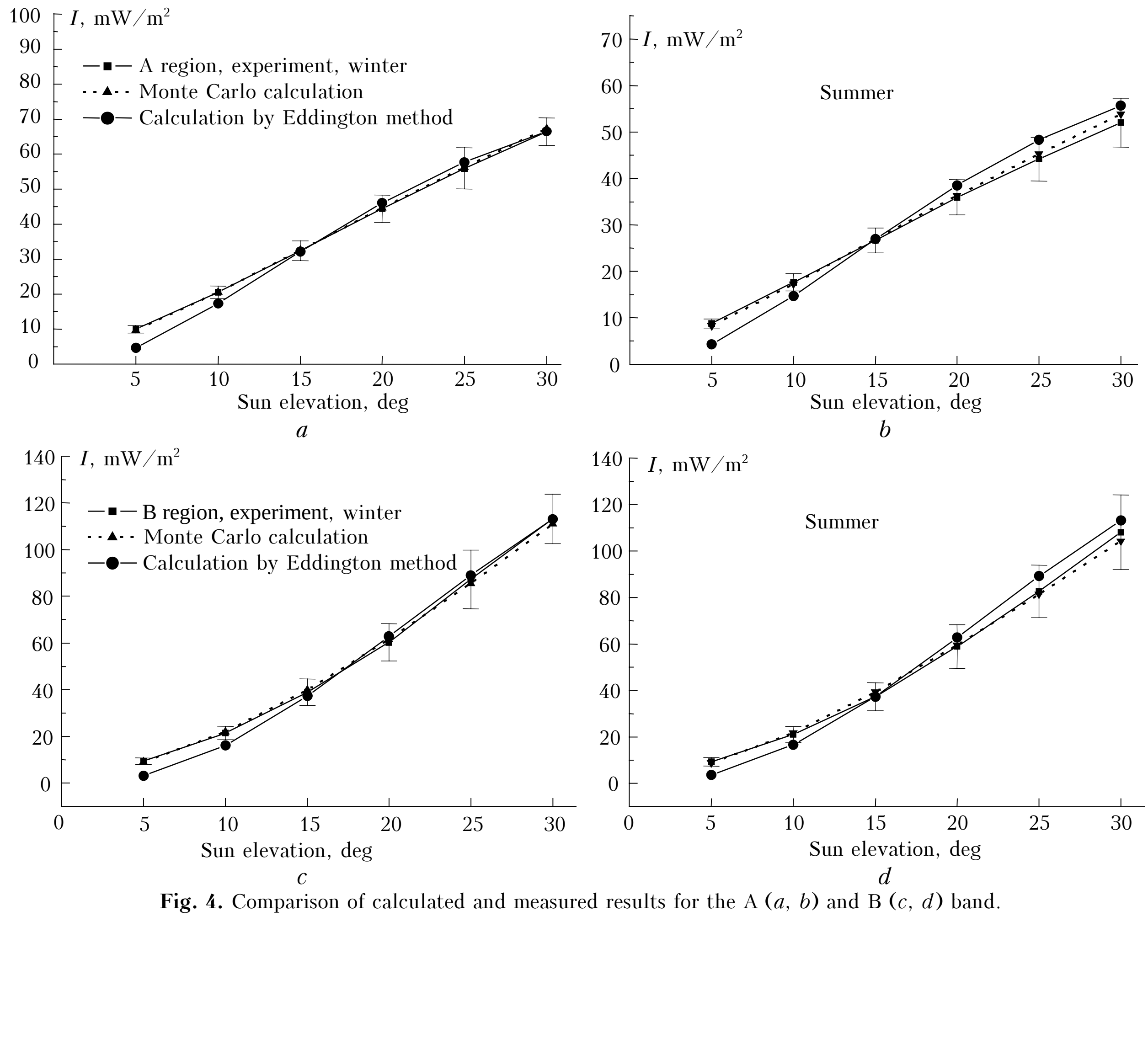

Fig. 4. Comparison of calculated and measured results for the A (*a*, *b*) and B (*c*, *d*) band.

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Appendix

Atmospheric reflection and transmission coefficients

Introduce the reflection $\rho(\tau,\mu,\eta,\varphi)$ and transmission $\sigma(\tau,\mu,\eta,\varphi)$ coefficients:

$$I^{\uparrow}(\tau,-\mu,\eta,\varphi) = S\eta\rho(\tau,-\mu,\eta,\varphi),$$
$$I^{\downarrow}(\tau,\mu,\eta,\varphi) = S\eta\sigma(\tau,\mu,\eta,\varphi),$$

where $I^{\uparrow}, I^{\downarrow}$ denote the intensity of upward and downward radiation.

The coefficients ρ and σ are the intensity ratios of the radiation diffusely scattered (or transmitted) by the atmosphere to the radiation $S\eta$ scattered by an absolutely white ortotropic screen set horizontally on the atmospheric top boundary.[6]

According to Eq. (3), the coefficients ρ and σ can be represented as

$$\rho(\tau,-\mu,\eta,\varphi) = \rho_0(\tau,-\mu,\eta) + \rho_1(\tau,-\mu,\eta)\cos\varphi$$

$$\sigma(\tau,\mu,\eta,\varphi) = \sigma_0(\tau,\mu,\eta) + \sigma_1(\tau,\mu,\eta)\cos\varphi$$

where $\rho_0, \rho_1, \sigma_0, \sigma_1$ are the azimuthally averaged parameters.

The equations for ρ_0 and ρ_1 follow from solution of Eq. (1) by the approximate method:

$$\rho_0(\tau,-\mu,\eta) = e^{-(\tau_0-\tau)/\mu} \times$$
$$\times A_0\left[C_1(1+b)e^{-k\tau_0} + C_2(1-b)e^{k\tau_0} + (1-L)e^{-\tau_0/\eta}\right] +$$
$$+\omega_0 C_1 \frac{1 - p_1\mu\dfrac{b}{2}}{1+\mu k}\left(e^{-\tau k} - e^{-(\tau_0-\tau)/\mu}e^{-\tau_0 k}\right) +$$
$$+\omega_0 C_2 \frac{1 + p\mu\dfrac{b}{2}}{1-\mu k}\left(e^{\tau k} - e^{-(\tau_0-\tau)/\mu}e^{\tau_0 k}\right) +$$
$$+\frac{\omega_0}{\eta+\mu}\left[D_1 + p_1\mu\eta\Phi + \frac{1}{4}p(-\mu,\eta)\right]\left(e^{-\tau/\eta} - e^{-(\tau_0-\tau)/\mu}e^{-\tau_0/\eta}\right),$$

$$\rho_1(\tau,-\mu,\eta) = \omega_0 p_1\sqrt{1-\mu^2}\left[\frac{C_3}{1+\mu k_1}\left(e^{-\tau k_1} - e^{-(\tau_0-\tau)/\mu}e^{-\tau_0 k_1}\right) +\right.$$
$$+\frac{C_4}{1-\mu k_1}\left(e^{\tau k_1} - e^{-(\tau_0-\tau)/\mu}e^{\tau_0 k_1}\right) +$$
$$\left.+\frac{4D_2 + \sqrt{1-\eta^2}}{4(\mu+\eta)}\left(e^{-\tau/\eta} - e^{-(\tau_0-\tau)/\mu}e^{-\tau_0/\eta}\right)\right],$$

where

$$p_1 = \frac{3}{2}\int_0^{\pi} p(\gamma)\cos\gamma\sin\gamma\,d\gamma$$

is the first expansion coefficient of the scattering phase function;

$$p(-\mu,\eta) = \frac{1}{2\pi}\int_0^{2\pi} p(\gamma)\,d\varphi$$

is the azimuthally averaged scattering phase function;

$$C_1 = \frac{\omega_0}{4}\frac{1}{1-k^2\eta^2}\frac{1}{\Delta}\left\{n_1[(1+b) - A_0(1-b)]e^{k\tau_0} +\right.$$
$$\left.+(1-b)(n_2 + A_0 n_1)e^{-\tau_0/\eta}\right\} - A_0\frac{1-b}{\Delta}e^{-\tau_0/\eta},$$

$$C_2 = -\frac{\omega_0}{4}\frac{1}{1-k^2\eta^2}\frac{1}{\Delta}\left\{(1+b)(n_2 + A_0 n_1)e^{-\tau_0/\eta} +\right.$$
$$\left.+ n_1[(1-b) - A_0(1+b)]e^{-k\tau_0}\right\} + A_0\frac{1+b}{\Delta}e^{-\tau_0/\eta},$$

$$C_3 = \frac{1}{R}\left\{\left(1 - \frac{2}{3}k_1\right)\left[\frac{\pi}{8}A_0 + m\left[\left(1 - \frac{2}{3\eta}\right) -\right.\right.\right.$$
$$\left.\left.\left.-A_0\left(1 + \frac{2}{3\eta}\right)\right]\right]e^{-\tau_0/\eta} - m\omega_2\left(1 + \frac{2}{3\eta}\right)e^{k_1\tau_0}\right\},$$

$$C_4 = -\frac{1}{R}\left\{\left(1 + \frac{2}{3}k_1\right)\left[\frac{\pi}{8}A_0 + m\left[\left(1 - \frac{2}{3\eta}\right) -\right.\right.\right.$$
$$\left.\left.\left.-A_0\left(1 + \frac{2}{3\eta}\right)\right]\right]e^{-\tau_0/\eta} - m\omega_1\left(1 + \frac{2}{3\eta}\right)e^{-k_1\tau_0}\right\};$$

$$D_1 = -\frac{\omega_0}{4}\frac{\eta^2(3 + (1-\omega_0)p_1)}{1-k^2\eta^2},\quad D_2 = -m\eta;$$

$$\Phi = \frac{\omega_0}{4}\frac{1 + (1-\omega_0)p_1\eta^2}{1-k^2\eta^2};$$

$$L = \frac{\omega_0}{4}\frac{n_1}{1-k^2\eta^2};$$

$$b = \frac{2k}{3-\omega_0 p_1};\quad k = \sqrt{(1-\omega_0)(3-\omega_0 p_1)};$$

$$n_1 = 2 + 3\eta + (1-\omega_0)p_1\eta(1+2\eta);$$

$$n_2 = 2 - 3\eta - (1-\omega_0)p_1\eta(1-2\eta);$$

$$m = \frac{3\pi^2}{128}p_1\omega_0\sqrt{1-\eta^2}\,\frac{\eta}{1-k_1^2\eta^2};$$

$$\Delta = (1+b)\left[(1+b) - A_0(1-b)\right]e^{k\tau_0} -$$
$$-(1-b)\left((1-b) - A_0(1+b)\right)e^{-k\tau_0};$$

$$k_1 = \sqrt{3\left(1 - \frac{\pi^2}{32}\omega_0 p_1\right)};$$

$$R = \omega_1\left(1 - \frac{2}{3}k_1\right)e^{-k_1\tau_0} - \omega_2\left(1 + \frac{2}{3}k_1\right)e^{k_1\tau_0};$$

$$\omega_1 = \left(1 - \frac{2}{3}k_1\right) - A_0\left(1 + \frac{2}{3}k_1\right),$$

$$\omega_2 = \left(1 + \frac{2}{3}k_1\right) - A_0\left(1 - \frac{2}{3}k_1\right).$$

The equations for the transmission coefficients σ_0 and σ_1 have the form

$$\sigma_0(\tau,\mu,\eta) = \omega_0\frac{1 + p_1\mu b/2}{1 - \mu k}C_1\left(e^{-\tau k} - e^{-\tau/\mu}\right) +$$

$$+\omega_0\frac{1 - p_1\mu b/2}{1 + \mu k}C_2\left(e^{\tau k} - e^{-\tau/\mu}\right) +$$

$$+\omega_0\left(D_1 - p_1\mu\eta\Phi + \frac{1}{4}p(\mu,\xi)\right)\frac{e^{-\tau/\mu} - e^{-\tau/\eta}}{\mu - \eta};$$

$$\sigma_1(\tau,\mu,\eta) = \omega_0 p_1\sqrt{1-\mu^2}\left(\frac{C_3}{1 - \mu k_1}(e^{-\tau k_1} - e^{-\tau/\mu}) +\right.$$

$$\left.+\frac{C_4}{1 + \mu k_1}(e^{\tau k_1} - e^{-\tau/\mu}) + \left(D_2 + \frac{1}{4}\sqrt{1-\eta^2}\right)\frac{(e^{-\tau/\mu} - e^{-\tau/\eta})}{\mu - \eta}\right).$$

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