ACTIVE RESTORATION OF IMAGES WITH MULTIPLICATIVE AMPLITUDE-PHASE DISTORTIONS OF A SPATIAL SIGNAL SPECTRUM

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The generalization is proposed for the active image restoration method that can be applied not only in the case of phase distortions of a spatial signal spectrum, but in the case of amplitude-phase distortions as well. The results obtained are illustrated by means of mathematical simulation.

The method of damping the phase distortions of a spatial signal spectrum was proposed and examined in Refs. 1–3. Notwithstanding the different origin of such distortions, similar mathematical models may be used for simulation. Thus, for example, in the optical range of the spectrum, phase fluctuations of signals are due to the turbulence of a medium, in particular, of the atmosphere. In antennas, in addition to atmospheric effects, phase distortions also can be introduced by heterogeneous elements of an aerial feeder channel. These distortions, as a rule, have longer correlation time than that caused by the atmospheric turbulence; nevertheless, in some cases it is convenient not to focus the attention of the researches on such peculiarities but to consider the problem in its most general form.

At the same time, the generalization allows one to note that the multiplicative interference cannot be reduced to phase distortions alone but also implies the multiplicative amplitude noise. In combination, the multiplicative amplitude-phase distortions make a solution of many important problems difficult, in particular, deteriorating the angular resolution. So it is feasible to develop the methods of compensation for not only phase but amplitude distortions as well.

Implementing ideas and designations from Refs. 1–3, in the present paper we propose the method of active restoration of images with multiplicative phase[1–3] and amplitude-phase distortions of a spatial signal spectrum without a reference source in the image plane of a target.

Let a spatiotemporal amplitude-phase distribution be formed in the plane ρ of a transmitting aperture

$$\varepsilon_{s.s} = \phi(\rho, t). \quad (1)$$

The condition of orthogonality

$$\int dt\, \phi(\rho_1, t)\, \phi^*(\rho_2, t) = \delta(\rho_1 - \rho_2), \quad (2)$$

is imposed on it, where $\delta(\rho_1 - \rho_2)$ is the Dirac delta function.

Assuming that a target is in the far field and using the Kirchhoff integral, we write down the distribution of a sounding signal over the image plane of the target in the Fraunhofer approximation to within factors that are insignificant for a subsequent analysis

$$E_{s.s}(r, t) = \int d\rho \exp\,[j\, 2\pi\, r\, \rho/(\lambda R)]\; \phi(\rho, t) \times$$

$$\times \exp\,(j\, \varphi_1(\rho))\, A_1(\rho), \quad (3)$$

where $\varphi_1(\rho)$ and $A_1(\rho)$ are the amplitude and phase distortions, respectively, introduced by a distorting medium (in optics, by the turbulent atmosphere) in the transition mode, and R is the distance to the target.

In the same way as in Refs. 1–3 we assume that the target is illuminated by a plain wave propagating along the line of sight. The image of the target $E(r)$ is taken to mean the spatiotemporal vector amplitude-phase distribution of the signal over the image plane of the target.

With illumination of the target, the spatiotemporal structure is formed in the image plane

$$E_s(r, t) = E(r)\, E_{s.s}(r, t). \quad (4)$$

In the plane of a receiving aperture this structure transforms to the following amplitude-phase distribution:

$$\varepsilon(\rho, t) = \exp\,(j\, \varphi_{a2}(\rho))\, A_2(\rho) \times$$

$$\times \int dr \exp\,[-\, j\, 2\pi\, r\, \rho/(\lambda R)]\; E_s(r, t). \quad (5)$$

Here, $A_2(\rho)$ and $\varphi_{a2}(\rho)$ are the amplitude and phase distortions of the spatial spectrum of a received signal, respectively. As earlier phase fluctuations,[1–3] now we consider the spatial structure of amplitude-phase distortions to be unchanged over periods of system operation (the condition of the frozen atmosphere, according to the terminology used in problems of wave propagation in random media). However, in the general case

$$A_1(\rho) \neq A_2(\rho), \qquad \varphi_{a1}(\rho) \neq \varphi_{a2}(\rho), \quad (6)$$

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which means that different distortions are possible in the transmission and reception modes.

In accordance with the approach developed in Refs. 1–3 let us multiply Eq. (5) by $\phi^*(\rho, t)$. After integration of the result over time, we obtain the equation

$$\varepsilon_{im}(\rho, \rho_1) = \int dt\, \varepsilon_s(\rho, t)\, \phi^*(\rho_1, t), \qquad (7)$$

which on account of Eqs. (3)–(5), can be transformed to the form

$$\varepsilon_{im}(\rho, \rho_1) = \int dt\, \phi^*(\rho_1, t) \exp(j\, \varphi_{a2}(\rho))\, A_2(\rho) \times$$
$$\times \int dr \exp[-j\, 2\pi\, r\, \rho/(\lambda R)]\, E(r) \times$$
$$\times \int d\rho' \exp[j\, 2\pi\, r\, \rho'/(\lambda R)] \times$$
$$\times\, \phi(\rho', t) \exp(j\, \varphi_{a1}(\rho'))\, A_1(\rho'). \qquad (8)$$

By changing the order of integration, we obtain

$$\varepsilon_{im}(\rho, \rho_1) = \exp(j\, \varphi_{a2}(\rho))\, A_2(\rho) \times$$
$$\times \int dr \exp[-j\, 2\pi\, r\, \rho/(\lambda R)]\, E(r) \times$$
$$\times \int d\rho' \exp[j\, 2\pi\, r\, \rho'/(\lambda R)] \exp(j\, \varphi_{a1}(\rho'))\, A_1(\rho') \times$$
$$\times \int dt\, \phi^*(\rho_1, t)\, \phi(\rho', t). \qquad (9)$$

Allowing for relation (2) and filtering property of a delta function, the last equation can be converted to the form

$$\varepsilon_{im}(\rho, \rho_1) = \exp(j\, \varphi_{a2}(\rho))\, A_2(\rho) \times$$
$$\times \int dr \exp[-j\, 2\pi\, r\, \rho/(\lambda R)]\, E(r) \times$$
$$\times \exp[j\, 2\pi\, r\, \rho_1/(\lambda R)] \exp(j\, \varphi_1(\rho_1))\, A(\rho_1) =$$
$$= \exp(j\, \varphi_{a2}(\rho))\, A_2(\rho)\, \varepsilon(\rho - \rho_1)\, A_1(\rho_1) \exp(j\varphi_{a1}(\rho_1)). \qquad (10)$$

Here,

$$\varepsilon(\rho) = \int dr \exp[-j\, 2\pi\, r\, \rho/(\lambda R)]\, E(r) \qquad (11)$$

is a spatial spectrum of a signal being reconstructed (a valid signal).

Multiplying Eq. (5) by $\phi^*(\rho_2, t)$, where $\rho_2 \neq \rho_1$, and then integrating it over time, we obtain in the same way the following result:

$$\varepsilon_{im}(\rho, \rho_2) = A_2(\rho) \exp(j\, \varphi_{a2}(\rho)) \times$$
$$\times\, \varepsilon(\rho - \rho_2)\, A_1(\rho_2) \exp(j\, \varphi_{a1}(\rho_2)). \qquad (12)$$

Because Eqs. (10) and (12) consider not only phase distortions but multiplicative amplitude-phase distortions as well, they form more general system of equations than that presented in Refs. 1–3.

Now we focus our attention on the fact that concerning the phase reconstruction nothing has been changed because the phase structure of the system components has not been changed.

So, it is worthwhile to write down the equations for the amplitudes. These equations can be reduced to the system

$$\left.\begin{aligned} |\varepsilon_{im}(\rho, \rho_1)| &= A_1(\rho_1)\, A_2(\rho)\, |\varepsilon(\rho - \rho_1)|, \\ |\varepsilon_{im}(\rho, \rho_2)| &= A_1(\rho_2)\, A_2(\rho)\, |\varepsilon(\rho - \rho_2)|. \end{aligned}\right\} \qquad (13)$$

For a particular case $\rho_1 = 0$ and $\rho_2 = \Delta\rho$, this system can be rewritten in the following way:

$$\left.\begin{aligned} |\varepsilon_{im}(\rho, 0)| &= A_1(0)\, A_2(\rho)\, |\varepsilon(\rho - 0)|, \\ |\varepsilon_{im}(\rho, \Delta\rho)| &= A_1(\Delta\rho)\, A_2(\rho)\, |\varepsilon(\rho - \Delta\rho)|. \end{aligned}\right\} \qquad (14)$$

Taking a natural logarithm of Eq. (14) we obtain

$$\left.\begin{aligned} &\ln|\varepsilon_{im}(\rho, 0)| = \\ &= \ln A_1(0) + \ln A_2(\rho) + \ln|\varepsilon(\rho - 0)|, \\ &\ln|\varepsilon_{im}(\rho, \Delta\rho)| = \\ &= \ln A_1(\Delta\rho) + \ln A_2(\rho) + \ln|\varepsilon(\rho - \Delta\rho)|. \end{aligned}\right\} \qquad (15)$$

Subtracting the first equation of system (15) from the second we find

$$\ln|\varepsilon_{im}(\rho, \Delta\rho)| - \ln|\varepsilon_{im}(\rho, 0)| = \ln A_1(\Delta\rho) -$$
$$- \ln A_1(0) + \ln|\varepsilon(\rho - \Delta\rho)| - \ln|\varepsilon(\rho)|. \qquad (16)$$

Dividing the left- and right-hand sides of Eq. (16) by $\Delta\rho$ and making $\Delta\rho$ approach zero with allowance for a derivative definition we obtain

$$\left.\frac{\partial \ln|\varepsilon_{im}(\rho, \Delta\rho)|}{\partial\, \Delta\rho}\right|_{\Delta\rho=0} = \left.\frac{\partial \ln A_1(\rho)}{\partial\, \rho}\right|_{\Delta\rho=0} - \frac{\partial \ln|\varepsilon(\rho)|}{\partial\, \rho}, \qquad (17)$$

The sought-for solution follows from this equation, namely

$$|\varepsilon(\rho)| = \exp\left(-\int_0^{\rho} d\rho \left.\frac{\partial \ln|\varepsilon_{0im}(\rho, \Delta\rho)|}{\partial\, \Delta\rho}\right|_{\Delta\rho=0}\right) \times$$
$$\times \exp\left(-\left.\frac{\partial \ln A_1(\rho)}{\partial\, \rho}\right|_{\Delta\rho=0} \rho\right). \qquad (18)$$

It is convenient to take the value of $\Delta\rho$ so small that $A_1(\Delta\rho) \approx A_1(0)$. This allows us to leave aside the problem on the negative influence of the second exponent in Eq. (18).

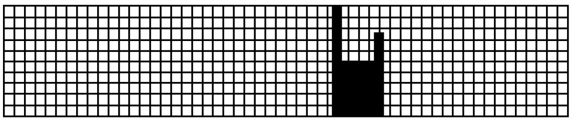

FIG. 1. Modulus of the initial signal.

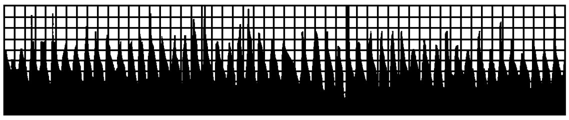

FIG. 2. The logarithm of the amplitude signal spectrum after differentiation.

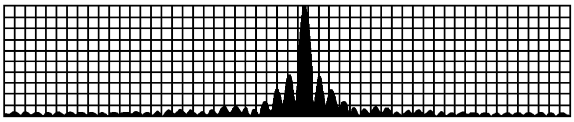

FIG. 3. Restored amplitude signal spectrum.

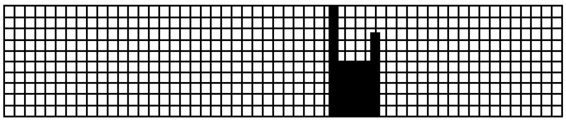

FIG. 4. Modulus of the restored signal.

The proposed algorithm is illustrated by the results of mathematical simulation (see Figs. 1–4).

As regards the phase spectrum, the results of its processing do not differ from that presented in Refs. 1–3 because the generalization of the method of active restoration proposed in the present paper does not include any changes of the algorithm of phase spectrum restoration.

So we can conclude that development of the active restoration method intended for solving the problems in real time has been proposed and illustrated by the example of mathematical simulation. With the use of this method not only phase distortions of the spatial signal spectrum[1–3] but amplitude-phase distortions as well can be reconstructed without a reference source in the image plane of a target.

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