FOMATION OF LATERAL SHEAR INTERFEROGRAMS WITH A DIFFUSELY SCATTERED FIELDS BY DOUBLE–EXPOSURE RECORDING OF FOURIER HOLOGRAMS

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Received March 25, 1992

A lateral shear interferometer operation based on double–exposure recording of a Fourier hologram of a matted screen illuminated with a diffusely scattered coherent light is analyzed. It is shown theoretically and experimentally that spatial filtering in the plane of the hologram enables one to check the lens or objective wave aberrations over the field.

In Ref. 1 a performance of a technique is described for constructing lateral shear interferograms in fringes of infinite width based on double‑exposure recording of a lens Fourier hologram of a matted screen illuminated with a diffusely scattered light field and using a compensation for a phase change introduced into the light wave due to transverse shift of the matted screen in the hologram plane by virtue of tilting the quasiplanar wavefront of the reference wave. In this technique a matted screen is illuminated with a coherent radiation of a quasiplanar light wave. As a result the recorded interference pattern characterizes, in the general case, the wave aberrations of a converging lens used for recording the hologram as well as the wave aberrations of the optical system forming the wavefront of light used to illuminate the matted screen. This same result is obtained at the double‑exposure recording of the Fourier transforms of a matted screen on a photographic plate when illuminating the screen with a converging quasispherical wave[2] as well as when forming the Fourier transform with a diverging lens.[3]

In Refs. 4 and 5 the double‑exposure recording of the Fourier hologram of a matted screen has been performed by illuminating the screen with a diverging spherical wave which can be formed aberrationless provided that a spatial filtering is used.[6]

As a consequence this enables recording the lateral shear interferograms that bear information only about the wave aberrations of a converging lens under control.

In this paper we consider the formation of the lateral shear interference patterns in fringes of infinite width using a double‑exposure recording of a lens Fourier hologram of a matted screen illuminated with a coherent diffusely scattered radiation that also allows one to eliminate the wave aberrations in the channel of the matted screen illumination.

As shown in Fig. 1*a* the matted screen *1* placed in the plane (x_1, y_1) is illuminated by a small–aperture laser beam so that diffusely scattered on it radiation illuminates the matted screen *2* placed at a distance L in the plane (x_2, y_2). Then the diffusely scattered on the screen *2* the radiation passes through the lens L_1 under control and the first exposure recording of the Fourier hologram of the matted screen *2* is performed on the photographic plate *3* using a quasispherical diverging reference wave *4*. Before making the second exposure recording the matted screen *2* is displaced in its plane, for example along the x axis by an amount a and the angle of incidence of the spatially limited reference beam is changed in the plane (x, z) from θ_1 to θ_2.

In the Fresnel approximation neglecting the amplitude and phase factors, which are constant, the complex amplitude of the field in the plane of the photographic plate (x_4, y_4) for the first exposure is

$$u_1(x_4, y_4) \sim \int\int\int\int\int\int_{-\infty}^{\infty} t_1(x_1, y_1)\, t_2(x_2, y_2) \times$$

$$\times \exp\{ik[(x_1 - x_2)^2 + (y_1 - y_2)^2]/2L\} \times$$

$$\times \exp\{ik[(x_2 - x_3)^2 + (y_2 - y_3)^2]/2l_1\}p_1(x_3, y_3) \times$$

$$\times \exp\{[-i\{k[(x_3^2 + y_3^2)/2f_1] - \varphi_1(x_3, y_3)\} \times$$

$$\times \exp\{ik[(x_3 - x_4)^2 + (y_3 - y_4)^2]/2l_2\}\, dx_1dy_1dx_2dy_2dx_3dy_3, \quad (1)$$

where k is the wave number, $t_1(x_1, y_1)$ and $t_2(x_2, y_2)$ are the complex transmission amplitudes of the matted screens *1* and *2*, respectively (these values are random functions of coordinates), $p_1(x_3, y_3)\exp i\varphi_1(x_3, y_3)$ is the generalized pupil function[7] of the lens L_1 allowing for its axial wave aberrations, f_1 is the focal length of the lens L_1, and l_1 and l_2 are the distances from the principal plane of the lens $L_1(x_3, y_3)$ to the plane of the matted screen *2* and to the plane of the photographic plate.

Under conditions that $1/l_1 - 1/f_1 + 1/l_2 = 1/M > 0$ and $L = l_1^2/(M - l_1)$ Eq. (1) can be reduced to the form

$$u_1(x_4, y_4) \sim \exp[ik(x_4^2 + y_4^2)/2l_2]\, 2\exp[-ik(x_4^2 + y_4^2)M/2l_2^2] \times$$

$$\times \{t_1(-\mu_1 x_4, -\mu_1 y_4) \exp[ik\mu_1^2(x_4^2 + y_4^2)/2L] \otimes$$

$$\otimes F[kx_4M/l_1l_2, ky_4M/l_1l_2] \otimes P_1(x_4, y_4)\}\ ; \quad (2)$$

where the symbol $\otimes$ denotes the operation of convolution, $\mu_1 = LM/l_1l_2$ is the scale transformation factor,

$$F[kx_4M/l_1l_2, ky_4M/l_1l_2] = \int\int_{-\infty}^{\infty} t_2(x_2, y_2) \times$$

$$\times \exp[-ik(x_2x_4 + y_2y_4)M/l_1l_2]\, dx_2dy_2$$

is the Fourier transform of the matted screen *2* transmission function, and

0235-6880/92/07 435-04 $02.00

$$P_1(x_4, y_4) = \int_{-\infty}^{\infty}\int p_1(x_3, y_3)\exp i\varphi_1(x_3, y_3)\exp[-ik(x_3x_4 + y_3y_4)/l_2]\mathrm{d}x_3\mathrm{d}y_3$$

is the Fourier transform of the generalized pupil function of a lens under control.

Within a region in the plane of the photographic plate with the diameter $D \le d_1l_1/M$, where d_1 is the diameter of the lens L_1 pupil, relation (2) takes the form[5]

$$u_1(x_4, y_4) \sim \exp[ik(x_4^2 + y_4^2)/(l_2 - M)/2l_2^2] \times$$
$$\times \{t_1(-\mu_1x_4, -\mu_1y_4)\exp[ik\mu_1^2(x_4^2 + y_4^2)/2L]\otimes$$
$$\otimes F[kx_4M/l_1l_2,\ ky_4M/l_1l_2]\otimes P_1(x_4, y_4)\}\ . \qquad (3)$$

Let the diameter of the laser beam in the plane of the matted screen *1* be d_0, then the size of the existence domain of the function $t_1(-\mu_1x_4, -\mu_1y_4)$ is determined by the value $d_0l_1l_2/LM$. Thus, as it follows from Eq. (3) the field distribution over the plane of the photographic plate corresponds to the Fourier transform of the matted screen *2* transmission function, which is convoluted with the function $t_1(-\mu_1x_4, -\mu_1y_4)\otimes p_1(x_4, y_4)$. Pulse response of the resultant transform is wider than the pulse response governed by the lens L_1 by the value of the existence domain size of the function $t_1(-\mu_1x_4, -\mu_1y_4)$. In addition, there is one more term, i.e., $\exp[ik(x_4^2 + y_4^2(l_2 - M)/2l_2^2]$, which describes the distribution of the phase of a diverging spherical wave with the radius of curvature $l_2^2/(l_2 - M)$ (see Ref. 5).

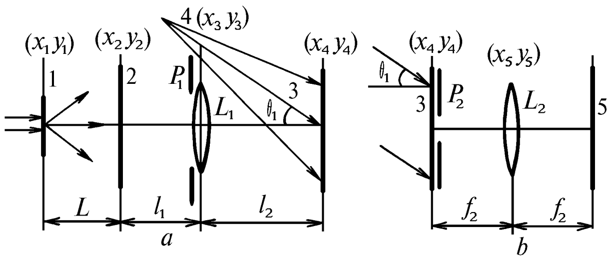

FIG. 1. The optical schemes used for recording (a) and reconstructing (b) a double–exposure Fourier hologram of a matted screen; 1 and 2) matted screen, 3) photographic plate hologram, 4) reference beam, and 5) plane of recording the interference pattern. L_1 and L_2 are lenses, p_1 is an aperture diaphragm; and, p_2 is a filtering diaphragm.

Based on the known property of the Fourier transform the distribution of the complex amplitude of the diffusely scattered field over the plane of the photographic plate, in the case of the second exposure, can be written by

$$u_2(x_4, y_4) \sim \exp[ik(x_4^2 + y_4^2)/(l_2 - M)/2l_2^2] \times$$
$$\times \{t_1(-\mu_1x_4, -\mu_1y_4)\exp[ik\mu_1^2(x_4^2 + y_4^2)/2L]\otimes\exp(ikaMx_4/l_1l_2)\times$$
$$\times F[kx_4M/l_1l_2,\ ky_4M/l_1l_2]\otimes P_1(x_4, y_4)\}\ . \qquad (4)$$

Within the approach used here the complex amplitudes of reference waves in the plane of the photographic plate take the following form

$$u_{01}(x_4, y_4) \sim \exp i\left[\frac{k}{2r}(x_4^2 + y_4^2) + kx_4\sin\theta_1 + \varphi_2(x_4, y_4)\right],$$

$$u_{02}(x_4, y_4) \sim \exp i\left[\frac{k}{2r}(x_4^2 + y_4^2) + kx_4\sin\theta_2 + \varphi_2(x_4 + b, y_4)\right],$$

where $r = l_2^2/(l_2 - M)$, $\varphi_2(x_4, y_4)$ is a function characterizing the phase distortions of the reference wave owing to wave aberrations of the optical system forming it, and b is the shift caused by the change of the slope angle of the spatially limited reference beam before the second exposure.

The distribution of light intensity over the double–exposure hologram is presented then as a sum of two intensity distributions being the interferograms produced by the object and reference beams, respectively

$$I(x_4, y_4) \sim [u_1(x_4, y_4) + u_{01}(x_4, y_4)]\,[u_1(x_4, y_4) +$$
$$+ u_{01}(x_4, y_4)]^* + [u_2(x_4, y_4) + u_{02}(x_4, y_4)] \times$$
$$\times [u_2(x_4, y_4) + u_{02}(x_4, y_4)]^*\ . \qquad (5)$$

Let us then assume the amplitude transmission coefficient of the hologram to be a linear function of the intensity. Let the hologram be illuminated with a plane monochromatic wave incident on the hologram at an angle θ_1 with respect to its plane, the complex amplitude of the wave being represented by the function $\exp ikx_4\sin\theta_1$. Then, as it follows from Eqs. (3), (4), and (5), the distribution of field over the hologram plane in the minus first order of diffraction is

$$u(x_4, y_4) \sim \exp - i\varphi_2(x_4, y_4)\{t_1(-\mu_1x_4, -\mu_1y_4) \times$$
$$\times \exp[ik\mu_1^2(x_4^2 + y_4^2)/2L]\otimes\{F[kx_4M/l_1l_2,\ ky_4M/l_1l_2]\otimes$$
$$\otimes P_1(x_4, y_4)\} + \exp i[kx_4\sin\theta_1 - kx_4\sin\theta_2 - \varphi_2(x_4 + b, y_4)] \times$$
$$\times \{t_1(-\mu_1x_4, -\mu_1y_4)\exp[ik\mu_1^2(x_4^2 + y_4^2)/2L]\otimes$$
$$\otimes\exp[ikaM/l_1l_2)\ F[kx_4M/l_1l_2,\ ky_4M/l_1l_2]\otimes P_1(x_4, y_4)\ . \qquad (6)$$

The period of the function $\exp(ikaMx_4/l_1l_2)$ entering into this formula is $\lambda l_1l_2/aM$, where λ is the wavelength of coherent light used for recording and reconstructing of the hologram. If this period is not less than the existence domain size of the function $t_1(-\mu_1x_4, -\mu_1y_4)$, what is valid when $a \le \lambda L/d_0$, or in other words, when the amount of the matted screen *2* shift before the second exposure does not exceed the size of a minimum objective speckle[8] in its plane, in Eq. (6) one can remove the factor characterizing linear distribution of the phase from the integrand of the integral of convolution with the function $t_1(-\mu_1x_4, -\mu_1y_4)\exp[ik\mu_1^2(x_4^2 + y_4^2)/2L]$. As a result, under condition that $\sin\theta_2 - \sin\theta_1 + aM/l_1l_2 = 0$ relation (6) is reduced to the form

$$u(x_4, y_4) \sim \exp - i\varphi_2(x_4, y_4)\ \{t_1(-\mu_1x_4, -\mu_1y_4) \times$$
$$\times \exp[ik\mu_1^2(x_4^2 + y_4^2)/2L] \otimes \{F[kx_4M/l_1l_2,\ ky_4M/l_1l_2] \otimes$$

$$\otimes P_1(x_4,\ y_4)\} + \exp - i\varphi_2(x_4 + b,\ y_4)\{t(-\mu_1 x_4,\ -\mu_1 y_4) \times$$
$$\times \exp\left[ik\mu_1^2(x_4^2 + y_4^2)/2L\right] \otimes F[kx_4 M/l_1 l_2,\ ky_4 M/l_1 l_2] \otimes$$
$$\otimes \exp\left[-ikaMx_4/l_1 l_2\right) P_1(x_4,\ y_4)\ . \qquad (7)$$

As can be seen from Eq. (7), the subjective speckle fields of two exposures coincide in the plane of hologram when the relative angle of slope between them is $\alpha = aM/l_1 l_2$. That means that in the hologram plane the interference pattern formed due to aberrations of the reference wave is localized.[5] If an opaque screen P_2 (see Fig. 1*b*) with a hole is placed in the hologram plane so that the hole center is on the optical axis and if the width of an interference fringe of the interference pattern localized in the hologram plane does not exceed the diameter of the hole, i.e., the condition that $\varphi_2(x_4 + b,\ y_4) - \varphi_2(x_4,\ y_4) \le \pi$ is fulfilled the diffraction field at the exit of the hole is

$$u(x_4,\ y_4) \sim p_2(x_4,\ y_4)\{t_1(-\mu_1 x_4, -\mu_1 y_4)\exp\left[ik\ {}_1^2(x_4^2 + y_4^2)/2L\right] \otimes$$
$$\otimes \{F[kx_4 M/l_1 l_2,\ ky_4 M/l_1 l_2] \otimes \left[1 +\right.$$
$$\left. + \exp\left(-ik\alpha Mx_4/l_1 l_2\right)\right] P_1(x_4,\ y_4)\}\ , \qquad (8)$$

where $p_2(x_4,\ y_4)$ is the transmission function of the screen with a round hole.[9]

Let the light field in the back focal plane of the lens L_2 (see Fig. 1*b*; f_2 is the lens L_2 focal length) be represented by the Fourier integral of the light field in the plane of spatial filtration, then, by making use of the basic Fourier integral relations, we have

$$u(x_5,\ y_5) \sim \{F(x_5,\ y_5) t_2(-\mu_2 x_5,\ -\mu_2 y_5)\, p_1(-\mu_3 x_5,\ -\mu_3 y_5) \times$$
$$\times \exp i\varphi_1(-\mu_3 x_5,\ -\mu_3 y_5) + F(x_5,\ y_5) t_2(-\mu_2 x_5,\ -\mu_2 y_5) \times$$
$$\times p_1(-\mu_3 x_5 - aM/l_1,\ -\mu_3 y_5) \exp i\varphi_1(-\mu_3 x_5 - aM/l_1,\ -\mu_3 y_5)\} \otimes$$
$$\otimes P_2(x_5,\ y_5)\ , \qquad (9)$$

where $\mu_2 = l_1 l_2/f_2 M$ and $\mu_3 = l_2/f_2$ are the coefficients of scale transformation, and $F(x_5,\ y_5) = \int\limits_{-\infty}^{\infty}\int t_1(-\mu_1 x_4,\ -\mu_1 y_4) \times$ $\times \exp\left[ik\mu_1^2(x_4^2 + y_4^2)/2L\right] \exp[-ik(x_4 x_5 + y_4 y_5)/f_2\}\mathrm{d}x_4\mathrm{d}y_4$, and

$$P_2(x_5,\ y_5) = \int\limits_{-\infty}^{\infty}\int p_2(x_4,\ y_4) \exp[-ik(x_4 x_5 + y_4 y_5)/f_2\}\mathrm{d}x_4\mathrm{d}y_4$$

are the Fourier transforms of the corresponding functions.

As follows from relation (9), the identical speckle fields of both exposures in the plane of the matted screen image coincide within the region in which the images of the lens L_1 pupil overlap. This means, in turn, that the interference pattern is localized in the plane $(x_5,\ y_5)$. Virtually, if the period of the function $\exp i\varphi_1(-\mu_3 x_5,\ -\mu_3 y_5) +$ $+ \exp i\varphi_1(-\mu_3 x_5 - aM/l_1 l_2,\ -\mu_3 y_5)$ is at least an order of magnitude[10] greater than the size of a speckle, determined by the width of the function $F(x_5,\ y_5) \otimes P_2(x_5,\ y_5)$, then in relation (9) this function can be removed from the integrand of the convolution integral. The superposition of the correlated speckle fields of both exposures yields, in the plane of recording the image of the matted screen *2*, the following distribution of the illumination:

$$I(x_5,\ y_5) \sim \{1 + \cos[\varphi_1(-\mu_3 x_5 - aM/l_1,\ -\mu_3 y_5) -$$
$$-\varphi_1(-\mu_3 x_5, -\mu_3 y_5)]\}\left| F(x_5,\ y_5)\ t_2(-\mu_2 x_5, -\mu_2 y_5)\ \otimes P_2(x_5,\ y_5)\right|^2 \qquad (10)$$

which describes the speckle structure modulated by interference fringes of the interference pattern that, in fact, is the lateral shear interferogram in the fringes of infinite width and characterizes the axial wave aberrations of the lens L_1. A shift of the filtering hole along the direction of the shift of the matted screen *2* enables controlling the lens L_1 over the field.[5]

As follows from expression (10) the size of the speckle in the plane of interference pattern recording is determined by the width of the function $F(x_5,\ y_5) \otimes P_2(x_5,\ y_5)$. If the diameter of the filtering diaphragm P_2 (see Fig. 1*b*) is greater than the existence domain size of the function $t_1(-\mu_1 x_4, -\mu_1 y_4)$, then the interference pattern modulates the objective speckle structure, and the size of a speckle $\lambda f_2 LM/d_0 l_1 l_2$ is determined by the width of the function $F(x_5,\ y_5)$. As a result one can improve the sensitivity of a lateral shear interferometer by reducing the diameter of a laser beam in the plane of the matted screen *1* (see Fig. 1*a*) since in this case the shift of the matted screen *2* can be increased. It should be noted, however, that in this case the size of an objective speckle in the plane of image recording of the matted screen *2* (see Fig. 1*b*) increases that leads to a decrease of the interference pattern contrast,[10] because the period of the interference fringes becomes comparable with the size of a speckle. On the other hand, a reduction of the objective speckle size in the plane of the matted screen *2* (see Fig. 1*a*) increases the contrast of the interference pattern in the plane[5] of its recording but decreases the sensitivity of the lateral shear interferometer.

As shown in Ref. 5, for spatial filtration to be performed on the optical axis in the plane of the image of the matted screen *2* one needs to record the interference pattern localized in the plane of the hologram.

In the experiment the double–exposure Fourier holograms of the matted screen were recorded on photographic plates of the type Mikrat–VRL using an LG–44 He–Ne laser at the wavelength 0.63 μm whose radiation was focused with an objective of 250–mm focal length onto the matted glass screen for forming a coherent diffusely scattered illumination field. As an example Fig. 2 shows interferograms obtained when performing spatial filtering in the plane of the hologram by reconstructing a hologram with a small aperture (≈ 2 mm) laser beam. The interference pattern shown in Fig. 2*a* characterizes the spherical aberration of the lens L_1 with $f_1 = 120$ mm and its pupil diameter $d_1 = 50$ mm with post focal defocusing, the interference pattern being reconstructed at a point on the optical axis. Using this lens under control the double–exposure recording of a hologram was performed for $l_1 = 55$ mm, $l_2 = 300$ mm, and $L = 145$ mm. In the channel forming the reference wave the laser beam was first expanded with a collimator and then transformed with an objective into a diverging quasispherical reference beam with the radius of curvature $r = 402$ mm in the plane of the photographic plate. Prior to making the second exposure recording the matted screen *2* (see Fig. 1*a*) was shifted perpendicularly to the optical axis by an amount $a = (0.15 \pm 0.002)$ mm and the slope angle of the reference beam was changed by an amount $\Delta\theta = 2'$ and $20'' \pm 10''$.

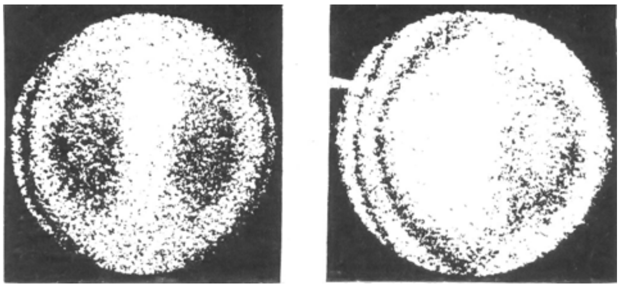

FIG. 2. Lateral shear interferograms recorded when performing spatial filtering in the plane of the hologram on the optical axis (a) and off the optical axis (b).

The interference pattern shown in Fig. 2*b*, characterizes the combination of axial (see Fig. 2*a*) and off-axis wave aberrations of the lens under control. This interference pattern is recorded when making spatial filtering at 26–mm distance from the optical axis at a point on the axis of the shift of the matted screen prior to the second exposure.

Thus, it follows from the above theoretical and experimental results that holographic lateral shear interferograms enabling the control of converging lenses and objectives can be formed in diffusely scattered fields using a double-exposure recording of a lens Fourier hologram of a matted screen illuminated by a coherent diffusely scattered light. Such a technique seems to be applicable to control lenses and objectives of large aperture and short focal length, when, owing to the possibility of increasing the lateral shift, no high sensitivity in the differential interferometry is required.

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