ESTIMATING THE INTENSITY OF AEROSOL PRECIPITATION FROM SMOKE PLUME OPTICAL THICKNESS DYNAMICS

V.V. Antonovich, G.O. Zadde, and A.V. Podanyov

Institute of Atmospheric Optics,
Siberian Branch of the USSR Academy of Sciences, Tomsk
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An equation is derived relating the intensity of aerosol precipitation by sedimentation and diffusion to the dynamics of the plume optical thickness. A relationship is obtained for estimating variations in the optical thickness field of the ith aerosol fraction in the smoke plume emitted by a point source. Schemes are suggested for practical application of this relationship.

The intensity of aerosol pollution around industrial enterprises and thermal power plants is given by the amount of aerosol precipitating to the ground per unit area per unit time. The scale of such pollution is determined by the intensity of the aerosol sources. Therefore it is important to develop techniques for estimating areal aerosol pollution and aerosol emission intensity.

Propagating through the atmosphere after emission, the aerosol cloud occupies a continuously increasing volumes as it moves away from the source, takes part in various *in situ* reactions, and finally precipitates to the underlying surface. Consequently its total optical thickness varies on the way. It is natural to try to use that easily measurable optical characteristic to estimate the above values and parameters. According to the theory of the dynamics of multiphase media[1] the equation of conservation for the *i*th aerosol component may be written in the form

$$\frac{\partial \rho_{ai}}{\partial t} + \mathrm{div}\left[\rho_{ai}\left(\vec{v} - \vec{\omega}_{ai}\right)\right] = \sum_{j=1}^{N_m} J_{ji}^{ma} + \sum_{j=1}^{N_a} J_{ji}^{aa}, \tag{1}$$

where ρ_{ai} is the mass density of the *i*th aerosol fraction; $\vec{v}$ is the velocity of the air-gas mixture; $\vec{\omega}_{ai} = \vec{v} - \vec{v}_{ai}$ is the diffusion velocity of the *i*th aerosol fraction ($\vec{v}_{ai}$ is its true velocity); J_{ji}^{ma} is the intensity of transformation of the *j*th molecular (gas) component into the ith aerosol component; J^{aa} is the intensity of the transformation of the j'th aerosol component into the *i*th such component as a result of coagulation, fractionation, and other processes.

Assuming the principal cause of the differences in the velocities between the gas and aerosol phases to be precipitation of the latter, under the influence of gravity, Eq. (1) may be rewritten in the form

$$\frac{\partial \rho_{ai}}{\partial t} + \frac{\partial \rho_{ai} v_x}{\partial x} + \frac{\partial \rho_{ai} v_y}{\partial y} + \frac{\partial}{\partial z}\left[\rho_{ai}\left(v_z - w_{aiz}\right)\right] = \sum_{j=1}^{N_m} J_{ji}^{ma} + \sum_{j=1}^{N_a} J_{ji}^{aa}. \tag{2}$$

We divide the above functions into mean and fluctuational components, calculate the means, and use the following representation of the fluctuating components[2]:

$$\overline{\rho'_{ai} v'_x} = -\mu \frac{\partial \rho_{ai}}{\partial x}, \quad \overline{\rho'_{ai} v'_y} = -\mu \frac{\partial \rho_{ai}}{\partial y},$$
$$\overline{\rho'_{ai} v'_z} = -\nu \frac{\partial \rho_{ai}}{\partial z}, \tag{3}$$

where $\mu \ge 0$ (and remains constant in the layer z = const) and $\nu \ge 0$ are the horizontal and vertical diffusion coefficients, respectively. Thus we obtain an equation for the mean values of the density of the *i*th aerosol component

$$\frac{\partial \rho_{ai}}{\partial t} + \frac{\partial \rho_{ai} v_x}{\partial x} + \frac{\partial \rho_{ai} v_y}{\partial y} + \frac{\partial}{\partial z}\left[\rho_{ai}\left(v_z - w_{aiz}\right)\right] = \mu\left(\frac{\partial^2 \rho_{ai}}{\partial x^2} + \frac{\partial^2 \rho_{ai}}{\partial y^2}\right) + \frac{\partial}{\partial z}\left(\nu \frac{\partial \rho_{ai}}{\partial z}\right) + \sum_{j=1}^{N_m} J_{ji}^{ma} + \sum_{j=1}^{N_a} J_{ji}^{aa}. \tag{4}$$

Assuming further that the aerosol particles are spherical in shape, we may use the following expressions for the volume coefficients of aerosol scattering $\kappa_{\lambda ai}$ and absorption $\sigma_{\lambda ai}$:

$$\kappa_{\lambda ai} = N_{ai} \pi r_i^2 K_{scatt}(r_i, m_i, \lambda) = \kappa^0_{\lambda ai} \rho_{ai},$$

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$$\sigma_{\lambda ai} = N_{ai}\pi r_1^2 K_{abs}(r_1, m_1, \lambda) = \sigma^0_{\lambda ai}\rho_{ai}; \quad (5)$$

Here N_{ai} is the number of particles of the ith aerosol fraction per unit volume; K_{abs} and are the absorption and scattering efficiency factors at wavelength λ for a particle of radius r_1 and complex index of refraction m_1. Note that the mass aerosol absorption and scattering coefficients

$$\sigma^0_{\lambda ai} = \frac{3}{4}\frac{K_{abs}}{r_1}\frac{1}{\rho^s_{ai}}, \quad \kappa^0_{\lambda ai} = \frac{3}{4}\frac{K_{scatt}}{r_1}\frac{1}{\rho^s_{ai}} \quad (6)$$

are constant for a selected fraction which is characterized both by its material properties and by its radius. Here ρ^s_{ai} is the density of the aerosol solid matter.

Multiplying Eq. (4) by $\sigma^0_{\lambda ai}$, $\kappa^0_{\lambda ai}$, or $\alpha^0_{\lambda ai} = \sigma^0_{\lambda ai} + \kappa^0_{\lambda ai}$ it is easy to obtain the conservation equation for the corresponding volume optical characteristic

$$\frac{\partial\varphi_{\lambda i}}{\partial t} + \frac{\partial}{\partial x}\left(\varphi_{\lambda i}v_x\right) + \frac{\partial}{\partial y}\left(\varphi_{\lambda i}v_y\right) + \frac{\partial}{\partial z}\left[\varphi_{\lambda i}\left(v_z - w_{aiz}\right)\right] = \mu\Delta\varphi_{\lambda i} + \frac{\partial}{\partial z}\left(\nu\frac{\partial\varphi_{\lambda i}}{\partial z}\right) + \sum_{j=1}^{N_m} J^{ma}_{ji}\varphi^0_{\lambda i} + \sum_{j=1}^{N_a} J^{aa}_{ji}\varphi^0_{\lambda i}, \quad (7)$$

where $\varphi_{\lambda i} = (\sigma^0_{\lambda ai}, \kappa^0_{\lambda ai}, \alpha_{\lambda ai})$.

Following Ref. 2 we may determine the average amount of aerosol (for the time interval over which our averaging is conducted) which precipitates out per unit area per unit time at $z = 0$.

We select a cylinder of height $z = H$ and radius $r = R$ and assume that

$$\rho_{ai} = \rho_{ai0} \text{ at } t = 0,\ \rho_{ai} = 0 \text{ at } r = \sqrt{x^2 + y^2} = R,$$
$$\frac{\partial\rho_{ai}}{\partial z} = \alpha\rho_{ai} \text{ at } z = 0,\ \rho_{ai} = 0 \text{ at } z = H. \quad (8)$$

We integrate Eq. (7) at $\varphi_{\lambda i} = \alpha_{\lambda ai}$ over the vertical coordinate z within the limits $0 \le z \le H$. Taking into account that

$$\int_0^H \alpha_{\lambda ai}dz = \tau_{\lambda ai}, \quad (9)$$

which expresses the aerosol optical thickness of the ith fraction, and neglecting the dependence of v_x and v_y on z, we obtain for the aerosol flux precipitating to the surface the following expression:

$$\rho_{ai}\left(w_{aiz} - \nu\alpha\right)\Big|_{z=0} = \frac{1}{\alpha^0_{\lambda ai}}\left[-\frac{\partial\tau_{\lambda ai}}{\partial t} - \frac{\partial}{\partial x}\left(\tau_{\lambda ai}v_x\right) - \frac{\partial}{\partial y}\left(\tau_{\lambda ai}v_y\right) + \mu\Delta\tau_{\lambda ai} + \sum_{j=1}^{N_m}\int_0^H J^{ma}_{ji}\alpha^0_{\lambda i}dz + \sum_{j=1}^{N_a}\int_0^H J^{aa}_{ji}\alpha^0_{\lambda i}dz\right]. \quad (10)$$

The resulting equation relates the intensity of the aerosol flux onto the surface due to sedimentation and diffusion with the dynamics of the respective optical thickness.

The first term on the right-hand side of Eq. (10) reflects the variations in the optical thickness due to nonstationarity of the process, and the second and third terms – due to aerosol advective transport. The fourth term is responsible for diffusion, and the last terms in Eq. (10) describe the gas-aerosol and aerosol-aerosol reactions.

APPENDIX

Let us consider the process of aerosol optical thickness transport following the emission of the aerosol from a point source under stationary conditions in which one may neglect the flux of aerosol to the ground together with all types of reactions in the cloud. Let the aerosol source be described by the δ-function $\delta(\vec{r} - \vec{r}_0)$, and v_x and v_y be constant. Then Eq. (10), which describes the variation of the optical thickness, simplifies and reduces to the form

$$v_x\frac{\partial\tau_{\lambda ai}}{\partial x} + v_y\frac{\partial\tau_{\lambda ai}}{\partial y} - \mu\Delta\tau_{\lambda ai} = Q_{\lambda ai}\delta\left(\vec{r} - \vec{r}_0\right), \quad (11)$$

where $Q_{\lambda ai}$ is the intensity of "emission" of the optical thickness of the ith aerosol component by the pollution point source, and is related to the mass intensity of emission of the ith aerosol component Q_{ai} by the expression

$$Q_{\lambda ai} = \int_0^H \alpha^0_{\lambda ai}Q_{ai}dz.$$

The solution of Eq. (11), presented in Ref. 2, has the form

$$\tau_{\lambda ai} = \frac{Q_{\lambda ai}}{2\pi\mu}\exp\left\{\frac{v_x\left(\vec{x} - \vec{x}_0\right) + v_y\left(\vec{y} - \vec{y}_0\right)}{2\mu}\right\}\times K_0\left[\frac{\sqrt{v_x^2 + v_y^2}}{2\mu}\sqrt{\left(x - x_0\right)^2 + \left(y - y_0\right)^2}\right], \quad (12)$$

where $K_0(x)$ is McDonald's function:

$$K_0(x) = \int_0^{\infty} \exp(-x\,\mathrm{ch}\zeta)\,d\zeta, \qquad x > 0.$$

We now transform the coordinate system so as to place the initial point at $\vec{r}_0$, and align the direction of the new x axis with the velocity vector $\vec{v}$. Then formula (12) simplifies to

$$\tau_{\lambda ai} = \frac{Q_{\lambda ai}}{2\pi\mu} \int_0^{\infty} \exp\left[\frac{U}{2\mu} \left[x - \sqrt{x^2 + y^2}\,\mathrm{ch}\zeta\right] \right] d\zeta,$$

and we have along the axis of the smoke plume

$$\tau_{\lambda ai}(x,\ 0) = \frac{Q_{\lambda ai}}{2\pi\mu} \int_0^{\infty} \exp\left[\frac{Ux}{2\mu} \left[1 - \mathrm{ch}\zeta\right] \right] d\zeta. \qquad (13)$$

Expanding chζ in a power series and keeping only the first term, we obtain for $\tau_{\lambda ai}(x,\ 0)$ the following simple expression:

$$\tau_{\lambda ai}(x,\ 0) = \frac{1}{2\sqrt{\pi}} \frac{Q_{\lambda ai}}{\sqrt{U\mu}} \frac{1}{\sqrt{x}}. \qquad (14)$$

According to this expression the optical thickness produced by the ith aerosol fraction falls off in inverse proportion to the square root of the distance from the emission source.

We now seek a spatial solution of our proble»in the form

$$\tau_{\lambda ai}(x,\ y) = \tau_{\lambda ai}(x,\ 0)\, f(y).$$

Taking into account that $v_y = 0$ and recalling that the source in expression (11) is equal to zero everywhere in the smoke plume, we may integrate Eq. (11) to obtain the following expression for $f(y)$:

$$f(y) = \cos(\sqrt{a_1}\; y),$$

where

$$a_1 = \left[\frac{3}{4} \frac{1}{x^2} + \frac{U}{2\mu} \frac{1}{x} \right].$$

The equation for $f(y)$ satisfies the following boundary conditions at $y = 0$ $\tau(x,\ y) = \tau(x,\ 0)$, and the maximum of $f(y)$ lies on the plume axis

$$\left[\frac{\partial f}{\partial y} = 0,\ \frac{\partial^2 f}{\partial y^2} < 0 \right].$$

Finally we arrive at the following simple approximate relationship for estimating the dynamics of the optical thickness of the ith aerosol fraction in the absence of precipitation and of gas-aerosol and aerosol-aerosol transformations

$$\tau_{\lambda ai}(x,\ y) = \frac{1}{2\sqrt{\pi U\mu x}} \int_0^{z} \alpha^0_{\lambda ai} Q_{ai}\, dz \times$$

$$\times \cos\left[\sqrt{\frac{3}{4}\frac{1}{x^2} + \frac{U}{2\mu x}}\; y \right], \qquad (15)$$

here U is the wind velocity; μ is the air-flow eddy viscosity; $\alpha^0_{\lambda ai}$ is the mass extinction coefficient of the ith aerosol fraction; Q_{ai} is the intensity of the point emission source of the ith aerosol fraction; x is the distance from the source along the smoke plume axis; y is the distance from the source along the smoke plume axis to the given point.

Figure 1 presents a graph showing the changes in the optical thickness of the ith aerosol fraction, computed from the exact (Eq. (13)) and the approximate (Eq. (14)) relationships. It can be seen that far enough from the pollution source the approximate relationship describes the actual solution quite well. The upper part of Fig. 1 shows the decrease in the approximation error with distance from the source.

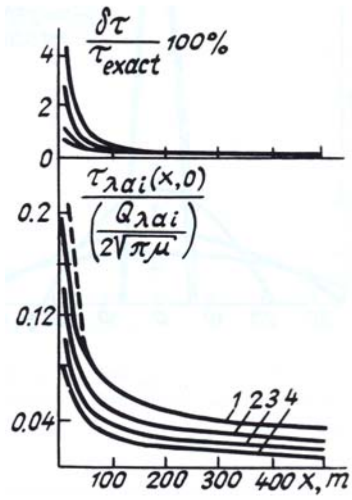

FIG. 1. The optical thickness $\tau_{\lambda ai}$ vs x and U/μ: solid curves — according to the exact relationship (13), dashed curve — according to the approximate relationship (14).

The variation of the optical thickness across the smoke plume is illustrated by Fig. 2. The dashed curve shows the optical thickness which corresponds to $a_1 = \dfrac{U}{2\mu}\dfrac{1}{x}$, for which the value of $\int_0^{y_s} \tau_{\lambda ai}\ dy$ is constant. Here $y_s = y_s(x) = \frac{\pi}{2\sqrt{a_1}}$, and the law of mass conservation is strictly obeyed.

1. Using formula (15) and having at one's disposal an instrument to measure the optical thickness of the *i*th aerosol fraction, one may estimate the amount of aerosol precipitating from the smoke plume, using the following algorithm: the optical thickness field is measured near the pollution source (up to where the aerosol begins to fall out). The needed parameters μ, U, and Q which enter into formula (15) are retrieved from these data. Then the optical thickness is measured within the region of aerosol precipitation. The difference between the measured optical thickness and that calculated using formula (15) yields the amount of precipitated aerosol.

2. Let the intensity of emission from the first of two sources Q_{a1} be known, and let the first source be positioned at the same level with the other source, whose intensity we want to estimate. Knowing the wind velocity U, we may retrieve the eddy viscosity coefficient μ from the value of Q_{a1}. With that coefficient at hand we estimate the intensity of the second source Q_{a1} using formula (15) and the shape of the second source plume.

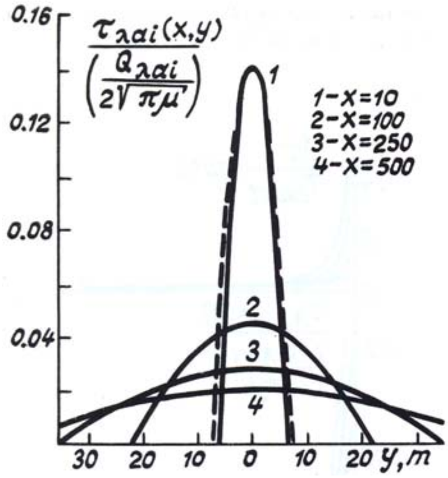

FIG. 2. Profiles of $\tau_{\lambda ai}$ *for various smoke plume cross sections (calculated according to formula (15)).*

REFERENCES

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