COMPARATIVE ANALYSIS OF TWO SCHEMES OF IMAGING THROUGH THE TURBULENT ATMOSPHERE

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This paper performs the comparative analysis of two point object imaging schemes: the scheme with a point light source and the scheme with object illumination by a focused light wave. It is shown that partial synchronization of phase fluctuations at different points of the receiving aperture occurs in the latter case, that results in higher image quality.

INTRODUCTION

Recently the effects associated with double passage of optical waves through the same (or strongly correlated with each other) inhomogeneities of the refractive index in randomly inhomogeneous medium are actively investigated. References 1 and 2 consider the most common regularities of statistics transformation of the radiation having passed through the same turbulent layer in forward and backward directions with reflection from a specular surface. The results of numerical simulation of phase characteristics of the reflected waves in the atmosphere are presented in Ref. 3. The analysis of the experimental data indicating the amplification of fluctuation intensity, when radiation is reflected from a specular surface, was carried out in Ref. 4. At last, Ref. 5 considers the scattering amplification effects in laser detection and ranging through the turbulent atmosphere, and in Ref. 6 an image quality of the coherently illuminated object in some randomly inhomogeneous medium was estimated.

At the same time, the problem of imaging through the turbulent atmosphere remains sufficiently urgent.[7,8] In this connection, of interest is the possibility to improve the image quality due to special illumination of the object. For instance, the image quality of the objects illuminated with coherent radiation in randomly inhomogeneous medium was considered in Ref. 6.

In this paper, we consider two imaging schemes of point objects in the turbulent atmosphere: the scheme A for a point source of radiation shown in Fig. 1*a* and the scheme B for an object illuminated by the wave focused at it shown in Fig. 1*b*. The latter scheme is the scheme with double passage of radiation through the medium, and it is of the special interest from our point of view. In the approximation of geometrical optics, for the scheme B we calculated the structural function of wave phase fluctuations, as well as the short- and long-exposure optical transfer functions (OTFs) for wave passing in the direction E toward the optical imaging system. The obtained results were compared with the well-known results for the scheme A. For correct comparison of two schemes, we believe that in the scheme A the wave passes in the medium the path $2z$ long.

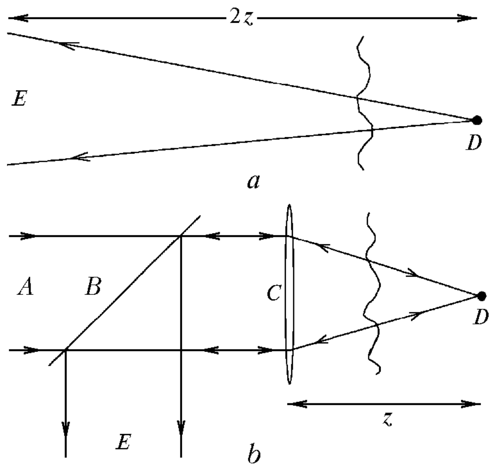

FIG. 1. The imaging schemes A (a) and B (b). In the scheme B, the wave from the radiation source A passes through the dividing plate B and is focused by the converging lens C at the point object D. The turbulent medium occupies the space to the right of the lens C. After reflection from the object D the wave passes through the lens C again, reflects from the dividing plate B, and goes in the direction E toward the imaging system.

CALCULATION OF THE STRUCTURAL FUNCTIONS

At the distance $2z$ from the point radiation source located within the thickness of turbulent medium, the structural function of the phase fluctuations φ of the spherical wave emitted by the object depends only on the difference in coordinates of the observation points $r = |\rho_1 - \rho_2|$ and has the well-known form,[9,10]

$$D_A(\rho_1, \rho_2) = \langle [\varphi(\rho_1, z) - \varphi(\rho_2, z)]^2 \rangle =$$

$$= D_A(r) = \frac{3}{8} \times 6.88 \left(\frac{r}{r_0}\right)^{5/3}, \qquad (1)$$

where

$$r_0 = 0.185 \left(\frac{\lambda^2}{2\, C_n^2\, z}\right)^{3/5} \qquad (2)$$

is the Fried radius calculated for the plane wave having passed through the layer of turbulent medium $2z$ thick, C_n^2 is the structural constant of the refractive index fluctuations in the medium, λ is wavelength of the radiation propagating through the medium.

Calculation of the structural function of the phase fluctuations for the scheme B carried out in approximation of the geometrical optics is more difficult. In this scheme the ray outgoing from the illumination source and reflected from the object passes along the path symmetrical about the axis of the optical system in the backward direction. Therefore, the phase shift along the ray coming to the point $\boldsymbol{\rho}_1$ of the receiving aperture of the imaging system can be written as

$$\varphi(\boldsymbol{\rho}_1) = k \int\limits_0^z \mathrm{d}\zeta\, [n_1(\boldsymbol{\rho}(\boldsymbol{\rho}_1, \zeta), \zeta) + n_1(-\boldsymbol{\rho}(\boldsymbol{\rho}_1, \zeta), \zeta)], \qquad (3)$$

where $\boldsymbol{\rho}(\boldsymbol{\rho}_1, \zeta) = \boldsymbol{\rho}_1 \times (1 - \zeta/z)$, $n_1(\boldsymbol{\rho}, z)$ is the fluctuating part of the refractive index, and $k = 2\pi/\lambda$ is the wavenumber. Having written the similar expression for the phase shift along the ray coming to the point $\boldsymbol{\rho}_2$ by subtracting them one from another, taking the square of their difference, and averaging over an ensemble of realizations, we can obtain the expression for the structural function of the phase fluctuations

$$D_B(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = k^2 \int\limits_0^z \mathrm{d}\zeta_1 \int\limits_0^z \mathrm{d}\zeta_2\, \{4\, [K_n(\zeta_1 - \zeta_2) -$$

$$- K_n(\sqrt{(\boldsymbol{\rho}(\boldsymbol{\rho}_1,\zeta_1) - \boldsymbol{\rho}(\boldsymbol{\rho}_2,\zeta_2))^2 + (\zeta_1 - \zeta_2)^2})] +$$

$$+ 4\, [K_n(\zeta_1 - \zeta_2) - K_n(\sqrt{(\boldsymbol{\rho}(\boldsymbol{\rho}_1,\zeta_1) + \boldsymbol{\rho}(\boldsymbol{\rho}_2,\zeta_2))^2 + (\zeta_1 - \zeta_2)^2})] -$$

$$- 2\, [K_n(\zeta_1 - \zeta_2) - K_n(\sqrt{4\rho^2(\boldsymbol{\rho}_1,\zeta_1) + (\zeta_1 - \zeta_2)^2})] -$$

$$- 2\, [K_n(\zeta_1 - \zeta_2) - K_n(\sqrt{4\rho^2(\boldsymbol{\rho}_2,\zeta_2) + (\zeta_1 - \zeta_2)^2})]\}, \qquad (4)$$

where $K_n(r) = \langle n_1(\mathbf{r}_1)\, n_1(\mathbf{r}_1 + \mathbf{r})\rangle$ is the correlation function of the refractive index fluctuations, which are considered statistically homogeneous and isotropic. For the Kolmogorov spectrum of the refractive index fluctuations the integral of every expression in square brackets in Eq. (4) is calculated in the same way as in Ref. 9, when calculating the structural function of phase fluctuations of the spherical wave. As a result, we obtain

$$D_B(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = D_A(|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|) + D_A(|\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2|) -$$

$$- \frac{1}{2}\, [D_A(2|\boldsymbol{\rho}_1|) + D_A(2|\boldsymbol{\rho}_2|)], \qquad (5)$$

where $D_A(\rho)$ is given by the expression (1). For the further analysis it is convenient to express Eq. (5) as the function of variables $\mathbf{R} = \dfrac{\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2}{2}$ and $\mathbf{r} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$

$$D_B(\mathbf{R}, \mathbf{r}) = D_A(r) + D_A(2R) -$$

$$- \frac{1}{2}\, [D_A(|2\mathbf{R} + \mathbf{r}|) + D_A(|2\mathbf{R} - \mathbf{r}|)]. \qquad (6)$$

While the structural function of the phase fluctuations for the scheme A given by Eq. (1) depends only on the absolute value of the distance r between the points $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ and monotonically increases with r in the power law, the behavior of the structural function (6) calculated for the scheme B is much more interesting. First, its value depends not only on r, but also on mutual position of the points $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$. Second, at least at not so large R the structural function D_B increases with r much slower than D_A, and at $R = 0$ it vanishes irrespective of r.

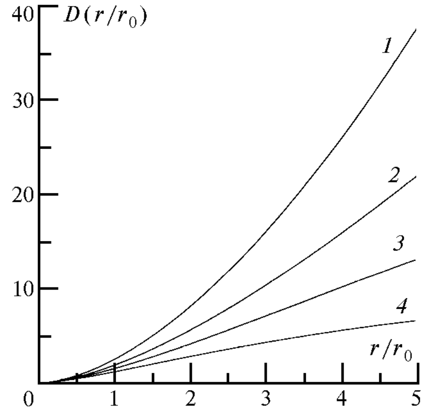

FIG. 2. The structural functions of the phase for the schemes A (1) and B at $\mathbf{R} \perp \mathbf{r}$ *and for different R:* $R/r_0 = 20$ (*2*), $R/r_0 = 5$ (*3*), *and* $R/r_0 = 2$ (*4*). At $R/r_0 \to \infty$ D_B coincides with D_A.

Figure 2 illustrates the behavior of D_A and D_B functions in the case, when the angle α between $\mathbf{R}$ and $\mathbf{r}$ vectors is equal to $\pi/2$, i.e., $\mathbf{R} \perp \mathbf{r}$. At arbitrarily oriented vectors $\mathbf{R}$ and $\mathbf{r}$, the distinction between the structural phase functions for the schemes A and B is a little less than in the case, when these vectors are perpendicular, but it remains nevertheless significant at least while $R < 20\, r_0$. The D_B dependence on r at different values of R and α remains qualitatively the same as at $\alpha = \pi/2$. That is why we do not show here the appropriate plots, restricting our consideration to that represented in Fig. 2. Let us point out that at $R \to \infty$ the expression (6) transforms into the expression (1) irrespective of the mutual orientation of the vectors $\mathbf{R}$ and $\mathbf{r}$.

So, as seen from Fig. 2, the phase fluctuations of the wave in the scheme B appear suppressed in comparison with the fluctuations in the scheme A, though in both cases the radiation passes the identical distance equal to $2z$ through the turbulent media. It is caused by the following reason. In the scheme B, the rays coming to the points $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ symmetric with respect to axis of the optical system ($\boldsymbol{\rho}_1 = -\boldsymbol{\rho}_2$, $R = 0$) passed through the same inhomogeneities of the medium refractive index, though in different directions. That is why the phase fluctuations at these points are identical. Even when the points $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are not absolutely symmetric ($R \neq 0$), the partial correlation of the wave phase fluctuations is retained at these points, though the corresponding rays pass through not the same, but close inhomogeneities.

OTF CALCULATION

Evidently, the above feature of the scheme B must result in higher quality of an image formed according to this scheme in comparison with the case, when the scheme A is used. Really, in the scheme B the rays coming to the points of the receiving aperture symmetric with respect to the axis of the optical system will interfere in the imaging plane as in the case with no turbulent medium, since the phase difference between them is zero. To confirm this result, we have carried out the calculation of long-exposure $\tau^{\mathrm{L}}(\Omega)$ and short-exposure $\tau^{\mathrm{S}}(\Omega)$ OTF for the schemes A and B, where Ω is the angular frequency.

Let us invoke the OTF definition through the generalized function of the optical system pupil $P(\mathbf{R}) = P_0(\mathbf{R}) \exp(i\varphi(\mathbf{R}))$ from Ref. 11, where P_0 is the pupil function and φ is the phase of the wave incident on the receiving aperture. If provided that the aperture of the optical system is circle-shaped with the diameter d_0, for long-exposure OTF we obtain the following:

$$\tau^{\mathrm{L}}_{\mathrm{A,B}}(\Omega) = \frac{1}{A}\int \mathrm{d}^2\mathbf{R} < P\left(\mathbf{R} - \frac{\lambda\mathbf{\Omega}}{2}\right) P^*\left(\mathbf{R} + \frac{\lambda\mathbf{\Omega}}{2}\right) > =$$

$$= \frac{1}{A}\int \mathrm{d}^2\mathbf{R}\, P_0\left(\mathbf{R} - \frac{\lambda\mathbf{\Omega}}{2}\right) P_0^*\left(\mathbf{R} + \frac{\lambda\mathbf{\Omega}}{2}\right) \times$$

$$\times \exp\left\{-\frac{1}{2} D_{\mathrm{A,B}}(\mathbf{R}, \lambda\mathbf{\Omega})\right\}, \qquad (7)$$

where the integral is taken over the whole plane $\{\mathbf{R}\}$, and $A = \pi\, d_0^2/4$ is the aperture area.

As D_{A} does not depend on $\mathbf{R}$, the well-known result[10,12] follows from Eq. (7) for the scheme A:

$$\tau^{\mathrm{L}}_{\mathrm{A}}(\Omega) = \tau_0(\Omega) \exp\left\{-\frac{1}{2} D_{\mathrm{A}}(\lambda\Omega)\right\}, \qquad (8)$$

where $\tau_0(\Omega)$ is the diffraction-limited OTF of the optical system, which falls down to zero at $\Omega = \Omega_0 = d_0/\lambda$ (Ref. 11).

For the scheme B, we failed to perform analytical integration in the expression (7), therefore the corresponding integral was found numerically using the expression (6) for $D_{\mathrm{B}}(\mathbf{R}, \mathbf{r})$.

In the case of short exposure for the scheme A, we have the well-known expression[12]

$$\tau^{\mathrm{S}}_{\mathrm{A}}(\Omega) = \tau_0(\Omega) \exp\left\{-\frac{1}{2}\left(D_{\mathrm{A}}(\lambda\Omega) - \frac{1}{2}\sigma^2_{\mathrm{A}}(\lambda\Omega)^2\right)\right\}, \qquad (9)$$

where σ^2_{A} is variance of wave front tilts[13]

$$\sigma^2_{\mathrm{A}} = \frac{64}{d_0^2}\int_0^1 \frac{4}{\pi}\left[(3u - 2u^3)\sqrt{1-u^2} - \right.$$

$$\left. - \arccos u\right] D_{\mathrm{A}}(ud_0)\, u\, \mathrm{d}u \simeq \frac{10.32}{r_0^{5/3}\, d_0^{1/3}}. \qquad (10)$$

The analysis shows that the variance of wave front tilts σ^2_{B} calculated for the scheme B appears identically equal to zero. This results in coinciding expressions for long-exposure and short-exposure OTF in the case B, i.e., $\tau^{\mathrm{S}}_{\mathrm{B}}(\Omega) \equiv \tau^{\mathrm{L}}_{\mathrm{B}}(\Omega)$. In other words, the quality of image formed using the scheme B may not essentially depend on exposure duration, while short exposure times are preferable for the scheme A. Besides, it is possible to show that for the scheme B the variances of all modes of odd order in polynomial expansion of the wave front shape are also equal to zero. At the same time, for the scheme A the variance monotonically decreases with increasing order and becomes negligible only for modes of sufficiently high order.[13,14]

Since the influence of random wave front tilts on the long-exposure image quality is the greatest, their absence in the scheme B is of fundamental importance. Besides, zero values of variances of all modes of odd order also results in increase in the image quality in comparison with the scheme A, especially at the rather large size of the aperture.

Figure 3 represent the long- and short-exposure OTFs for the schemes A and B. It is seen that the width of OTF corresponding to the scheme B essentially exceeds the width of the long-exposure OTF for the scheme A. The curves representing the short-exposure OTF for the scheme A and OTF for the scheme B are rather close, however, the latter goes somewhat above the former. It is especially noticeable at angular frequencies $\Omega \sim \Omega_0/2$. Thus, the use of the scheme B even at prolonged exposure can allow obtaining of higher-quality images than in the scheme A with short exposure.

At last, we should note one more interesting feature of the scheme B. It is known[1] that in the scheme A the integration resolution

$$R = 2\pi \int_0^\infty \tau^{\mathrm{L,S}}_{\mathrm{A}}\, \Omega\, \mathrm{d}\Omega \qquad (11)$$

is saturated at the level $R = \pi/4\ (r_0/\lambda)^2$ with increasing aperture diameter d_0. It is caused by the fact that phases of waves coming to far separated parts of the receiving aperture are practically noncorrelated, consequently the constructive interference of these waves in the image plane appears impossible. Therefore, increasing of the aperture diameter up to values larger than several r_0 does not lead to increase in the number of constructively interfering waves in the image plane, so it is inexpedient.

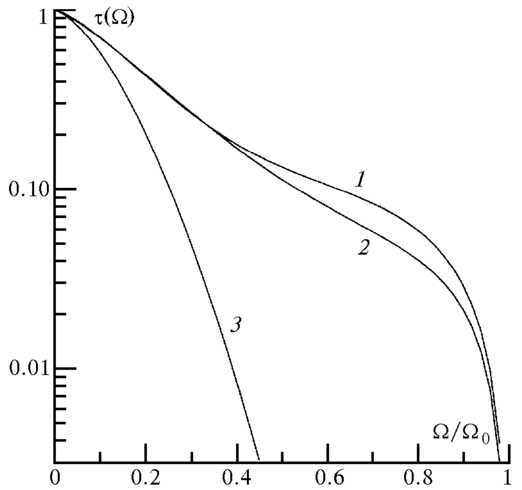

FIG. 3. The optical transfer functions at $d_0 = 5\ r_0$ for the scheme B (1) and the scheme A: short exposure (2), long exposure (3).

However, when the scheme B is used, the situation is changed, because the phase shift along the rays coming to the points of the receiving aperture symmetric with respect to the axis of the optical system is always the same. It allows such rays to constructively interfere in the image plane, no matter how large is the distance between the points of their arrival to the receiving aperture plane. Therefore, there are strong grounds for stating that the scheme B must ensure infinite increase of the imaging system resolution when the diameter of its receiving aperture tends to infinity.

CONCLUSION

In this paper, we carried out the comparative analysis of two imaging schemes for point objects in the turbulent atmosphere, namely, the scheme with radiation source and the scheme with an object illuminated by a focused wave. The results obtained for the structural function of the phase fluctuations show that in the latter case the partial suppression of fluctuations of the phase difference of the waves incident on different parts of the imaging system aperture takes place. Because the rays coming to the receiving aperture points symmetric with respect to the axis of the optical system pass through the same refractive index inhomogeneities in the medium, phase fluctuations at these points are completely correlated and their difference identically vanishes. For points nonsymmetrical with respect to the axis of the optical system, this effect is retained only partially.

Under conditions, when the distance the ray passes in the turbulent medium is fixed, the scheme with illumination is preferable, because it can ensure better vision conditions. Our calculations show that the image quality in the scheme with illuminated object must weakly depend on the exposure time, and in all cases it appears higher than in the scheme with radiation source and short exposure. There is reason to expect that for the scheme with object illumination with the wave focused at it, the integral resolution will not saturate at a constant level with aperture diameter tending to infinity, in contrast to the scheme with radiation source object.

REFERENCES

1. V.P. Lukin, *Atmospheric Adaptive Optics* (Nauka, Novosibirsk, 1986), 286 pp.
2. V.A. Banakh and V.L. Mironov, *Lidar in a Turbulent Medium* (Artech House, Boston-London, 1987), 185 pp.
3. P.A. Konyaev, V.P. Lukin, and S.Yu. Tabakaev, Atm. Opt. **4**, No. 12, 910–911 (1991).
4. A.S. Gurvich, A.P. Ivanov, S.S. Kashkarov, G.Ya. Patrushev, and A.P. Rostov, Atmos. Oceanic Opt. **5**, No. 1, 29–32 (1992).
5. V.A. Banakh and V.L. Mironov, Atmos. Oceanic Opt. **8**, Nos. 1–2, 23–32 (1995).
6. V.A. Banakh and B.N. Chen, Atmos. Oceanic Opt. **7**, Nos. 11–12, 827–830 (1994).
7. V.P. Lukin, Atmos. Oceanic Opt. **8**, Nos. 1–2, 145–150 (1995).
8. V.P. Lukin, Atmos. Oceanic Opt. **8**, No. 3, 152–173 (1995).
9. D.L. Fried, J. Opt. Soc. Am. **56**, No. 10, 1380–1384 (1966).
10. J.W. Goodman, *Statistical Optics* (John Wiley and Sons, New York, 1985)
11. D. Goodman, *Introduction to Fourier Optics* (McGraw-Hill, New York, 1968).
12. D.L. Fried, J. Opt. Soc. Am. **56**, No. 10, 1372–1379 (1966).
13. D.L. Fried, J. Opt. Soc. Am. **55**. No. 11, 1427–1435 (1965).
14. R.J. Noll, J. Opt. Soc. Am. **66**, No. 3, 207–211 (1976).