ON THE RESOLUTION OF AN ADAPTIVE SYSTEM WITH FOUR-SEGMENT COMPENSATION OF RANDOM WAFEFRONT TILTS

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An adaptive optical system, with a continuous flexible mirror is modeled. The random wavefront tilts are estimated from the shifts of the centers of gravity of the images formed by four sections of the aperture. The dependence of the resolution of the system on the ratio of the aperture diameter to the size of the correlation length of atmospheric distortions is determined. The potential resolution of the system is estimated. The characteristics of the system mentioned above are compared with the characteristics of a systems with compensation of the general tilt and a four-segment mirror. It is shown that the system studied is 2.1 times more efficient than a system with compensation of the general tilt and 1.2 times more efficient than a system with a four-segment mirror.

Different adaptive systems for estimating and compensating atmospheric distortions of the received radiation for the purpose of forming an image of the observed object are now being actively studied throughout the world.[1] One of the simplest systems is a system based on measurement of the wavefront tilts based on the shifts of the corresponding short-exposure images (SEIs) and on their compensation in the process of formation of a long-exposure image (LEI). Theoretical analysis of this system shows that such compensation can increase the resolution by almost a factor of four, but for comparatively small receiving apertures D. Real experiments have confirmed this.[2,3,4] The maximum gain is achieved with $D \approx 4r_0$, where r_0 is Fried's parameter, which characterizes the correlation length of the distortions of the radiation field.[2] For $D > 6r_0$ the efficiency of the system drops sharply.

An obvious way to extend the boundaries of the practical efficiency is to determine and compensate the partial wavefront tilts on separate segments of the aperture. In Ref. 5 mathematical modeling showed that in a four-segment system the gain in resolution can be increased by a factor of 4.5 and the range of effectiveness of the system can be extended up to $10r_0$. However, the practical implementation of this system presupposes the use of not four rigid segments (sectors of the circular aperture), but rather a standard compensator of the general tilt and a flexible mirror with 16 pushers. These pushers, distributed almost uniformly over the aperture (over four per segment) make it possible to compensate the partial tilts, which are estimated, as previously, from the shifts of the SEIs, formed in parallel from the corresponding sections of the wavefront of the received radiation. This made it necessary to refine the results of the preceding work.[5]

We performed mathematical modeling of the process of formation of LEIs of a point source for the above-described adaptive system under the following conditions:

– averaging of 100 SEIs is sufficient to obtain an LEI;

– the function $F_j(\vec{v})$ describing the response of the mirror to the action of the jth pusher has the form

$$F_j(\vec{v}) = A_j \exp\left\{-\frac{\left[\vec{v}-\vec{v}_j\right]^2}{2\sigma^2}\right\},$$

where A_j is the amplitude of the action, $\vec{v}_j$ is the point of application of the jth pusher, and the parameter σ characterizes the width of the response;

– the angular resolution R is estimated as the ratio of the intensity at the center of the LEI to its total energy and is compared with the limiting resolution R_∞, achieved in the limit $D/r_0 \to \infty$ and is equal to[2]

$$R_\infty = \frac{\pi}{4}\left[\frac{r_0}{\lambda}\right]^2,$$

where λ is the wavelength.

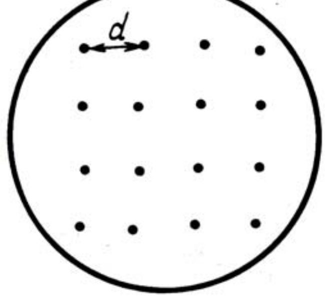

FIG. 1. Arrangement of the pushers over the aperture.

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The arrangement of the 16 pushers over the aperture is shown in Fig. 1. The step $d \sim D/5$.

As we have already mentioned, to implement the system under study in practice it is proposed that the instantaneous amplitudes A_j be determined from the condition that the wavefront tilt on the corresponding ith segment be determined in the form shown below from the condition that the wavefront tilt on the corresponding ith segment be compensated:

$$A_j = (\vec{a}_i - \vec{a})(\vec{v}_j - \vec{v}_{ci}),$$

where $\vec{v}_{ci}$ is the geometric center of the segment;

$$a_i = S_i^{-1}\int d\vec{v}\, W_i(\vec{v})\, \mathrm{grad}\varphi(\vec{v})$$

is the partial tilt of the wavefront; $W_i(\vec{v})$ is the aperture function of the ith segment; $S_i = \int d\vec{v}\, W_i(\vec{v})$ is the area of the ith segment; $\varphi(\vec{v})$ is the instantaneous distribution of the atmospheric distortions of the phase of the radiation field; and, $\vec{a} = \frac{1}{4}\sum_{i=1}^{4}\vec{a}_i$ is the independent compensated general tilt. To determine the possibilities of this system we also investigated the case when the amplitudes A_j are determined from the condition

$$\int d\vec{v}\, W_1(\vec{v})\left[\varphi(\vec{v}) - \sum_{1=1}^{16} F_j(\vec{v})\right]^2 = \min.$$

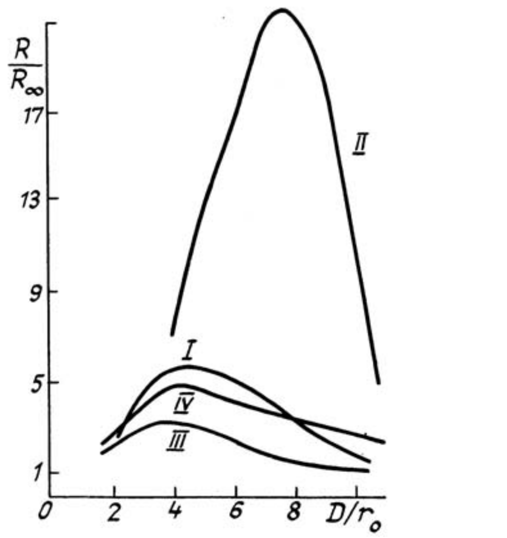

FIG. 2. R/R_∞ versus D/r_0 for the following system.: I is the system under study with $\sigma \neq 3$; II is the "potential" system with $\sigma = 6$; III is the system with compensation of the general tilt; and, IV is the system with a four-segment mirror.

In the process of modeling, up to 100 random realizations of the distortions $\varphi(\vec{v})$ with a "5/3" structure function were played out for values of D/r_0 ranging from 1 to 15. The system was investigated for different values of the parameter σ. The results are shown in Figs. 2 and 3, where the analogous characteristics of the previously investigated systems are presented for comparison. The following notation was used here: I is the system under study, II is the "potential" system, III is the system with compensation of the general tilt over the entire aperture, and IV is a system with a four-segment mirror.

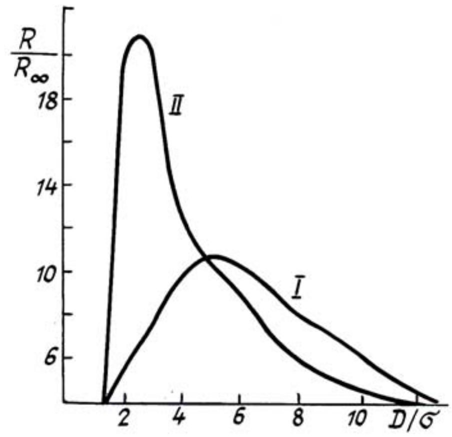

FIG. 3. R/R_∞ versus D/r_0 with $D/r_0 = 4$ for the system I and $D/r_0 = 7$ for the system II.

It is not difficult to verify that 1) the maximum resolution of system I is achieved with $D/r_0 = 4$ and $D/\sigma = 5$ and is equal to $5.65R_\infty$; 2) the maximum gain of system I over the system III is equal to 2.1 with $D/r_0 = 6$, while compared with system IV the gain is 1.2 with $D/r_0 = 4$; and 3) the maximum resolution of the system II is achieved with $D/r_0 = 7$ and $D/\sigma = 3$ and is equal to $20.6R_\infty$, which is 3.5 times larger than the resolution of the system I.

Thus it can be concluded that in estimating the wavefront tilts from the shift of the center of gravity of the SEIs the adaptive system with a continuous flexible mirror is not as effective as a system with a segmented mirror. However this system has great potential, and by developing appropriate algorithms the resolution of the adaptive system studied can be substantially increased.

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