SYNTHESIS OF AN OPTIMAL CONTROL ALGORITHM FOR A PHASE–CONJUGATED ADAPTIVE OPTICAL SYSTEM

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An algorithm for control with the wavefront corrector of an adaptive optical system, which minimizes the quadratic quality functional, is synthesized for the problem of correction of the distortions of the wavefront from a remote point source.

Introduction. One of the serious problems in the development of the adaptive optical systems (AOS) is the construction of the effective algorithms for control with the final control elements of these systems, namely, the wavefront correctors (WFC's). At present, search for the optimal solution, which is optimal according to some statistical criterion, and the problems of compensating for the wavefront distortions of the phase–conjugated AOS are superseded with an euristic construction of the efficient algorithms[1,2] which, essentially, do not take into account the dynamics of variation in the statistical parameters of the atmospheric distortions and the dynamic properties of the final control elements of the correction system.

In this connection, the use of the algorithms of the stochastic theory of optimal control, which make it possible to formulate a law for control with the final control elements of the AOS, which is optimal according to the criterion of the minimum of the integral quadratic quality functional, is promising.

For this reason, the formulation and the solution of the problem of synthesizing the law for control by the AOS, which is optimal according to the criterion of the minimum of the quality functional and takes into account the errors of correction of the wavefront distortions and the expense of the control, is of scientific and practical interest.

1. The problem formulation. Let the phase–conjugated AOS with the given part (the wavefront corrector)

$$\dot{\mathbf{z}}(t) = A\mathbf{z}(t) + Bu(t)\ , \tag{1}$$

$$\tilde{\alpha}(t) = C\mathbf{z}(t)\ , \tag{2}$$

$(\alpha = \psi,\ \theta)$

be intended for monitoring the fluctuations of the tilt $\alpha(t)$ of the total wavefront from a remote point source caused by atmospheric turbulence

$$\dot{\alpha}(t) = -\beta\alpha(t) + \sqrt{2\beta d}\ \upsilon(t) \tag{3}$$

$(\alpha = \psi,\ \theta)$

when the observations from the outputs of the sensor of the wavefront distortions are available

$$\mathbf{u}_{\text{ob}}(t) = k[\gamma(t) - \tilde{\gamma}(t)] + \mathbf{u}_f(t)\ , \tag{4}$$

where $\mathbf{z}(t)$ is the k–dimensional vector of the state of the controllable object, A is the $k\times k$ matrix, B is the $k\times 1$ column vector, G is the $1\times k$ row vector, $\tilde{\alpha}(t)$ is the controllable quantity, which corresponds to the WFC tilt angles with respect to the plane of the input pupil of the optical system in the two orthogonal directions $(\alpha = \psi,\ \theta)$; $u(t)$ is the voltage applied at the WFC actuators, $\beta = 1/\tau_0$, τ_0 is the time constant of the correlation of the random process $\alpha(t)$, $\upsilon(t)$ is the "white" noise with zero mean value and the spectral density matrix $S_\upsilon = 1$, d is the variance of the fluctuations of the random quantities $\alpha(t)$, k is the slope of the sensor characteristic with respect to the wavefront tilt, $\gamma(t) = \begin{bmatrix}\psi(t)\\ \theta(t)\end{bmatrix}$ is the combined column vector of the tilts of the wavefront incident on the receiving aperture, $\tilde{\gamma}(t) = \begin{bmatrix}\tilde{\psi}(t)\\ \tilde{\theta}(t)\end{bmatrix}$ is the combined vector of the shifts in the field of the wavefront tilts introduced by the corrector, and $\mathbf{u}_f(t) = \begin{bmatrix}\mathbf{u}_{f\psi}(t)\\ \mathbf{u}_{f\theta}(t)\end{bmatrix}$ is the column vector of the measurement noise (the noise is assumed to be "white").

We should find the law for control with the WFC of the AOS $u^*(t)$, which is optimal according to the criterion of the minimum of the quality functional

$$J = M\left\{\int\limits_{t_0}^{t_\text{k}}\left[\frac{\Delta\alpha^2(t)}{\Delta\alpha^2_{\text{ac}}} + \frac{u^2(t)}{\Delta u^2_{\text{ac}}}\right]\text{d}t\right\}, \tag{5}$$

where $M\{\cdot\}$ is the mathematical expectation operator, $\Delta\alpha(t) = \alpha(t) - \tilde{\alpha}(t)$ is the correction error of the total wavefront tilt of the AOS, $\Delta\alpha^2_{\text{ac}}$ is the acceptable variance of the correction error, and $\Delta\mathbf{u}^2_{\text{ac}}$ is the squared acceptable voltage applied at the actuator of the total wavefront tilt.

2. The solution of the problem. In accordance with the separation theorem of the optimal control theory[3] in the systems, which are synthesized according to the quality criterion (5), the optimal controlling inputs on a linear object are linear combinations of the optimal estimates of the vector–parameters, which determine the state of the wavefront incident upon the aperture. For this reason, the problem of forming the optimal controlling inputs involves the problem of optimal estimating the wavefront state and the problem of forming the WFC controlling inputs based on the estimates obtained. The solution of the former problem was given in Ref. 4.

Making use of Eq. (3) which describes the variation of the total wavefront tilt and Eqs. (1) and (2) describing the WFC performance let us introduce the generalized equations of the state and of the error of correction of the total tilt

$$\dot{\mathbf{z}}_\text{g}(t) = A_\text{g}\mathbf{z}_\text{g}(t) + B_\text{g}u(t) + F_\text{g}\upsilon(t)\ ;$$

$$\Delta\alpha(t) = \alpha(t) - \tilde{a}(t) = C_\text{g}\mathbf{z}_\text{g}(t)\ ;\ (\alpha = \psi,\ \theta) \tag{6}$$

where

$$A_g = \begin{bmatrix} -\beta & 0 \\ 0 & A \end{bmatrix}; \quad B_g = \begin{bmatrix} 0 \\ B \end{bmatrix}; \quad F_g = \begin{bmatrix} \sqrt{2\beta d} \\ 0 \end{bmatrix};$$

$C_g = [1 - C]$, and $\mathbf{z}_g(t) = \begin{bmatrix} \alpha(t) \\ \mathbf{z}(t) \end{bmatrix}$.

Using Pontryagin's principle[3] of maximum one can show that in this case the optimal controlling voltage $u^*(t)$, minimizing functional (5), will satisfy the condition

$$u^*(t) = -K^*(t)\,\hat{\mathbf{z}}_g(t)\,, \tag{7}$$

where $K^*(t)$ is the matrix (row vector) of the optimal gain coefficients, $\hat{\mathbf{z}}_g(t)$ is the column vector, whose components are the estimate of the preset input $\hat{\alpha}(t)$, which fed from the output of the state estimator,[4] and $\mathbf{z}(t)$ is a vector of the controllable object state, the information about which is obtained from the sensor of the state of the WFC actuator.

The matrix $K^*(t)$ is calculated in accordance with the relation[3]

$$K^*(t) = B_g^T \Gamma_g(t)/u_{ac}^2\,, \tag{8}$$

where $\Gamma_g(t)$ is the matrix, which satisfies the Riccati matrix differential equation (the upper index T denotes the transposition operation),

$$\Gamma_g(t) = -\Gamma_g(t)A_g(t) - A_g^T\Gamma_g(t) + $$

$$+ \Gamma_g(t)B_gB_g^T\Gamma_g(t)/u_{ac}^2 - C_gC_g^T/\Delta\alpha_{ac}^2\,. \tag{9}$$

The main difficulty in realization of this algorithm is connected with the solution of Eq. (9) in reverse time, since the boundary condition is superposed on the value $\Gamma_g(t)$ at the time t_c in which the control has ceased. It is well known that as $t_c \to \infty$, the solution of Eq. (9) does not depend on the boundary condition, while for the constant matrices A_g, B_g, and C_g the matrix Γ_g has constant coefficients, which is searched out from an algebraic equation derived on the basis of Eq. (9) with $\Gamma_g(t) = 0$. Thus, the given solution can be found beforehand and, consequently, the needed gain coefficients of the summators of the signals, which fed from the outputs of the wavefront state estimator, of the sensor of the state of the WFC actuators, and of the electronic amplifiers can be also preset.

Conclusion. The proposed approach makes it possible to formalize the problem of operation of the phase–conjugated AOS with an account of the dynamic parameters of the phase fluctuations caused by the atmospheric turbulence, and of the final control elements of the correction system based on the theory of optimal control by the dynamic systems. The synthesized algorithm provides the best performance quality for the phase–conjugated AOS within the frameworks of the formalized quality description defined by functional (5), which characterizes the figure of performance of the correction system and energy consumption for the WFC control.

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