

## NUMERICAL STUDY OF THE AEROSOL EXTINCTION AT $\lambda = 10.6 \mu\text{m}$ DURING THE EVOLUTION OF A STRATUS CLOUD

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*Based on a simple mathematical model of the evolution of stratus clouds the behavior of the aerosol extinction coefficient is investigated in this paper.*

Fogs and low-lying stratus clouds are the most frequent atmospheric formations responsible for heavy turbidity in the atmosphere. In well-developed low-lying clouds the scattering coefficient, averaged over a cloud, can reach  $100 \text{ km}^{-1}$ , which exceeds the coefficient of molecular light scattering by about four orders of magnitude.

At present it is possible to find dozens of mathematical models in the literature<sup>1</sup> describing the evolution of clouds, which take into account the microstructure of cloud-forming processes to different extents. The subject of the present study is the dynamics of the aerosol extinction coefficient at the  $\text{CO}_2$ -laser wavelength ( $\lambda = 10.6 \mu\text{m}$ ) occurring during the formation of a cloud layer. The investigation used the simplest hydrodynamic model of the evolution of the cloud and temperature fields in a moving cyclone<sup>2</sup> assuming the size-distribution function of the cloud droplets to be known.

The model<sup>2</sup> uses the following system of equations for heat and moisture transfer in the turbulent atmosphere:

$$\frac{\partial \Pi}{\partial t} + \omega \frac{\partial \Pi}{\partial z} = \frac{\partial}{\partial z} k \frac{\partial \Pi}{\partial z} - \omega \gamma_a; \quad (1)$$

$$\frac{\partial s}{\partial t} + \omega \frac{\partial s}{\partial z} = \frac{\partial}{\partial z} k \frac{\partial s}{\partial z}; \quad (2)$$

$$\Pi = T + \frac{L}{C_p} q; \quad (3)$$

$$s = q + \delta, \quad (4)$$

where  $\omega = 4\omega_m z / H(1 - z/H)$  is the vertical speed,  $\gamma_a$  is the dry adiabatic gradient,  $k$  is the turbulence coefficient,  $q$  is the specific humidity of air,  $\delta$  is the liquid water content in a cloud,  $s$  is the specific moisture content in air,  $\Pi$  is the equivalent temperature,  $L$  is the latent heat of condensation of water vapor,  $C_p$  is the specific heat of air at constant pressure, and  $T$  is the air temperature.

Taking the cloud formation process into account, system of equations (1)–(4) was supplemented by the following equation

$$q = q_m(T, p) = 0.622 E(T)/p, \quad (5)$$

where  $E$  is the water vapor saturation pressure, and  $p$  is the pressure.

The initial distributions of the values entering the system of equations were set by the following dependences:

$$T(z, 0) = T_1 - \gamma_0 z; \quad p(z, 0) = p_1 (1 - \gamma_0 z/T)^{g/R\gamma_0}$$

$$\delta(z, 0) = 0; \quad q(z, 0) = 0.622 f_1 E(z, 0)/p,$$

where  $\gamma_0$  is the vertical temperature gradient,  $g$  is the acceleration due to gravity, and  $R$  is the universal gas constant.

The boundary conditions at the ground ( $z = 0$ ) were given by the following equations:

$$\delta(0, t) = s(0, t) - q(0, t);$$

$$T(0, t) = T_2 + (T_1 - T_2) \exp(-t/t_k);$$

$$s(0, t) = f_1 q_m(T_1, p_1) [r_2 + (1 - r_2) \exp(-t/t_k)],$$

( $f_1$  is the relative humidity and  $r_2$  is a parameter), and at  $z = H$  by

$$\partial T / \partial z = 0; \quad \partial s / \partial z = 0; \quad \delta = 0.$$

The air pressure at all altitudes of interest as well as at the ground level ( $z = 0$ ) was assumed to be constant in time:

$$p(z, 0) = p(z, t).$$

Thus, the dynamics of the air density at different heights was assumed to be determined only by changes in the temperature  $T$  and the specific humidity  $q$

$$\rho(z, t) = \frac{p(z, 0)}{R \cdot T(z, t) \cdot (1 + 0.608 q(z, t))}. \quad (6)$$

The size distribution of the cloud droplets was described by the  $\Gamma$ -distribution:

$$f(r) = \frac{1}{\Gamma(\alpha)r_0^\alpha} r^{\alpha-1} \exp(-r/r_0). \tag{7}$$

from which it follows that the volume concentration is given by

$$N = \frac{3\delta\rho\Gamma(\alpha)}{4\pi\rho_1 r_0^3 \Gamma(\alpha+3)}, \tag{8}$$

where  $\rho$  and  $\rho_1$  are the density of air and water, respectively;  $\Gamma(\alpha)$  is the  $\Gamma$ -function; and,  $r_0$  is the parameter of the  $\Gamma$ -distribution;

The expression for the averaged aerosol extinction coefficient  $K_\lambda(\mu^{-1})$  (Ref. 3) can then be represented in the form

$$K_\lambda = \frac{3\delta\rho}{4\rho_1 r_0^3 \Gamma(\alpha+3)} \int_0^\infty K(\mu) \left(\frac{\mu}{\mu_0}\right)^{\alpha+1} \exp\left[-\frac{\mu}{\mu_0}\right] d\left(\frac{\mu}{\mu_0}\right), \tag{9}$$

where  $\mu = 2\pi r/\lambda$  is the dimensionless Mie parameter.

To facilitate the calculations, Eq. 9 was represented as the sum of a constant and a variable component, thus:  $K(\mu) = K_m + K$ ,  $K_m = 2$ .

$$K_\lambda = K_m \frac{3\delta\Gamma(\alpha+2)\rho}{4r_0^3 \Gamma(\alpha+3)\rho_1} + \Delta K_\lambda; \tag{10}$$

$$\Delta K_\lambda = \frac{3\delta\beta}{4r_0^3 \Gamma(\alpha+3)\rho_1} \int_0^\infty \tilde{K} \left(\frac{\mu}{\mu_0}\right)^{\alpha+1} \exp\left[-\frac{\mu}{\mu_0}\right] d\left(\frac{\mu}{\mu_0}\right). \tag{11}$$

The integral was calculated by the Simpson method. The upper integration limit was assumed to be  $r_{\max} = 2r_0(\lambda + 1)$ . Further increase of  $r_{\max}$  yielded only a 1–1.5% growth of the integral. Calculations of the equivalent temperature  $\Pi$  and specific moisture content  $s$  were made using a difference scheme based on the scalar-run iteration-interpolation method<sup>4</sup>. The results obtained for the temperature and the water content show good agreement with the results obtained in Ref. 2. The values of  $K(\mu)$  were taken from a look-up table with an increment of  $\Delta\mu = 0.01$ . The computation time for one version (at  $\lambda = 10.6 \mu\text{m}$ ) was 2–3 minutes on the BESM-6 computer.

In the implemented dynamic model of cloud genesis the varying parameters characterizing the moving cyclone were the turbulence coefficient  $k$ , the maximum value of the convective speed  $\omega_m$ , the time constant  $t_k$ , on the order of a day in value (the relaxation time of the temperature and moisture content at ground level, after which their values have relaxed from  $T_1$  and  $q_1$  to  $T_k = (T_2 - T_1)/e + T_2$  and  $q_k = (q_1 - q_2)/e + q_2$ ), and the height  $H$  of the layer taking part in the convective motion. The calculations were made for two versions of the initial parameters. In the first version taken from Ref. 2 the

formation of thick clouds was simulated using the following initial parameters:  $H = 11 \text{ km}$ ,  $K = 5 \text{ m}^2/\text{s}$ ,  $\omega_m = 0.025 \text{ m/s}$ ,  $h = 0.7$ ,  $r_2 = 0.9$ , and  $t_k = 24 \text{ h}$ . The initial data ( $H = 1.9 \text{ km}$ ,  $K = 0.9 \text{ m}^2/\text{s}$ ,  $\omega_m = 0.01 \text{ m/s}$ ,  $f_1 = 0.75$ , and  $r_2 = 0.9$ ) in the second version were chosen so that the obtained clouds were mean-statistical *St–Sc* clouds of the lower level.<sup>7</sup>

For given liquid water content  $\delta$  and known  $r_0$  and  $\alpha$  the system of equations is closed. Following the results of Ref. 5, it was assumed that  $r_0$  is constant while  $\alpha$  varies from  $\alpha_1 = 3$  at the lower boundary of the cloud to  $\alpha_u = 7$  at the upper boundary.

This variation of parameters characterizes the increase of the mean radius of the aerosol particles from the lower to the upper boundary of the cloud layer. Such a height dependence of the size distribution was maintained at each time step in the first version. In the second version  $\alpha_u$  was assigned by the following linearly increasing function of time:

$$\alpha_u(t) = \alpha_1 + \frac{\alpha_u - \alpha_1}{t_k - t_1}(t - t_k),$$

where  $t_1$  is the time at which cloud formation starts inside the cloud the parameter  $\alpha$  varies linearly from  $\alpha_1$  to  $\alpha_u(t)$ .

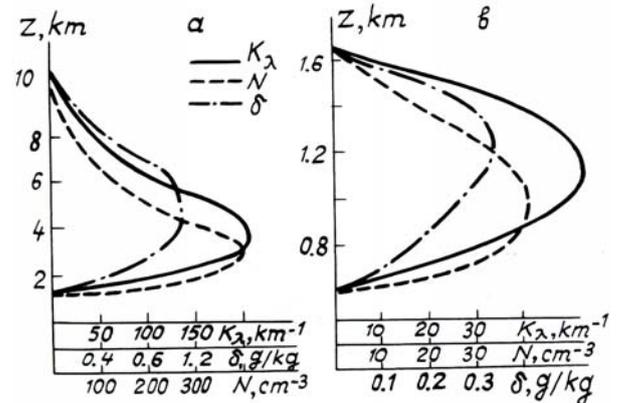


FIG. 1.  $K_\lambda$ ,  $N$ , and  $\delta$  as functions of height 24 hours after the beginning of the simulation: a) variant I; b) variant II.

Figure 1 depicts the dynamics of the growth of the extinction coefficient  $K_\lambda$  and the volume concentration  $N$  of water droplet aerosol during cloud formation, calculated according to the first (Fig. 1a) and second (Fig. 1b) versions.

In the first version a thick cloudiness forms which occupies the larger part of the troposphere and is significantly asymmetric with height due to the decrease of density with height. In both versions the cloud begins to form near the height of the maximum vertical wind speed after 5 hours in the first version and after 15 hours in the second one. Characteristics of the cloudiness which has developed after 24 hours are presented in Fig. 2.

We see from this figure that the maxima of  $K_\lambda$ ,  $N$ , and  $\delta$  are displaced toward the lower part of the cloud in the first version, and are located in the middle of the cloud according to the second version. In addition, in the first case the maximum of the aerosol extinction coefficient  $K_\lambda$  is 1 km below the maximum of the liquid water content  $\delta$ , and about 1 km above the maximum of the droplet concentration  $N$ . In the second case these figures are 160 m and 70 m, respectively.

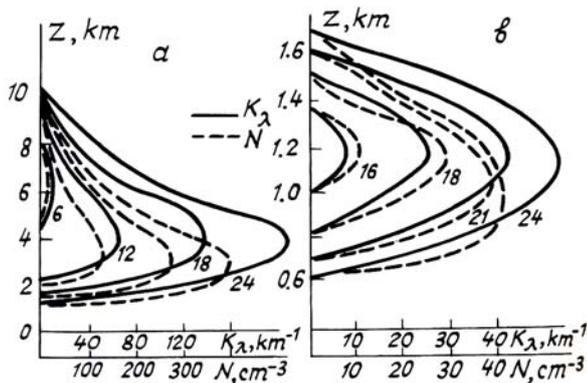


FIG. 2. Evolution of the extinction coefficient  $K_\lambda$  and volume concentration  $N$  during the development of cloudiness. The numbers labeling the curves give the time in hours from the beginning of the simulation: a) variant I; b) variant II.

After 24 hours, the maximum values of the cloud characteristics for the first (second) version are  $K_{\max} = 201 \text{ km}^{-1}$  ( $53 \text{ km}^{-1}$ ),  $N_{\max} = 400 \text{ cm}^{-3}$  ( $40 \text{ cm}^{-3}$ );  $\delta_{\max} = 1.11 \text{ g/kg}$  ( $0.34 \text{ g/kg}$ ). Average values for the entire cloud layer are given in Table I.

TABLE I

Summary table of cloudiness characteristics for the calculated variants.

Characteristics	Variant	time, h		
		12.00	18.00	24.00
$\bar{K}$ , $\text{km}^{-1}$	I	29.0	61.6	96.0
	II	—	15.3	34.0
$\delta$ , $\text{g/kg}$	I	0.20	0.40	0.63
	II	—	0.07	0.21
$\bar{N}$ , $\text{cm}^{-3}$	I	47.0	104	158
	II	—	17.7	25.3
$h_{\text{cloud}}^*$ , m	I	8000	8800	9400
	II	—	670	1100
$\tau^{**}$	I	232	542	902
	II	—	11.8	37.4

$h_{\text{cloud}}^*$  — cloud thickness,  
 $\tau^{**}$  — optical depth.

Figure 3 shows the character of the growth of the extinction coefficient  $K_\lambda$  with time at several representative altitudes. It can be seen that in the first

version the growth of  $K_\lambda$  is linear almost everywhere with the exception of a short period at the beginning of the growth.

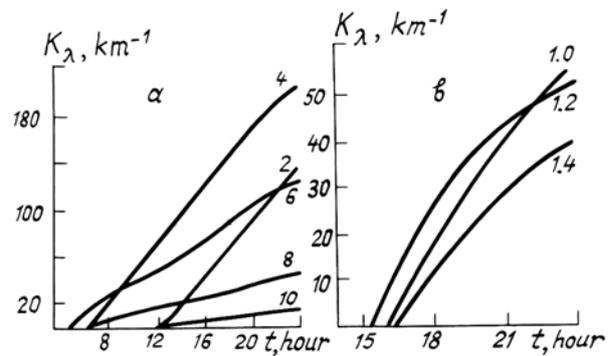


FIG. 3. Growth of the extinction coefficient at a fixed level (the numbers labeling the curves are altitude in kilometers): a) variant I; b) variant II.

It can be seen in the version of development of the mean-statistical clouds that the time course of  $K_\lambda$  deviates from linear in the final stage, which is connected with the variation of  $\alpha_{li}$  with time.

We note in conclusion that the values obtained in the first version of the calculations are much more characteristic of a thick, developed cumulonimbus cloud than the values obtained in the second version, which better describe *St-Sc* low-level clouds, as far as the characteristics under consideration are concerned.

The conclusion which we have arrived at on the linearity of the growth of  $K_\lambda$  for a given level of cloudiness may serve as a basis for forecasting the optical transmissivity of a cloud during its formation. It should be noted, however, that this conclusion has been arrived at within the framework of a simple model and, therefore, requires additional theoretical and experimental validation.

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