Formation of interference patterns in diffusely scattered fields by use of spatial filtering of the diffraction field of double-exposure quasi-Fourier and Fourier holograms. Part II

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In this paper I analyze the sensitivity of an interferometer based on the double-exposure recording by means of a positive lens of the quasi-Fourier and Fourier holograms, to the cross or longitudinal motions of a flat diffuse surface. It is shown that interference patterns are localized in the hologram plane and in the Fourier-transform plane. For their recording, a spatial filtering of the diffraction field is necessary. The experimental results agree with the theoretical preconditions.

As shown in Ref. 1 the double-exposure recording of the quasi-Fourier hologram by means of a negative lens aimed at monitoring of the cross and longitudinal motions of a flat light diffuse surface is accompanied by the formation of interference patterns, localized in the hologram plane and in the Fourier plane at the stage of the record reconstruction. Spatial filtering of the diffraction field in the corresponding planes provides an opportunity for determining the interferometer sensitivity. Thus, it depends on both sign and magnitude of the radius of curvature of a spherical wave front of a coherent radiation used for illumination of the diffuser and for the interference pattern localized in the Fourier plane when the cross motion monitoring of the diffuser is carried out. In its turn, in the case of the longitudinal motion monitoring of the diffuser, the interferometer sensitivity does not depend on sign of the radius of curvature. Besides, the dynamics of interference patterns behavior during spatial filtering of the diffraction field off the optical axis is as follows: a shift of the interference patterns takes place due to the uniform or non-uniform displacement of the subjective speckles corresponding to the second exposure in the plane of the quasi-Fourier hologram.

For the double-exposure Fourier hologram, when diffuse surface is moved across the optical axis before the photographic plate re-exposure, the absence of the uniform displacement of subjective speckles corresponding to the second exposure in the hologram plane is typical as well as the localization of the interference pattern in it. In its turn, in the case of monitoring the longitudinal motion of the diffuser, a spatial filtering of the diffraction field is necessary for recording the interference pattern localized in the hologram plane owing to the non-uniform displacement of subjective speckles corresponding to the second exposure.

In the present paper, I analyze conditions and features of the interference patterns formation at the double-exposure recording by means of a positive lens of the quasi-Fourier and Fourier holograms in order to determine the interferometer sensitivity to the cross and longitudinal motions of a flat diffuse surface.

According to Fig. 1, a matte screen *1* in the plane (x_1, y_1), is illuminated by a coherent radiation of a diverging spherical wave with the radius of curvature R. Diffusely scattered radiation after the passage of a thin positive lens L with a focal length f is recorded with an off-axis reference wave on a photographic plate *2*. This photographic plate is placed in the plane (x_3, y_3) during the first exposure. The axis of an angular narrow beam makes an angle θ with the normal to a plane of the photographic plate. Both the radius of curvature, r, and sign of the spherical reference wave will be set below by the parameters f, l_1, l_2, where l_1 is the distance between the matte screen and the principal plane (x_2, y_2) of the lens L; l_2 is the distance between the planes (x_2, y_2), (x_3, y_3). Before the re-exposure, in case of the longitudinal displacement monitoring, the matte screen is moved by the distance a in the plane of its location, for example, along the direction of the x-axis.

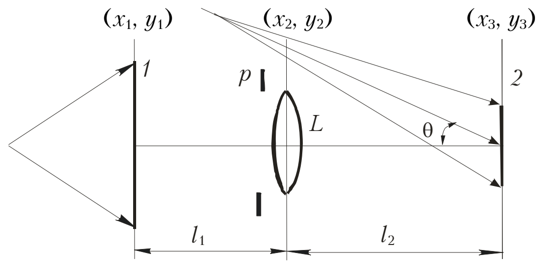

Fig. 1. Diagram of the double-exposure recording of the quasi-Fourier hologram: *1* is the matte screen; *2* is the photographic plate; L is the positive lens; p is the aperture diaphragm.

Let us use the Fresnel approximation omitting the constant factors in determining the distribution function of the complex field amplitude $u_1(x_3, y_3)$, in the subjective channel, over the plane of the

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photographic plate corresponding to the first exposure. Then, with regard for the angular limitedness of the diffraction field we have

$$u_1(x_3,y_3) \sim \iiint\int_{-\infty}^{\infty} t(x_1,y_1)\exp\left[\frac{ik}{2R}\left(x_1^2+y_1^2\right)\right]\times$$
$$\times\exp\left\{\frac{ik}{2l_1}\left[(x_1-x_2)^2+(y_1-y_2)^2\right]\right\}\times$$
$$\times p(x_2,y_2)\exp\left[-\frac{ik}{2f}\left(x_2^2+y_2^2\right)\right]\times$$
$$\times\exp\left\{\frac{ik}{2l_2}\left[(x_2-x_3)^2+(y_2-y_3)^2\right]\right\}dx_1dy_1dx_2dy_2, \quad (1)$$

where k is the wave number; $t(x_1, y_1)$ is the complex amplitude of the matte screen transmission, being a random function of coordinates; $p(x_2, y_2)$ is the pupil's function[2] of a positive lens L.

Expression (1) due to the Fourier transform takes the following form:

$$u_1(x_3,y_3) \sim \exp\left[\frac{ik}{2l_2}\left(x_3^2+y_3^2\right)\right]\left\{\exp\left[-\frac{ikL_p}{2l_2^2}\left(x_3^2+y_3^2\right)\right]\times\right.$$
$$\left.\times\left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_p^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\right\}\otimes P(x_3,y_3)\right\}, \quad (2)$$

where $\otimes$ is the symbol of convolution operator; L_p is the geometrical parameter of an optical system in the subjective channel satisfying the condition $1/L_p = 1/l_1 - 1/f + 1/l_2 > 0$, that is $f > l_1l_2/(l_1+l_2)$. Thus, $L_p < \infty$, as the condition $L_p = \infty$ corresponds to the real image formation of the diffuser in a plane of the photographic plate.[3] Also $1/l = 1/R + 1/l_1 - L_p/l_1^2$ is the symbol introduced for the sake of abbreviating the notation; $F(x_3, y_3)$ is the Fourier-image of the function $t(x_1, y_1)$ with the spatial frequencies $L_px_3/\lambda l_1l_2$, $L_py_3/\lambda l_1l_2$, λ is the wavelength of a coherent radiation used for the hologram recording and reconstruction; $P(x_3, y_3)$ is the Fourier mage of the function $p(x_2, y_2)$ with the spatial frequencies $x_3/\lambda l_2$, $y_3/\lambda l_2$.

If in the limits of the domain of the function $P(x_3, y_3)$ existence, Ref. 4, the phase change of the diverging spherical wave with the radius of curvature l_2^2/L_p does not exceed π, this condition will hold for the photographic plate area with the diameter[5] $D \le d\frac{l_2}{L_p} = d\left(1+\frac{l_2}{l_1}-\frac{l_2}{f}\right)$, where d is the diameter of a positive lens L (see Fig. 1). Therefore, the distribution of the complex field amplitude in the above-stated plane (x_3, y_3) is determined by the expression

$$u_1(x_3,y_3) \sim \exp\left[\frac{ik}{2r}\left(x_3^2+y_3^2\right)\right]\times$$
$$\times\left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_p^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes P(x_3,y_3)\right\}, \quad (3)$$

where $r = l_2 + fl_1/(f - l_1)$ is the radius of curvature of the phase distribution of a spherical wave. Thus, $r > 0$, if $l_2 > fl_1/(l_1 - f)$; $r < 0$, if $l_2 > fl_1/(f - l_1)$. If $l_1 = f$, then $r = \infty$.[6]

As follows from Eq. (3)

$$\exp\left[-\frac{iklL_p^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right] \ne \delta(x_3,y_3),$$

where $\delta(x_3, y_3)$ is the Dirac delta function. The quasi-Fourier image of the function $t(x_1, y_1)$ is formed in the plane (x_3, y_3), within the area of diameter D; every point is widened to the size of a subjective speckle, determined by the width of the function $P(x_3, y_3)$, if the diameter D_0 of the illuminated area of the matte screen *1* (see Fig. 1) satisfies the condition[5] $D_0 \ge d\left(1+\frac{l_1}{l_2}-\frac{l_1}{f}\right)$. It is necessary for the angular limitedness of the diffuse field confined within the pupil of the positive lens L. Besides, the phase distribution of a spherical wave with the radius of curvature r and $r = \infty$ at $l_1 = f$ is superposed on the subjective speckle-field.

Since the general expression (3) includes a particular case ($l_2 = f$), which bears specific features in the formation of the interference patterns, characterizing the cross or longitudinal motions of a flat diffuse surface, then below in parallel with the general case, I shall carry out analysis of the formation of the interference patterns corresponding to this particular case. For this case, distribution of the complex field amplitude, corresponding to the first exposure in the plane of the photographic plate within the diameter $\tilde{D} \le df/l_1$ at $\tilde{D}_0 \ge d$, takes the following form

$$\tilde{u}_1(x_3,y_3) \sim \exp\left[\frac{ik}{2\tilde{r}}\left(x_3^2+y_3^2\right)\right]\times$$
$$\times\left\{\tilde{F}(x_3,y_3)\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_3^2+y_3^2\right)\right]\otimes\tilde{P}(x_3,y_3)\right\}, \quad (4)$$

where $\tilde{r} = f^2/(f - l_1)$ is the radius of curvature of the phase distribution of a spherical wave; $\tilde{F}(x_3,y_3)$, $\tilde{P}(x_3,y_3)$ are the Fourier images of the functions $t(x_1, y_1)$ and $p(x_2, y_2)$ with the spatial frequencies $x_3/\lambda l_2$, $y_3/\lambda l_2$.

According to the expression (4), at $R \ne \infty$, the quasi-Fourier image of the function $t(x_1, y_1)$ is formed in the plane of the photographic plate, within the area of diameter $\tilde{D}$. Thus, the phase distribution of the diverging spherical wave is superposed on the subjective speckle-field, if $f > l_1$, or of the converging spherical wave, if $f < l_1$, and $\tilde{r} = \infty$ ($l_1 = f$).[7]

The distribution of the complex field amplitude, corresponding to the second exposure in the subjective channel, in a plane of the photographic plate, is written as follows

$$u_2(x_3,y_3) \sim \iiiint_{-\infty}^{\infty} t(x_1+a,y_1)\exp\left[\frac{ik}{2R}\left(x_1^2+y_1^2\right)\right]\times$$
$$\times\exp\left\{\frac{ik}{2l_1}\left[(x_1-x_2)^2+(y_1-y_2)^2\right]\right\}\times$$
$$\times p(x_2,y_2)\exp\left[-\frac{ik}{2f}\left(x_2^2+y_2^2\right)\right]\times$$
$$\times\exp\left\{\frac{ik}{2l_2}\left[(x_2-x_3)^2+(y_2-y_3)^2\right]\right\}\mathrm{d}x_1\mathrm{d}y_1\mathrm{d}x_2\mathrm{d}y_2, \quad (5)$$

which takes the following form:

$$u_2(x_3,y_3) \sim \exp\left[\frac{ik}{2r}\left(x_3^2+y_3^2\right)\right]\exp\left(\frac{ikL_\mathrm{p}ax_3}{l_1l_2}\right)\times$$
$$\times\left\{F(x_3,y_3)\otimes\exp\left(-\frac{ikL_\mathrm{p}ax_3}{l_1l_2}\right)\times\right.$$
$$\left.\times\left\{\exp\left[-\frac{ikL_\mathrm{p}^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes P(x_3,y_3)\right\}\right\}. \quad (6)$$

Since

$$\exp\left[-\frac{iklL_\mathrm{p}^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes\exp\left[\frac{iklL_\mathrm{p}^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]=\delta(x_3,y_3),$$

due to the integral representation of the convolution operation, one obtains the proof of the following identity, as in Ref. 1:

$$\exp\left[\frac{iklL_\mathrm{p}^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes\exp\left(\frac{-ikL_\mathrm{p}ax_3}{l_1l_2}\right)\times$$
$$\times\left\{\exp\left[\frac{-iklL_\mathrm{p}^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes P(x_3,y_3)\right\}=$$
$$=\exp\left(\frac{-ika^2}{2l}\right)\exp\left(\frac{-ikL_\mathrm{p}ax_3}{l_1l_2}\right)P\left(x_3+\frac{l_1l_2}{lL_\mathrm{p}}a,y_3\right).$$

Therefore, taking into account this condition, the distribution of the complex field amplitude, corresponding to the second exposure, in the subjective channel in the plane of the photographic plate is determined by the expression

$$u_2(x_3,y_3) \sim \exp\left[\frac{ik}{2r}\left(x_3^2+y_3^2\right)\right]\exp\left(-\frac{ika^2}{2l}\right)\exp\left(\frac{ikL_\mathrm{p}ax_3}{l_1l_2}\right)\times$$
$$\times\left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_\mathrm{p}^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes\right.$$
$$\left.\otimes\exp\left(-\frac{ikL_\mathrm{p}ax_3}{l_1l_2}\right)P\left(x_3+\frac{l_1l_2}{lL_\mathrm{p}}a,y_3\right)\right\}. \quad (7)$$

As follows from Eq. (7), in contrast to the distribution of the complex field amplitude that corresponds to the first exposure, here the subjective speckles are displaced by $al_1l_2/(lL_\mathrm{p})$, and tilted by the angle $aL_\mathrm{p}/(l_1l_2)$.

If $l_2 = f$, then

$$\tilde{u}_2(x_3,y_3) \sim \exp\left[\frac{ik}{2\tilde{r}}\left(x_3^2+y_3^2\right)\right]\exp\left(\frac{ikax_3}{f}\right)\times$$
$$\times\exp\left(-\frac{ika^2}{2R}\right)\left\{\tilde{F}(x_3,y_3)\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_3^2+y_3^2\right)\right]\otimes\right.$$
$$\left.\otimes\exp\left(-\frac{ikax_3}{f}\right)\tilde{P}\left(x_3+\frac{f}{R}a,y_3\right)\right\} \quad (8)$$

and for the subjective speckles, corresponding to the second exposure, the displacement by af/R and tilt by the angle a/f occurs.

Under conditions of double-exposure recording of the quasi-Fourier hologram, within the linear section of the photographic material blackening curve, using a spherical reference wave with the radius of curvature r, the distribution of the complex amplitude of its transmission, corresponding to the (-1)st diffraction order, takes the form

$$\tau(x_3,y_3) \sim \exp(-ikx_3\sin\theta)\left\{F(x_3,y_3)\otimes\right.$$
$$\otimes\exp\left[-\frac{iklL_\mathrm{p}^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes P(x_3,y_3)+\exp\left(-\frac{ika^2}{2l}\right)\times$$
$$\times\exp\left(\frac{ikL_\mathrm{p}ax_3}{l_1l_2}\right)\left\{F(x_3,y_3)\otimes\exp\left[-\frac{iklL_\mathrm{p}^2}{2l_1^2l_2^2}\left(x_3^2+y_3^2\right)\right]\otimes\right.$$
$$\left.\left.\otimes\exp\left(-\frac{ikL_\mathrm{p}ax_3}{l_1l_2}\right)P\left(x_3+\frac{l_1l_2}{lL_\mathrm{p}}a,y_3\right)\right\}\right\}, \quad (9)$$

and when $l_2 = f$ and the radius of curvature of a spherical reference wave equals $\tilde{r}$, one obtains

$$\tilde{\tau}(x_3,y_3) \sim \exp(-ikx_3\sin\theta)\left\{\tilde{F}(x_3,y_3)\otimes\right.$$
$$\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_3^2+y_3^2\right)\right]\otimes\tilde{P}(x_3,y_3)+\exp\left(-\frac{ika^2}{2R}\right)\times$$
$$\times\exp\left(\frac{ikax_3}{f}\right)\left\{\tilde{F}(x_3,y_3)\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_3^2+y_3^2\right)\right]\otimes\right.$$
$$\left.\left.\otimes\exp\left(-\frac{ikax_3}{f}\right)\tilde{P}\left(x_3+\frac{f}{R}a,y_3\right)\right\}\right\}. \quad (10)$$

As in Ref. 1, assume that at the stage of the hologram reconstruction in its plane on the optical axis, there is applied a spatial filtering of the diffraction field by means of a round aperture in an opaque screen p_0 (Ref. 1, Fig. 2). Thus, within the limits of the filtering aperture diameter, the phase change $kL_p ax_3/l_1l_2$ does not exceed π. Therefore, distribution of illumination over the focal plane (x_4, y_4) (Ref. 1, Fig. 2) is described by the expression

$$I(x_4,y_4) \sim \left[1+\cos\left(\frac{kl_1}{lL_p}ax_4+\frac{ka^2}{2l}\right)\right]\times$$

$$\times\left|p(x_4,y_4)t\left(-\frac{l_1}{L_p}x_4,-\frac{l_1}{L_p}y_4\right)\times\right.$$

$$\left.\times\exp\left[\frac{ikl_1^2}{2lL_p^2}(x_4^2+y_4^2)\right]\otimes P_0(x_4,y_4)\right|^2, \qquad (11)$$

where $P_0(x_4, y_4)$ is the Fourier image of the transmission function $p_0(x_3, y_3)$ of a spatial filter[8] with the spatial frequencies $x_4/\lambda l_2$, $y_4/\lambda l_2$. Moreover, here and further when deriving the expressions, for the sake of brevity notation, I shall use the assumptions, formulated in Ref. 1.

It follows from expression (11) that in the plane of the diffuser image formation, limited by size of the positive lens L pupil (see Fig. 1), the subjective speckle-structure with the size of speckle, determined by the function width $P_0(x_4, y_4)$, is modulated by the interference fringes that periodically change along the x-axis. Measurement of the period of the interference fringes at known values of λ, l_1, l, and L_p provides a possibility of determining the transverse motion of a flat diffuse surface.

If $l_2 = f$, in carrying out a spatial filtering of the diffraction field on the optical axis in the hologram plane within the limits of the filtering aperture diameter, the phase change kax_3/f does not exceed π, the distribution of illumination over the focal plane (x_4, y_4) (Ref. 1, Fig. 2) takes the following form

$$\tilde{I}(x_4,y_4) \sim \left[1+\cos\left(\frac{kax_4}{R}+\frac{ka^2}{2R}\right)\right]\times$$

$$\times\left|p(x_4,y_4)t(-x_4,-y_4)\exp\left[\frac{ik}{2R}(x_4^2+y_4^2)\right]\otimes \tilde{P}_0(x_4,y_4)\right|^2, (12)$$

where $\tilde{P}_0(x_4,y_4)$ is the Fourier image of the transmission function $p_0(x_3, y_3)$ of a spatial filter with the spatial frequencies $x_4/\lambda f$ and $y_4/\lambda f$.

According to Eq. (12), in the considered particular case, a period of the interference fringes modulating the subjective speckle-structure with the size of speckle determined by the function width $\tilde{P}_0(x_4,y_4)$, depends only on the values of λ, a, and R.

Let, as in Ref. 1, at the stage of the double-exposure quasi-Fourier hologram reconstruction, spatial filtering of the diffraction field is being carried out on the optical axis in the frequency plane (x_4, y_4) of the optical system such as the Kepler telescope (Ref. 1, Fig. 3), by means of which the hologram image is formed. Thus, assume that in the limits of the filtering aperture diameter, the phase change kl_1ax_4/lL_p does not exceed π. Hence, distribution of illumination over the plane (x_5, y_5) of the hologram image formation is determined by the expression

$$I(x_5,y_5) \sim \left[1+\cos\left(\frac{-kL_p ax_5}{l_1l_2}+\frac{ka^2}{2l}\right)\right]\times$$

$$\times\left|F(-x_5,-y_5)\otimes\exp\left[-\frac{iklL_p^2}{2l_1^2l_2^2}(x_5^2+y_5^2)\right]\otimes P_0(x_5,y_5)\right|^2, \quad (13)$$

where $P_0(x_5, y_5)$ is the Fourier image of the transmission function $p_0(x_4, y_4)$ of a spatial filter with the spatial frequencies $x_5/\lambda l_2$ and $y_5/\lambda l_2$.

As follows from Eq. (13), in the plane (x_5, y_5), the interference pattern is formed as the periodically alternating interference fringes along the x-axis, modulating the subjective speckle-structure with the size of speckle determined by the function width $P_0(x_5, y_5)$. Thus, as in Ref. 1, the frequency of interference fringes in the limits of the quasi-Fourier image does not depend on the radius of curvature of a spherical wave of a coherent radiation used for illumination of the diffuser at the stage of the hologram recording. Besides, unlike Ref. 1, at $l_1 = f$, when $D_0 \geq df/l_2$, $D \leq d$, the period of the interference fringes equidistantly located in the plane (x_5, y_5) is determined only by the values of λ, a, and f.

If $l_2 = f$, in carrying out the spatial filtering of the diffraction field in the plane (x_4, y_4) with the filtering aperture diameter, being within the limits where the phase change kax_4/R does not exceed π, the distribution of illumination over the plane of the hologram image formation, takes the following form

$$\tilde{I}(x_5,y_5) \sim \left[1+\cos\left(\frac{-kax_5}{f}+\frac{ka^2}{2R}\right)\right]\times$$

$$\times\left|\tilde{F}(-x_5,-y_5)\otimes\exp\left[-\frac{ikR}{2f^2}(x_5^2+y_5^2)\right]\otimes \tilde{P}_0(x_5,y_5)\right|^2, \quad (14)$$

where $\tilde{P}_0(x_5,y_5)$ is the Fourier image of the transmission function $p_0(x_4, y_4)$ of a spatial filter with the spatial frequencies $x_5/\lambda f$ and $y_5/\lambda f$.

According to Eq. (14), in the considered particular case, the frequency of interference fringes, modulating the subjective speckle-structure with the size of speckle determined by the function width $\tilde{P}_0(x_5,y_5)$, depends only on the values of λ, a, and f.

Comparison of the expressions (11) and (13) shows that for the interference pattern, localized in the plane of the diffuser image formation, the interferometer sensitivity to its cross motion changes by $G_1 = [l_2(l_1 - L_p)/L_p^2] + l_1^2 l_2/(L_p^2 R)$ times. Moreover, if $l_1 < L_p$ ($l_2 > f$), then for R decreasing in the limits of $l_1^2/(L_p - l_1) \leq R \leq \infty$, the value of G_1 reduces due to the reduction of the displacement in the hologram plane of subjective speckles, corresponding to the second exposure, compared to the speckles of the first exposure. Thus, $G_1 = 0$ $[R = l_1^2/(L_p - l_1)]$ corresponds to the condition of the Fourier-image formation of the function $t(x_1, y_1)$ in the hologram plane and to the absence of the speckle displacement.[5] In this case, the "frozen" interference pattern is localized in the hologram plane and its view does not change when changing the observation angle. The further reduction of R leads to an increase in the displacement of speckles of the second exposure and, consequently, to the increase in the interferometer sensitivity to the cross motion of the diffuser. As an example, the dependence of G_1 on R of a diverging spherical wave is presented in Fig. 2 for the fixed values of $l_1 = 220$ mm, $f = 220$ mm, $l_2 = 300$ mm, which have been used in the experiment.

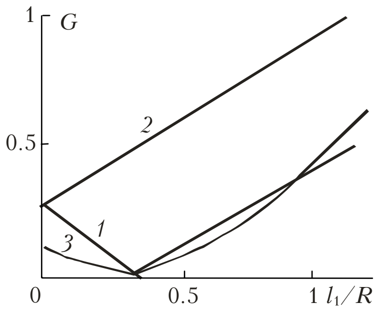


Fig. 2. Dependences of the interferometer sensitivity factors at the fixed values of $f = 220$ mm, $l_1 = 220$ mm, $l_2 = 300$ mm: G_1 (*1*), G_2 (*2*), G_3 (*3*).

At illumination of the matte screen *1* (see Fig. 1) with a coherent radiation of a diverging spherical wave, for the case of interference pattern localized in a plane of the diffuser image formation, the interferometer sensitivity to its cross motion changes by $G_2 = [l_2(l_1 - L_p)/L_p^2] - l_1^2 l_2/(L_p^2 R)$ times. Thus, it increases when reducing R (see Fig. 2) due to the motion increase in the plane of subjective speckles' hologram, corresponding to the second exposure.

When $l_1 > L_p$ ($l_2 < f$) and the matte screen *1* (see Fig. 1) is illuminated with a coherent radiation of diverging spherical wave, for the case of interference pattern, localized in the plane of the diffuser image formation, the interferometer sensitivity to its cross motion changes by $G'_2 = [l_2(l_1 - L_p)/L_p^2] - l_1^2 l_2/(L_p^2 R)$ times. Thus, it increases with the reduction of the R due to the motion increase in the plane of subjective speckles' hologram, corresponding to the second exposure. As an example, the dependence of G'_1 on R of the divergent spherical wave, is presented in Fig. 3 for the fixed values of $l_1 = 220$ mm, $f = 220$ mm, $l_2 = 180$ mm, which were used in the experiment.

At illumination of the matte screen *1* (see Fig. 1) with a coherent radiation of a converging spherical wave, the interferometer sensitivity to the cross motion of the diffuser changes by $G'_2 = [l_2(l_1 - L_p)/L_p^2] - l_1^2 l_2/(L_p^2 R)$ times. Thus, with the reduction of R in the limits of $[l_2(l_1 - L_p)]/L_p^2 \le R \le \infty$, the value of G'_2 reduces due to the reduction of displacement of the subjective speckles corresponding to the second exposure in the hologram plane, compared to the speckles of the first exposure. In addition, $G'_2 = 0$ $[R = l_1^2/(l_1 - L_p)]$ corresponds to the condition of the Fourier-image hologram formation in the hologram plane of the function $t(x_1, y_1)$ and to the absence of speckle displacement.[9] This case is similar to that presented in Ref. 1, where the Fourier hologram recording is possible only at illumination of the diffuser with a coherent radiation of a converging spherical wave. The further reduction of R leads to an increase in the displacement of the subjective speckles, corresponding to the second exposure in the hologram plane, and, consequently, to the increase in the interferometer sensitivity (see Fig. 3) to the cross motion of the diffuser.

If $l_2 = f$, then at $R = \infty$ in the hologram plane, the Fourier image is formed of $t(x_1, y_1)$ and we can observe the "frozen" interference fringes in it. The recurrence period of the interference fringes is determined by the values of λ, a, and f.

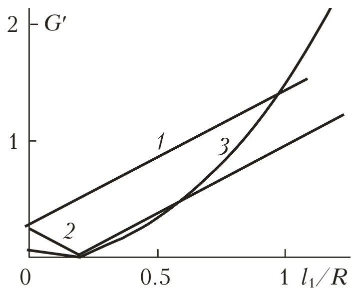


Fig. 3. Dependences of the interferometer sensitivity coefficients at the fixed values of $f = 220$ mm, $l_1 = 220$ mm, $l_2 = 180$ mm: G'_1 (*1*), G'_2 (*2*), G'_3 (*3*).

As follows from the above-stated analysis of the interference pattern formation, with a positive lens, characterizing the cross motion of a flat diffuse surface the double-exposure recording of the quasi-Fourier hologram is carried out, they are localized in two planes, as shown in Ref. 1: in the hologram plane and in the far-field diffraction region, where the diffuser image is formed. Similar explanation consists in that, on the one hand, there is a uniform displacement of subjective speckles, corresponding to the second exposure in the hologram plane, with respect to the speckles of the first exposure. Then, at the stage of the double-exposure hologram reconstruction when carrying out a spatial filtering of the diffraction field, full overlap of identical speckles of the two exposures is provided in the far-field diffraction region. On the other hand, the tilt angle of subjective speckles corresponding to the second exposure in the hologram plane, relative to the speckles of the first exposure, causes the interference pattern localization in it when carrying out a spatial filtering of the diffraction field in the Fourier plane.

Let us now, before the photographic plate *2* re-exposure (see Fig. 1), a matte screen *1* be displaced along the z-axis by the distance $\Delta l \ll l_1$, R. Then in the approximation used, the complex amplitude distribution of the double-exposure hologram transmission, corresponding to the (−1)st diffraction order, takes the following form

$$\tau'(x_3,y_3) \sim \exp(-ikx_3 \sin\theta)\Big\{F(x_3,y_3)\otimes \otimes\exp\left[-\frac{iklL_p^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]\otimes P(x_3,y_3)+ +\exp(ik\Delta l)\exp\left[-\frac{ik\Delta l L_p^2}{2l_1^2l_2^2}(x_3^2+y_3^2)\right]\Big\{F'(x_3,y_3)\otimes \otimes\exp\left[-\frac{ikl'L_p'^2}{2(l_1+\Delta l)^2 l_2^2}(x_3^2+y_3^2)\right]\otimes P(x_3,y_3)\Big\}\Big\}, \quad (15)$$

where

$$L_p' = L_p\left(1+\frac{L_p\Delta l}{l_1^2}\right),\quad \frac{1}{l'} = \frac{1}{R-\Delta l}+\frac{1}{l_1+\Delta l}-\frac{L_p'}{(l_1+\Delta l)^2}$$

are the introduced symbols for the abbreviated notation; $F'(x_3, y_3)$ is the Fourier image of the function $t(x_1, y_1)$ with the spatial frequencies $\frac{L_p' x_3}{\lambda(l_1+\Delta l)l_2}$ and $\frac{L_p' y_3}{\lambda(l_1+\Delta l)l_2}$.

As follows from Eq. (15), the subjective speckles, corresponding to the second exposure, are displaced in the hologram plane along the radius from the optical axis, relative to the speckles of the first exposure due to the difference in scales of the Fourier images of the function $t(x_1, y_1)$. Moreover, the change along the radius from the optical axis of their tilt angle causes break of the correlation of the subjective speckle-structures of the two exposures. Besides, the factor $\exp[-ikL_p^2\Delta l(x_3^2+y_3^2)/(2l_1^2l_2^2)]$ points to the tilt angle of the subjective speckles corresponding to the second exposure relative to the speckles of the first exposure in the hologram plane, independent of the radius of curvature of a spherical wave front of a coherent radiation, used for illumination of the diffuser, and, changing along the radius from the optical axis.

If $l_2 = f$, the distribution of the complex amplitude of the double-exposure hologram transmission, corresponding to the (−1)st diffraction order, is determined by the expression

$$\tilde\tau'(x_3,y_3) \sim \exp(-ikx_3 \sin\theta)\Big\{\tilde F(x_3,y_3)\otimes \otimes\exp\left[-\frac{ikR}{2f^2}(x_3^2+y_3^2)\right]\otimes \tilde P(x_3,y_3)+ +\exp(ik\Delta l)\exp\left[-\frac{ik\Delta l}{2f^2}(x_3^2+y_3^2)\right]\Big\{\tilde F(x_3,y_3)\otimes \otimes\exp\left[-\frac{ik(R-\Delta l)}{2f^2}(x_3^2+y_3^2)\right]\otimes \tilde P(x_3,y_3)\Big\}\Big\}. \quad (16)$$

According to Eq. (16), in the considered particular case, there is a typical feature, which consists in the fact that the Fourier images of the function $t(x_1, y_1)$ appear to be the same in the first and second exposures.

As in Ref. 1, we assume that at the stage of double-exposure hologram reconstruction, in its plane on the optical axis, spatial filtering of the diffraction field is performed by means of a round aperture in an opaque screen p_0 (Ref. 1, Fig. 2). Thus, within the aperture diameter, the phase change $kL_p^2\Delta l(x_3^2+y_3^2)/(2l_1^2l_2^2)$ does not exceed π. Besides, assume that the filtering aperture diameter satisfies the condition $d_f \le 2\lambda l_1^2 l_2/d(l_1-L_p)\Delta l$. Then, as in Ref. 1, we can obtain distribution of illumination over the Fourier plane (x_4, y_4) (Ref. 1, Fig. 2), which takes the form

$$I'(x_4,y_4) \sim \left\{1+\cos\left[k\Delta l+\frac{kM\Delta l}{2l_1^2}(x_4^2+y_4^2)\right]\right\}\times \times\left|p(x_4,y_4)t\left(-\frac{l_1}{L_p}x_4,-\frac{l_1}{L_p}y_4\right)\times \times\exp\left[\frac{ikl_1^2}{2lL_p^2}(x_4^2+y_4^2)\right]\otimes P_0(x_4,y_4)\right|^2, \quad (17)$$

where $M = [l_1^4 - R^2(L_p - l_1)^2]/(R^2L_p^2)$ is the symbol introduced for brevity.

As follows from Eq. (17), the subjective speckle-structure, in the plane of the diffuser image formation, is modulated by the fringes of equal tilt restricted by the lens L pupil (see Fig. 1), i.e., by a system of concentric interference fringes. Moreover, measurement of their radii in the adjacent orders of interference provides an opportunity of determining the longitudinal motion of the flat diffuser having known the values of λ, R, l_1, and L_p.

In the particular case of $l_2 = f$ and a spatial filtering of the diffraction field being carried out on the optical axis in the hologram plane, in the limits of the filtering aperture diameter, where the phase change $k\Delta l(x_3^2+y_3^2)/2f^2$ does not exceed π, the distribution of illumination over the plane of the diffuser image formation, is determined by the expression

$$\tilde I'(x_4,y_4) \sim \left\{1+\cos\left[k\Delta l+\frac{k\Delta l}{2R^2}(x_4^2+y_4^2)\right]\right\}\times \times\left|p(x_4,y_4)t(-x_4,-y_4)\exp\left[\frac{ik}{2R}(x_4^2+y_4^2)\right]\otimes P_0(x_4,y_4)\right|^2, \quad (18)$$

As follows from this expression, the radii of interference fringes depend only on the values of λ, Δl, and R.

If spatial filtering of the diffraction field is carried out on the optical axis in the plane of the diffuser image formation, then, as in Ref. 1, we can obtain the distribution of illumination over the plane (x_5, y_5) of the hologram image formation, by means of the collimating optical system, such as the Kepler telescope, which at $l_2 \neq f$ takes the form

$$I'(x_5, y_5) \sim \left\{1 + \cos\left[k\Delta l - \frac{kL_p^2 \Delta l}{2l_1^2 l_2^2}\left(x_5^2 + y_5^2\right)\right]\right\} \times$$

$$\times \left| F(-x_5, -y_5) \otimes \exp\left[-\frac{iklL_p^2}{2l_1^2 l_2^2}\left(x_5^2 + y_5^2\right)\right] \otimes P_0(x_5, y_5)\right|^2 . \quad (19)$$

According to Eq. (19), in the plane of the hologram image formation, the subjective speckle-structure with the speckle size determined by the function width $P_0(x_5, y_5)$, is modulated by the fringes of equal tilts, i.e., by the system of concentric fringes. Thus, their radii do not depend on the radius of curvature of a spherical wave of a coherent radiation used for the matte screen *1* illumination (see Fig. 1) at stage of the hologram recording. Besides, it should be noted that in comparison with the cross motion monitoring of the diffuser, in the case of the interference pattern recording, localized in the hologram plane, a spatial filtering of the diffraction field is also necessary for the Fourier-image formation of the function $t(x_1, y_1)$ in the hologram plane too. It takes place due to the displacement of subjective speckles in it, corresponding to the second exposure, along the radius from the optical axis. If $l_2 = f$ and within the limits of the filtering aperture diameter in the plane of the diffuser image formation, the phase change $k\Delta l(x_4^2 + y_4^2)/(2R)$ does not exceed π, therefore, distribution of the illumination over the plane of the hologram image formation, is determined by the expression

$$\tilde{I}'(x_5, y_5) \sim \left\{1 + \cos\left[k\Delta l - \frac{k\Delta l}{2f^2}\left(x_5^2 + y_5^2\right)\right]\right\} \times$$

$$\times \left| \tilde{F}(-x_5, -y_5) \otimes \exp\left[-\frac{ikR}{2f^2}\left(x_5^2 + y_5^2\right)\right] \otimes \tilde{P}_0(x_5, y_5)\right|^2 . \quad (20)$$

As follows from this expression, the radii of interference fringes depend only on the values of λ, Δl, and f. Moreover, at the Fourier-image formation of the function $t(x_1, y_1)$ in the hologram plane, which takes place in case of the matte screen *1* illumination (see Fig. 1) by a collimated beam, there is no need in spatial filtering of the diffraction field. It takes place since the "frozen" interference pattern is formed in the hologram plane as in the case of cross motion monitoring of the diffuser.

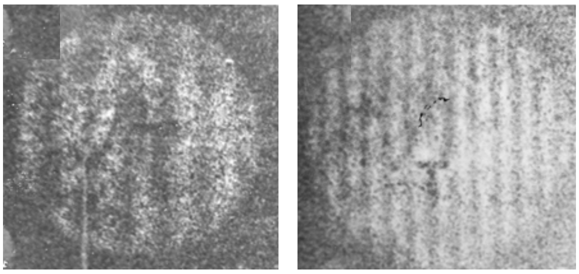
As follows from the comparison of expressions (17) and (19), for the interference pattern, localized in the plane of the diffuser image formation, the interferometer sensitivity to its longitudinal motion changes by $G_3 = Ml_2^2/L_p^2$ times. In addition, it does not depend on the sign of the radius of curvature R. It follows from the fact that at the matte screen *1* illumination (see Fig. 1) with a coherent radiation of a converging spherical wave in the above-stated analysis of the interference pattern formation, it is necessary to substitute l for the quantity, satisfying the condition $\frac{1}{l} = -\frac{1}{R} + \frac{1}{l_1} - \frac{L_p}{l_1^2}$, and l' for the quantity, satisfying the condition

$$\frac{1}{l'} = -\frac{1}{R + \Delta l} + \frac{1}{l_1 + \Delta l} - \frac{L'_p}{(l_1 + \Delta l)^2},$$

that leads to the above-mentioned value of the G_3 coefficient. Thus, $G_3 = 0$, when the Fourier image of the function $t(x_1, y_1)$ is formed in the hologram plane, at illumination of the diffuser with a coherent radiation of a diverging spherical wave, if $l_2 > f$, or of a converging spherical wave, if $l_2 < f$. The calculated dependence of the G_3 coefficient on the radius of curvature R for the above-mentioned fixed values of l_1, l_2, and f is presented in Figs. 2 and 3.

It follows from analysis of the interference patterns formation, characterizing the longitudinal motion of a flat diffuser, when a positive lens is used for the double-exposure quasi-Fourier hologram recording, these interference patterns are localized in two planes: in the hologram plane and in the far-field region of the diffraction, where the diffuser image is formed. It is explained by the fact that, on the one hand, there is a tilt angle of the subjective speckles, corresponding to the second exposure, relative to the speckles of the first exposure, changing along the radius from the optical axis in the hologram plane. As a result, the interference pattern is localized in the hologram plane if a spatial filtering of the diffraction field is carried out in the Fourier plane, which provides obtaining identical speckles of two exposures in the plane of the interference pattern recording. On the other hand, the orientation of subjective speckles in the hologram plane is such that an additional variation of the tilt angle of the subjective speckles, corresponding to the second exposure, relative to the speckles of the first exposure takes place along the radius from the optical axis. Moreover, in this case, obtaining of identical speckles of two exposures is possible in the far-field region of diffraction by use of a spatial filtering of the diffraction field in the hologram plane.

In the experiment, the double-exposure quasi-Fourier and Fourier holograms were recorded on the photographic plates of a Micrat-VRL type by means of radiation of the He–Ne laser at the wavelength of 0.63 μm. Technique of the experimental investigations consisted in comparing the hologram recorded at the fixed values of both cross ($a = (0.04 \pm 0.002)$ mm), and longitudinal ($\Delta l = (1 \pm 0.002)$ mm) shifts. Thus, a positive lens with the focal length of $f = 220$ mm and the pupil's diameter of $d = 35$ mm was used. Distance l_1 (see Fig. 1) made 220 mm, but l_2 corresponded to 300, 180, and 220 mm. For a plane-wave reference beam of 50-mm diameter, $\theta = 10°$. Different radii of curvature of the diverging or converging spherical waves of coherent radiation for illumination of the matte screen were chosen in the limits from $R = \infty$ to $|R| = 200$ mm. The diameter of the illuminated spot on the matte screen was 60 mm. The interference patterns, localized in the plane of the diffuser image formation restricted by the pupil of the positive lens are presented in Fig. 4 as an example, when $l_2 = 300$ mm, and characterizing its cross motion.

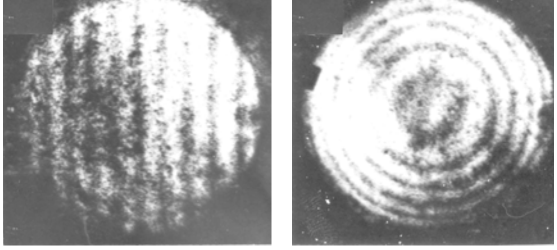


a *b*

Fig. 4. Interference patterns, localized in the plane of the diffuser image formation and characterizing its cross motion: the illumination of the diffuser with the radiation of a diverging spherical wave (*a*); of a converging spherical wave (*b*).

A mark in the form of the letter "T" was preliminary drawn on the matte screen, and the Russian letter "Л" – on the lateral surface of a positive lens L (see Fig. 1). Interference patterns were recorded while carrying out spatial filtering of the diffraction field in the hologram plane by means of its reconstruction using small-aperture (≈ 2-mm diameter) laser beam. The matte screen was illuminated with a coherent radiation of a diverging (Fig. 4*a*) and converging (Fig. 4*b*) spherical waves with $R = 200$ mm.

In these two cases, as well as in the subsequent ones, connected with the magnitude and sign variation of R, the interference patterns, localized in the hologram plane, had identical frequency of the interference fringes, corresponding to the fringe frequency presented in Fig. 5*a*.



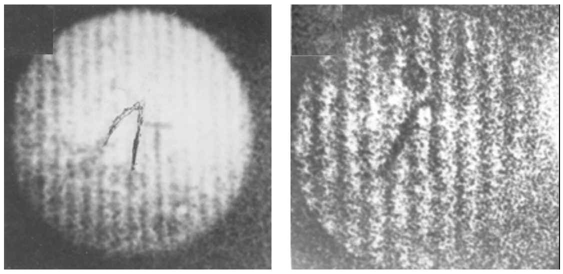
a *b*

Fig. 5. Interference patterns, localized in the hologram plane and characterizing: the cross motion of the diffuser (*a*); the longitudinal motion of the diffuser (*b*).

Recording of the interference patterns in Fig. 5 was carried out as in Refs. 1 and 7, at the hologram illumination using a collimated beam with a spatial filtering of the diffraction field in the frequency plane of the optical system like a Kepler telescope. Moreover, a spatial size of the interference pattern, localized in the hologram plane, made 35 mm that corresponds to the calculated value.

By measuring the interference fringe periods, the G_1 and G_2 coefficients were determined (in addition to that those can be determined by measuring f, l_1, l_2, and R). The values of G_1 and G_2 derived in that way correspond to Fig. 2 within the experimental error of 10%.

At $l_2 = 180$ mm, the interference patterns (Fig. 6) are localized in the plane of the diffuser image formation.



a *b*

Fig. 6. Interference patterns, localized in the plane of the diffuser image formation and characterizing its cross motion: for the illumination of the diffuser with radiation of a diverging spherical wave (*a*); for the case of illumination with a converging spherical wave (*b*).

Matte screen was illuminated with a coherent radiation of a diverging (Fig. 6*a*) or a converging (Fig. 6*b*) spherical wave with the radius of curvature $R = 200$ mm. Recording of the interference patterns in Fig. 6 was carried out, as in Fig. 4, while spatially filtering the diffraction field in the hologram plane, where the interference patterns with the frequency of interference fringes corresponding to that in Fig. 5*a* were localized. In addition, the values of the G'_1 and G'_2 coefficients derived from measured periods of interference fringes, correspond to those in Fig. 3.

The interference patterns presented in Fig. 7 are localized in the plane of the diffuser image formation within the limits of the positive lens' L pupil (see Fig. 1) and characterizing its longitudinal motion, when at the stage of the hologram recording, the matte screen *1* was illuminated with a coherent radiation of a diverging spherical wave with the radius of curvature of the wave front $R = 200$ mm.

Recording of the interference patterns (see Fig. 7) was carried out while spatially filtering the diffraction field in the hologram plane, where the interference patterns, corresponding to Fig. 5*b*, were localized. In these two cases, as well as in others, connected with the variation of both the magnitude of the radius of curvature of a spherical wave front, and its sign, the values of the G_3 and G'_3 coefficients,

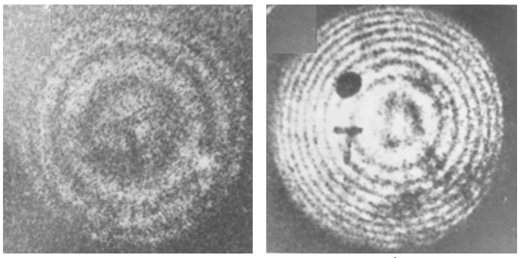


Fig. 7. Interference patterns, localized in the plane of the diffuser image formation and characterizing its longitudinal motion: for the distance $l_2 = 300$ mm (*a*); for $l_2 = 180$ mm (*b*).

determined from measured radii of the interference fringes in the adjacent orders of interference (in addition to the fact that these can be determined from measured values of f, l_1, l_2, and R), correspond to Figs. 2 and 3 within 10% error of experimental measurements.

For the double-exposure quasi-Fourier hologram at $l_2 \neq f$, as in Ref. 1, the similar behavior of the interference patterns recorded in the far zone of diffraction takes place when shifting the center of the filtering aperture in the hologram plane. Therefore, if the hologram characterizes the cross motion along the x-axis of a flat diffuser, the behavior dynamics of the recorded interference fringes is as follows. At a displacement of the filtering aperture along the x-axis, the interference pattern moves relative to a fixed image of the diffuser. Besides, the phase of interference pattern varies from 0 to π, when the center of the filtering aperture moves from the minimum of the interference pattern, localized in the hologram plane, to its maximum ("living" interference fringes). If the hologram characterizes the longitudinal motion of a flat diffuser, then the displacement of the filtering aperture center relative to the optical axis is accompanied by a non-uniform displacement of the interference fringes relative to the fixed image of the diffuser. This takes place due to the non-uniform displacement, across the optical axis, of subjective speckles, corresponding to the second exposure compared to the speckles of the first exposure, in the hologram plane. Besides, as in case of the cross motion monitoring of the diffuser, the phase of interference pattern varies from 0 to π, when the center of a filtering aperture moves from the minimum of the interference pattern, localized in the hologram plane to its maximum.

Let in the case of $l_2 = f$, the double-exposure quasi-Fourier hologram characterizes the cross motion of a flat diffuser and the center of a filtering aperture in the hologram plane has the coordinates of x_{03}, 0. Then, the distribution of the complex amplitude at the output of the spatial filter at the stage of the hologram reconstruction, takes the form

$$\tilde{u}(x_3,y_3) \sim p(x_3,y_3)\Big\{\tilde{F}(x_3+x_{03},y_3)\otimes$$

$$\otimes\exp\left\{-\frac{ikR}{2f^2}\left[(x_3+x_{03})^2+y_3^2\right]\right\}\otimes\exp\left(\frac{ikx_{03}x_3}{f}\right)\tilde{P}(x_3,y_3)+$$

$$+\tilde{F}(x_3+x_{03},y_3)\otimes\exp\left\{-\frac{ikR}{2f^2}\left[(x_3+x_{03})^2+y_3^2\right]\right\}\times$$

$$\times\exp\left(-\frac{ikax_3}{f}\right)\otimes\exp\left[-\frac{ik(x_{03}-a)x_3}{f}\right]\tilde{P}(x_3,y_3)\Big\}. \quad (21)$$

Upon the transformation, the Fourier distribution of the complex field amplitude in the Fourier plane (x_4, y_4) is determined by the expression

$$\tilde{u}(x_4,y_4) \sim \Big\{p(x_4-x_{03},y_4)t(-x_4,-y_4)\exp\left(\frac{2ikx_{03}x_3}{f}\right)\times$$

$$\times\exp\left[\frac{ik}{2R}\left(x_4^2+y_4^2\right)\right]+p(x_4-x_{03}+a,y_4)\times$$

$$\times t(-x_4,-y_4)\exp\left(\frac{2ikx_{03}x_3}{f}\right)\exp\left(\frac{ikx_{03}a}{f}\right)\times$$

$$\times\exp\left\{\frac{ik}{2R}\left[(x_4+a)^2+y_4^2\right]\right\}\Big\}\otimes\tilde{P}_0(x_4,y_4). \quad (22)$$

Because of smallness of the diffuser cross motion and of the function $\tilde{P}_0(x_4,y_4)$ width,[10] as compared with the period of the function

$$1+\exp ik\left(\frac{ax_4}{R}+\frac{a^2}{2R}+\frac{ax_{03}}{f}\right)$$

the distribution of illumination over the Fourier plane takes the following form

$$\tilde{I}(x_4,y_4) \sim \left\{1+\cos\left[\frac{ka}{R}\left(x_4+\frac{R}{f}x_{03}\right)+\frac{ka^2}{R}\right]\right\}\times$$

$$\times\Big|p(x_4-x_{03},y_4)t(-x_4,-y_4)\exp\left(\frac{i2kx_{03}x_4}{f}\right)\times$$

$$\times\exp\left[\frac{ik}{2R}\left(x_4^2+y_4^2\right)\right]\otimes\tilde{P}_0(x_4,y_4)\Big|^2. \quad (23)$$

From the comparison of expressions (12) and (23), it follows that, as in the general case, when $l_2 \neq f$, displacement of the interference patterns relative to the fixed image of a flat diffuse, takes place.

Let in the case of $l_2 = f$, the double-exposure quasi-Fourier hologram characterizes longitudinal motion of the diffuser and the center of a filtering aperture in the hologram plane, within the limits of which the phase change $\dfrac{k\Delta l(x_3^2+y_3^2)}{2f^2}$ does not exceed π, has the coordinates of x_{03}, 0. Then, the distribution of the complex field amplitude at output of a spatial filter, is determined by the expression

$$\tilde{u}'(x_3,y_3) \sim p(x_3,y_3)\Big\{\tilde{F}(x_3+x_{03},y_3)\otimes$$

$$\otimes\exp\left[-\frac{ikR}{2f^2}\left[(x_3+x_{03})^2+y_3^2\right]\right]\otimes\exp\left(\frac{ikx_{03}x_3}{f}\right)\tilde{P}(x_3,y_3)+$$

$$+\exp(ik\Delta l)\Bigg\{\tilde{F}(x_3 + x_{03}, y_3) \otimes$$

$$\otimes \exp\left\{-\frac{ik(R-\Delta l)}{2f^2}\left[(x_3+x_{03})^2 + y_3^2\right]\right\} \otimes$$

$$\otimes \exp\left(\frac{ikx_{03}x_3}{f}\right)\tilde{P}(x_3, y_3)\Bigg\}\Bigg\}. \quad (24)$$

Upon the transformations, the Fourier distribution of the complex field amplitude in the Fourier plane, takes the form

$$\tilde{u}'(x_4, y_4) \sim \Bigg\{p(x_4 - x_{03}, y_4)t(-x_4, -y_4)\exp\left(\frac{i2kx_{03}x_4}{f}\right) \times$$

$$\times \exp\left[\frac{ik}{2R}\left(x_4^2 + y_4^2\right)\right] + \exp(ik\Delta l)p(x_4 - x_{03}, y_4)t(-x_4, -y_4) \times$$

$$\times \exp\left(\frac{i2kx_{03}x_4}{f}\right)\exp\left[\frac{ik}{2(R-\Delta l)}\left(x_4^2 + y_4^2\right)\right]\Bigg\} \otimes \tilde{P}_0(x_4, y_4), \quad (25)$$

based on which, with the allowance made for the above-mentioned condition, the distribution of illumination over the Fourier plane, is determined by the expression

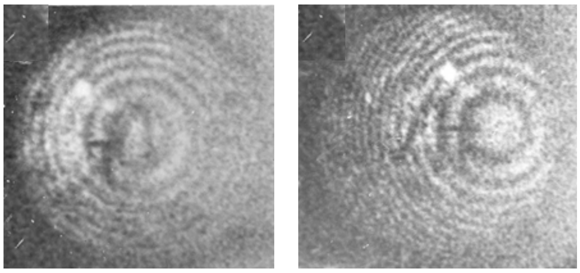
$$\tilde{I}(x_4, y_4) \sim \left\{1 + \cos\left[k\Delta l + \frac{k\Delta l}{2R^2}\left(x_4^2 + y_4^2\right)\right]\right\} \times$$

$$\times \Bigg| p(x_4 - x_{03}, y_4)t(-x_4, -y_4)\exp\left(\frac{i2kx_{03}x_4}{f}\right) \times$$

$$\times \exp\left[\frac{ik}{2R}\left(x_4^2 + y_4^2\right)\right]\Bigg|^2. \quad (26)$$

As follows from the comparison of expressions (18) and (26), in the considered particular case ($l_2 = f$) when performing the spatial filtering outside the optical axis in the plane of the double-exposure quasi-Fourier hologram characterizing longitudinal motion of a flat diffuser, the position of center of the interference fringes relative to the fixed image of the diffuser, remains invariable. This is explained by the fact that there is no displacement of subjective speckles, corresponding to the second exposure, with respect to the speckles of the first exposure in the hologram plane. Thus, Fig. 8*a* corresponds to the spatial filtering of the diffraction field on the optical axis, while Fig. 8*b* to that at a distance of x_{03} = 5 mm off from the axis. The double-exposure quasi-Fourier hologram recording was carried out at the following geometric parameters of the scheme: $l_1 = l_2 = f$ = 220 mm, R = 220 mm.

It should be noted that in this case, the behavior dynamics of "living" interference fringes consists only in that the phase of interference pattern will vary from 0 up to π, when the center of the filtering aperture moves from the minimum of the interference pattern, localized in the hologram plane, to its maximum value.



a *b*

Fig. 8. Interference patterns, localized in the plane of the diffuser image formation, characterizing its longitudinal motion and recorded while performing spatial filtering of the diffraction field in the hologram plane: the filtering on the optical axis (*a*); for that outside the optical axis (*b*).

As is obvious from the above-stated analysis of the interference patterns formation, characterizing the cross or longitudinal motions of a flat diffuser in the diffuse light fields, in case of the double-exposure Fourier hologram recording ($l_2 = f$), the "frozen" interference fringes are localized in its plane. Consequently, if combining the cross and longitudinal motions of the diffuser in the plane of Fourier hologram, the "frozen" interference pattern is formed in this case, since only the combination of uniform and non-uniform tilts of the subjective speckles, corresponding to the second exposure, with respect to the speckles of the first exposure, takes place in it.

As an example Fig. 9 shows the interference pattern, localized in the plane of Fourier hologram obtained at $l_1 = l_2 = f$ = 220 mm and characterizing cross shift of the diffuser by the distance $a = (0.04 \pm 0.002)$ mm and longitudinal motion $\Delta l = (1 \pm 0.002)$ mm.

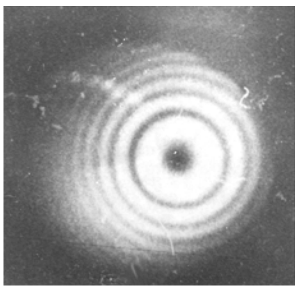


Fig. 9. Interference pattern, localized in the plane of the Fourier hologram and characterizing the cross and longitudinal motions of a flat diffuser.

It is worth noting that in this case, the hologram was recorded using a photographic objective without any spatial filtering of the diffraction field. In the course of this recording, the spatial extent of the interference pattern in the hologram plane was limited by the diameter of the reference beam, which was equal to 40 mm at the stage of the hologram recording.

Thus, the results of investigation have shown the following. As in the case with the double-exposure recording of the quasi-Fourier hologram by means of a negative lens, the interferometer sensitivity to the cross or longitudinal motion of a flat diffuser does not depend, for the interference pattern localized in the hologram plane, on the radius of curvature of a spherical wave of a coherent radiation used for the diffuser illumination. At the same time, it depends both on the magnitude and sign of the radius of curvature, in the case of the interference pattern localized in the Fourier plane and characterizing cross motion of the diffuser at $l_2 \neq f$ (for $l_2 = f$ there is no dependence of the interferometer sensitivity on the sign of the curvature radius).

In its turn, for the interference pattern characterizing longitudinal motion of a flat diffuser and localized in the Fourier plane, there is no sensitivity dependence on the sign of the curvature radius.

Besides, for the interferogram recording, spatial filtering of the diffraction field in the corresponding planes is needed for the theoretical estimation of sensitivity to the motions of the diffuser.

Specific features of the recording by means of a positive lens of the double-exposure quasi-Fourier hologram for the cross or longitudinal motion monitoring of a flat diffuser are, in fact, caused by that for the fixed values of d, f, l_1, the magnitude and sign of R, the interferometer sensitivity is always higher if recording of the interference pattern localized in the Fourier plane is being done at $l_2 < f$, as compared with the case of $l_2 > f$, owing to the smaller size of the subjective speckle in the hologram plane.

As in the case of the double-exposure recording by means of a negative lens of the Fourier hologram, characterizing the cross motion of a flat diffuser, the interference pattern is localized only in the hologram plane, where there is no any displacement of the subjective speckles, corresponding to the second exposure with respect the speckles of the first exposure.

In its turn, for the Fourier hologram, characterizing longitudinal motion of the diffuser, a spatial filtering of the diffraction field is needed at recording the interference pattern localized in its plane due to the non-uniform displacement of the subjective speckles of the second exposure at $l_2 \neq f$.

In the particular case of $l_2 = f$, the interference pattern is localized only in the hologram plane and there is no need in a spatial filtering of the diffraction field for its recording because of the absence of any displacement of the subjective speckles of the second exposure in the plane of the Fourier hologram, both at the cross and longitudinal motions of the diffuser.

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