PULSED SENSING OF THE FOAM-COVERED SEA SURFACE THROUGH THE ATMOSPHERE: THE OPTICAL SIGNAL POWER

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This study considers the power of the echo-signal detected during pulsed sensing of the partially foam-covered sea surface through the atmosphere.

Analytical expressions are derived for the average received power and for the echo-pulse delay and width during nadir sensing of the sea surface through both the optically transparent and dense aerosol atmospheres.

The presence of sea surface foam is shown to distort the shape of the recorded echo-signal quite significantly.

Continuous lidar sensing of the partially foam-covered sea surface was treated in Ref. 1. Below we study the energy characteristics of the echo-signal for the case of pulsed lidar sensing of such a surface through the atmosphere.

Let the radiation wavelength be small compared to the characteristic curvature radius and roughness elevations of the sea surface, and be in the IR range, where the main part of the received radiation is specularly reflected from the air-sea interface. We also assume that one may neglect any changes in the sea surface profile during its interaction with the pulse.

Recall that radiation pulses reflected from the clean sea surface and from the foam-covered surface add together incoherently

$$P(t) = \left(1 - S_f\right) P_0(t) + S_f P_f(t), \tag{1}$$

where S_f is the sea surface coverage by foam and white-caps; $P(t)$, $P_0(t)$, and $P_f(t)$ are, respectively, the average power received by the lidar from the partially foam-covered sea surface, from that free from foam, and from that completely foam-covered.

$$P_{0,f}(t) \simeq \int\limits_{-\infty}^{\infty} d\zeta\, W(\zeta) \int\limits_{S_0} E_s(\mathbf{R})\, E_d(\mathbf{R}) \times$$
$$\times K_{0,f}\left[\overline{\gamma_x^2},\ \overline{\gamma_y^2},\ R_x,\ R_y\right] f\left[t - \frac{2L}{c} + \frac{2\zeta}{c} - \frac{R^2}{cL}\right] d\mathbf{R} \tag{2}$$

In relation (2) we have for the locally Lambertian surface

$$K_f\left[\overline{\gamma_x^2},\ \overline{\gamma_y^2},\ R_x,\ R_y\right] = K_f\left[\overline{\gamma_x^2},\ \overline{\gamma_y^2}\right] =$$
$$= \frac{A a^{3/4} \exp(1/2a)}{2\pi\left[\overline{\gamma_x^2}\ \overline{\gamma_y^2}\right]^{1/2}} \sum_{k=0}^{\infty} \frac{a^{-k}}{k!} \left[\frac{\beta}{2}\right]^{2k} \frac{\Gamma(2k+1)}{\Gamma(k+1)} \times$$
$$\times W_{-k-1/4,\,k+1/4}\left[\frac{1}{a}\right],$$
$$a = 4\left[\frac{1}{\overline{\gamma_x^2}} + \frac{1}{\overline{\gamma_y^2}}\right]^{-1}; \quad \beta = \frac{\Delta a}{2}; \quad \Delta = \frac{1}{2\overline{\gamma_x^2}} - \frac{1}{2\overline{\gamma_y^2}};$$

and for the locally specular surface

$$K_0\left[\overline{\gamma_x^2},\ \overline{\gamma_y^2},\ R_x,\ R_y\right] =$$
$$= \exp\left\{-\frac{R_x^2}{2\overline{\gamma_x^2}L^2} - \frac{R_y^2}{2\overline{\gamma_y^2}L^2}\right\} K_0\left[\overline{\gamma_x^2},\ \overline{\gamma_y^2}\right];$$
$$K_0\left[\overline{\gamma_x^2},\ \overline{\gamma_y^2}\right] = \frac{V^2}{8\pi\left[\overline{\gamma_x^2},\ \overline{\gamma_y^2}\right]^{1/2}},$$

As a model of the foam-free sea surface we assume the model of a randomly rough, locally specular surface (see Ref. 2). For a completely foam-covered sea surface we consider two, models: one — that of the randomly rough, locally Lambertian surface, and another — that of a flat Lambertian surface.[1]

Let us consider the average power received by a lidar from a randomly rough surface with locally Lambertian and locally specular scattering phase functions, assuming that their slope distribution coincides with that of the sea wave slopes.[1] Assume also that the source and detector are collocated in a single unit, and that nadir sensing geometry is employed.

As in Refs. 3 and 4 we may write an integral expression for the echo-signal power for the case of nadir sensing of the randomly rough surface S, employing the averaging technique from Ref. 5, neglecting shadowing effects (in which some elements of the surface are shadowed by others), and considering the distribution

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density for surface heights and slopes to be Gaussian, where $\overline{\gamma^2_{x,y}}$ is the variance of the sea surface slopes; $W_{n,m}(x)$ is the Whittaker function; $\Gamma(k)$ is the gamma-function; $f(t)$ describes the shape of the sensing pulse; S_0 is the projection of the surface upon the $z = 0$ plane; R is radial distance in the S_0 plane; ζ is the height of the randomly rough surface S at the point R; $W(\zeta)$ is the distribution density of the elevations of the randomly rough surface S; $E_s(\mathbf{R})$ and $E_d(\mathbf{R})$ are the atmospheric irradiances (for the case of continuous sensing) in the planes normal to the source and detector optical axes, respectively, due to the real source and a fictious source with parameters identical to those of the detector;[6] L is the distance from the observation sector center (on the surface S_0) to the locator; A is the albedo of a foam-covered surface element; V^2 is the Fresnel coefficient for the flat sea surface during nadir sensing.

Using the expressions for $E_s(\mathbf{R})$ and $E_d(\mathbf{R})$ from Ref. 6 for a narrow illuminating beam we obtain the following analytical expression for the "surfaces ensemble" averaged echo-signal from the randomly rough surface sensed through the aerosol atmosphere

$$P_{0,f}(t) \simeq \frac{a_s a_d}{L^4} \frac{\tau_s cL\pi}{2} K_{0,f}\left[\overline{\gamma^2_x}, \overline{\gamma^2_y}\right] \times$$
$$\times \left\{1 - \Phi\left[N_{0,f}p^{1/2} - \frac{t'cL}{2p^{1/2}}\right]\right\} \times$$
$$\times \exp\left\{N^2_{0,f}p - t'N_{0,f}Lc\right\}. \quad (3)$$

We assume here that the illuminated spot and the observation sector at the surface are much larger than $\left(\overline{\zeta^2}\right)^{1/2}$ and the sensing pulse shape is Gaussian: $f(t) = \frac{2}{\sqrt{\pi}}\exp\left\{-\frac{4t^2}{\tau_s^2}\right\}$. The values that refer to the locally specularly reflecting surface are indexed "0" in Eq. (3) and those that refer to the locally Lambertian surface are indexed "f"

$$N_0 = N_f + \frac{1}{4\overline{\gamma^2_x}L^2} + \frac{1}{4\overline{\gamma^2_y}L^2}; \quad N_f = C_s + C_d;$$

$$p = \frac{\tau_s^2c^2L^2}{16} + 2\overline{\zeta^2}L^2; \quad t' = t - \frac{2L}{c}.$$

We have for a transparent aerosol atmosphere[6]

$$a_s = \frac{P_0}{\pi\alpha_s^2}\exp\left[-\int_0^L \sigma(z)\,dz\right];$$

$$a_d = \pi r_d^2 \exp\left[-\int_0^L \sigma(z)\,dz\right];$$

$$C_{s,d} = (\alpha_{s,d}L)^{-2}.$$

while for an optically dense atmosphere[6]

$$a_s = \frac{P_0C_sL^2}{\pi}\exp\left\{-\int_0^L (1-\lambda)\,\varepsilon(z)\,dz\right\};$$

$$a_d = \pi r_d^2C_d(L\alpha_d)^2\exp\left\{-\int_0^L (1-\lambda)\varepsilon(z)\,dz\right\};$$

$$C_{s,d} = \left[(\alpha_{s,d}L)^2 + \mu L^2\right]^{-1};$$

$$\mu = L^{-2}\int_0^L \tilde{\sigma}(z)\,\langle\gamma^2(z)\rangle\,(L-z)^2\,dz; \quad \lambda = \frac{\tilde{\sigma}}{\varepsilon},$$

where $\varepsilon(z)$ and $\sigma(z)$ are the atmospheric extinction and scattering coefficients; $\langle\gamma^2(z)\rangle$ is the variance of the beam deflection angle arising during an elementary scattering act; $\tilde{\sigma}(z)$ is the effective scattering coefficient; $\tilde{\tau} = (1 - x_0)\sigma$, where x_0 is the isotropic part of the scattering phase function[6]; r_d is the effective radius of the detector aperture; $2\alpha_s$ and $2\alpha_d$ are the divergence angle of the source and the detector field-of-view angle, respectively; P_0 is the power emitted by the source; $\overline{\zeta^2}$ is the variance of the sea surface elevations; $\Phi(x)$ is the Fresnel integral.

Expression (3) was derived within the approximation $\beta \ll 1$, which is satisfied quite well for the sea surface wind-generated roughness (for surface wind velocities in the range 1–20 m/s β does not exceed 0.21).

The expression for $P_f(t)$ is obtained for a flat Lambertian surface from relation (3) by setting $\overline{\zeta^2_x}$ and $\overline{\gamma^2_{x,y}}$ equal to 0.

If the detector field-of-view angle is much larger than the lidar source divergence angle, and the roughness is isotropic $\left(\overline{\gamma^2_x} = \overline{\gamma^2_y}\right)$ the expression for $P_0(t)$ agrees well with the results from Ref. 2.

We will now evaluate the effect of foam on the structure of the echo-signal received by the detector.

Using relations (1) and (3) we obtain for the average echo-signal power in the case of nadir sensing of the foam-covered sea surface

$$P(t) = C_1\left\{C_2\exp\left\{N_0^2p - t'N_0Lc\right\} \times\right.$$
$$\times\left\{1 - \Phi\left[N_0p^{1/2} - \frac{t'cL}{2p^{1/2}}\right]\right\} + C_3\exp\left\{N_f^2\tilde{p} - t'N_fLc\right\} \times$$
$$\times \left.\left\{1 - \Phi\left[N_f\tilde{p}^{1/2} - \frac{t'cL}{2\tilde{p}^{1/2}}\right]\right\}\right\}, \quad (4)$$

where

$$C_1 = \frac{\alpha_s\ \alpha_f}{L^4}\ \frac{\tau_s c L\pi}{2}\ ; \quad C_2 = \left(1 - S_f\right) K_0\left[\ \overline{\gamma_x^2},\ \overline{\gamma_y^2}\ \right]\ ;$$

For foam modeled as a randomly rough Lambertian surface

$$C_3 = S_f K_f\left[\ \overline{\gamma_x^2},\ \overline{\gamma_y^2}\ \right]; \qquad \tilde{p} = p;$$

For foam modeled as flat Lambertian surface

$$C_3 = S_f\ \frac{A}{\pi}; \qquad \tilde{p} = p\ \left[\overline{\zeta^2} = 0\right].$$

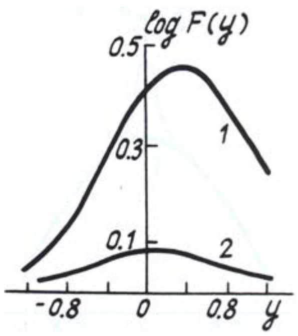

FIG. 1. Echo-pulse shape for the sea surface at $\alpha_s = 8.7 \cdot 10^{-3}$, $\tau_s = 10^{-8}$ s.

Figures 1 and 2 show the computational results of the shape of echo-signals for various surface wind velocities U. The values of $F(y) = \dfrac{P(t)}{C_1 C_2}$ were computed from expression (4) for foam modeled as a randomly rough Lambertian surface (solid lines) and as a flat Lambertian surface (dashed lines) with following values of the parameters entering into Eq. (4) $\left(y = t\dfrac{cL}{2p^{1/2}}\right)$

$L = 10$ km, $\alpha_d = 2.9 \cdot 10^{-2}$;
$\alpha_s = 8.7 \cdot 10^{-3}$, $\tau_s = 10^{-8}$ s (Fig. 1);
$\alpha_s = 10^{-3}$, $\tau_s = 10^{-9}$ s (Fig. 2);

$U = 2$ m/s (curve 1), $U = 14$ m/s (curve 2, Fig. 1); $U = 14$ m/s (Fig. 2).

Here and below the values $\overline{\gamma^2_{x,y}}$ were computed from the Cox—Munk formulas,[7] and the following expressions were used for S and $\overline{\zeta^2}$ (see Refs. 8 and 2): $S = 0.09U^3 - 0.3296U^2 + 4.549U - 21.33$; $\left(\overline{\zeta^2}\right)^{1/2} = 0.016\ U^2$. Here U is the surface wind velocity in m/s.

It can be seen from these figures that the presence of foam, which appears on the surface at high wind velocities, strongly affects the value shape of the echo-signals. The echo-signal power for a laser beam with $\alpha_s = 2.9 \cdot 10^{-3}$ depends only slightly on the foam model chosen (solid and dashed lines merge in Fig. 1). The echo-signal shape for a sufficiently narrow laser beam ($\alpha_s = 10^{-3}$) depends significantly on the model used to describe the foam (see Fig. 2).

The most important parameters for the temporal trend of the detected signal power are the echo-signal delay and width. Measured values of these parameters are used to retrieve the profiles and the statistical parameters of the heights and slopes of the sensed surface (see, e.g., Ref. 2).

We define the delay T (from the moment the signal pulse is emitted) and the width τ of the echo-pulse as follows[2]

$$T = \frac{\int\limits_{-\infty}^{\infty} dt\ tP(t)}{\int\limits_{-\infty}^{\infty} dt\ P(t)}\ ; \quad \tau^2 = \frac{\int\limits_{-\infty}^{\infty} dt(t - T)^2\ P(t)}{\int\limits_{-\infty}^{\infty} dtP(t)}\ . \tag{5}$$

It follows from the relation (4) that

$$T = T_0 K_0 + T_f K_f; \tag{6}$$

$$\tau^2 = \tau_0^2 K_0 + \tau_f^2 K_f, \tag{7}$$

where

$$K_0 = \left(1 + a_f\right)^{-1}; \quad K_f = \left(1 + a_f\right)^{-1} a_f;$$

$$T_0 = \frac{2L}{c} + \frac{1}{cL2}\left[\frac{1}{a_1} + \frac{1}{a_2}\right]\ ;$$

$$\tau_0^2 = \frac{\tau_s^2}{8} + \frac{4\overline{\zeta^2}}{c^2} + \frac{1}{2c^2L^2}\left[a_1^{-2} + a_2^{-2}\right];$$

$$a_{1,2} = N_f + 0.5\ \left[\ \overline{\gamma^2_{x,y}}\ L^2\right]^{-1};$$

For the randomly rough locally Lambertian foam model we have

$$a_f = \frac{K_f\left[\ \overline{\gamma^2_x},\ \overline{\gamma_y^2}\ \right]\ S_f\left(a_1 \cdot a_2\right)^{1/2}}{K_0\ \left[\ \overline{\gamma_x^2},\ \overline{\gamma_y^2}\ \right]\ \left(1 - S_f\right)N_f}\ ;$$

$$\tau_f^2 = \frac{\tau_s^2}{8} + \frac{4\overline{\zeta^2}}{c^2} + \left(c^2L^2N_f^2\right)^{-1}; \quad T_f = \frac{2L}{c} + \frac{1}{cLN_f}\ ;$$

and for the flat Lambertian foam model

$$a_f = \frac{A\ S_f\left(a_1 \cdot a_2\right)^{1/2}}{K_0\ \left[\ \overline{\gamma_x^2},\ \overline{\gamma_y^2}\ \right]\ \left(1 - S_f\right)N_f}\ ;$$

$$T_f = \frac{2L}{c} + \frac{1}{cLN_f}; \quad \tau_f^2 = \frac{\tau_s^2}{8} + \frac{1}{c^2L^2N_f^2},$$

where (T_0, τ_0) and (T_f, τ_f) are the delay and width of the echo-signal from the foam-free sea surface and from the completely foam-covered sea surface, respectively.

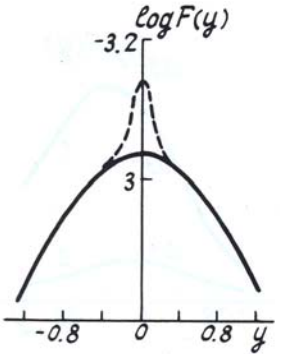

FIG. 2. Echo-pulse shape for the sea surface at $\alpha_s = 10^{-3}$, $\tau_s = 10^{-9}$.

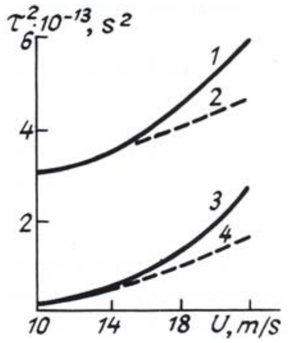

FIG. 3. Duration of the echo-pulse from the sea surface in a transparent and an optically dense aerosol atmosphere.

Figure 3 presents the computed widths for echo-pulses from the sea surface at various surface wind velocities, the computations of τ^2 carried out for the foam modeled as a randomly rough locally Lambertian surface (solid lines) and as a flat Lambertian surface (dashed lines) for the following parameter values: L = 10 km, $\alpha_d = 2.9 \cdot 10^{-2}$; $\alpha_s = 10^{-3}$; $\tau_s = 10^{-9}$ s; $\mu = 0$ (curves 3 and 4); $\mu = 3 \cdot 10^{-3}$ (curves 1 and 2).

It can be seen from Fig. 3 that the duration of the echo-pulse depends significantly on both the surface wind velocitiy and the chosen foam model. However, the latter dependence is manifested only for high surface wind velocities. Atmospheric turbulence results in a longer delay r and a weaker effect of the foam on the echo-signal.

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