ESTIMATE OF THE POTENTIAL RESOLUTION OF PASSIVE METHODS OF IMAGE FORMATION THROUGH A TURBULENT ATMOSPHERE. III. INFRARED SPECKLE INTERFEROMETRY

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Received February 5, 1990

Expressions for the potential resolution in the case of image formation from infrared radiation emitted by the heated surface of an object are derived based on the relation for the accuracy of the estimate of the squared modulus of the spatial spectrum of the image of the object from a series of short-exposure images distorted by the atmosphere. It is shown that this observational regime is preferable for objects whose angular dimensions are larger then the atmospheric limit of the resolution.

In our preceding papers[1,2] we evaluated the possibility of speckle-interferometric methods of image formation based on the visible solar radiation reflected by the object. We showed that for angular size of the object $\theta_0 \gtrsim \theta_a$, where $\theta_a = 4\lambda/(\pi \cdot r_0)$ is the average "atmospheric" resolution,[3] λ is the wavelength, and r_0 is Fried's parameter, characterizing the region of correlation of the atmospheric distortions of the field on the aperture,[3] a resolution of $\sim 10^{-7}$ rad is achievable, while for $\theta_0 \ll \theta_a$ a resolution of up to $2 \cdot 10^{-8}$ rad is achievable. The latter value corresponds to a diameter D of the telescope aperture of about 30 m, which presupposes the use of synthesis telescopes (STs).[5] For this diameter it is possible to form effectively an image of an object at infrared wavelengths in the transmission windows of the atmosphere (for example, 8–12 μm) based on the thermal radiation from the solar-heated surface of the object. This range is characterized by the following positive characteristics:

– the possibility of round-the-clock observation owing to the fact that for $\lambda \geq 4$ μm there is no scattering of sunlight in the earth's atmosphere;[8]

– there is no occlusion of the thermal images; and,

– the effect of atmospheric distortions is significantly reduced.

To illustrate the latter feature we note that according to the 5/3 law for the structure function of the distortions $D_a(v)$ the size r_0 depends on the wavelength λ as $\lambda^{6/5}$ and increases from 0.1 m at $\lambda = 0.6$ μm to 2.5 m at $\lambda = 10$ μm. In reality, apparently, the correlation region reaches a size of 3 ... 5 m. The point is that the law $D_a(v) = 6.88(v/r_0)^{5/3}$ is valid only if $v \ll L_a$, where $L_a \sim 10$ m is the saturation interval of the structure function (the outer scale of turbulence). The slowing down of the growth of $D_a(v)$ at $v \sim L_a$, which is significant for telescope apertures $D \gtrsim L_a$, is what leads to the increase in the correlation size. At the same time that r_0 increases the variance ($\tau_a^2 = 1/2D_a(\infty)$) of the phase distortions decreases as λ^{-2}. According to the estimates of Ref. 7, at 0.6 μm it equals $\sim 10^3$rad^2 and at 10 μm it equals about 4 rad^2. Finally, the correlation time interval of the distortions T_c increases substantially in the IR region. Since $T_c \sim r_0/\sigma_V$,[8] where σ_V is the average value of the fluctuations of the fluctuations of the transport velocities of separate atmospheric layers, in the IR range T_c increases from 0.01 to 0.3 ... 0.5 sec. All these features make IR observations very promising, and this is what encouraged us to estimate the corresponding potential resolution.

Following the method adopted in Ref. 2 we shall define the maximum resolution θ_r as the inverse of the maximum spatial frequency f_r at which $Q(\vec{f}) \geq 5$. Here $Q(\vec{f})$ denotes the signal-to-noise ratio (SNR), which characterizes the accuracy of the estimate of the squared modulus of the spatial spectrum (SS) $|O(\vec{f})|^2$ of the image of the object based on the recorded series of M short-exposure (distorted by the atmosphere and recording noise) images (Sis). The SNR $Q(\vec{f})$ satisfies a general expression of the form

$$Q(\vec{f}) = \sqrt{M}\,\frac{E^2}{\sigma_r^2}\,\frac{\sqrt{S_0 \cdot S_J}}{S_r}\,|O(\vec{f})|^2 \langle |H(\vec{f})|^2 \rangle, \qquad (1)$$

where E is the average energy of one short-exposure image; σ_r^2 is the variance of the SS of the recording noise; S_0, S_J, and S_r are the areas of the regions of correlation of the spatial spectrum of the distributions, respectively, of the undistored image of the object, the short-exposure image, and the recording noise; and, $\langle |H(f)|^2 \rangle$ is the optical transfer function (OTF) of the "atmosphere-telescope" system. Expanding the terms appearing in Eq. (1), according to Ref. 2, we obtain the following more detailed representation:

$$Q(f) = \sqrt{M}\,\xi\eta\,\frac{\rho_0^2\cdot\theta_0^2 S_{SK}\sqrt{S_0 S_J}}{\rho_0\theta_0^2 + \rho_b S_{SK}} \times$$

$$\times \frac{\Delta\lambda\cdot\Delta\lambda_c}{\Delta\lambda + \Delta\lambda_c}\,\frac{TT_c}{T+T_c}\,\frac{0.34 r_e^2}{2\pi^2 f^2}\,B(f), \qquad (2)$$

where ξ is the transmission coefficient of the optical system of the telescope; η is the quantum efficiency of the detector; ρ_0 and ρ_b are the spectral densities of the received fluxes of the signal radiation from the object and the background radiation, respectively (it is assumed that the detector operates in the background-limited regime); $S_{SK} = \left(\theta_0 + \dfrac{4\lambda}{\pi r_e}\right)^2$ is the average angular size of the short-exposure image, $S_0 = \dfrac{\pi}{8\cdot\theta_0^2}$, and

$$S_J = \frac{\pi}{4}\min\left[\frac{1}{\theta_0},\ \frac{\lambda}{r_e}\right]\cdot\min\left[\frac{1}{2\theta_0},\ \frac{\Delta\lambda}{\lambda}\cdot f\right]; \qquad (3)$$

$\Delta\lambda_c = R_e/f$ is the spectral correlation interval of the spatial spectrum of the short-exposure image; $\Delta\lambda$ is the spectral range of the recorded radiation; T is the exposure time of the short-exposure image; r_e is the effective size of the spatial region of correlation of the atmospheric distortions, equal approximately to

$$r_e = \begin{cases} \dfrac{3.8\cdot D_T\cdot r_0}{D_T + 3.8\cdot r_0} & \text{at } D_T \le 3.8\cdot r_0, \\ r_0\cdot\left(1 + 1.43\sqrt{r_0/D_T}\right) & \text{at } D_T \ge 3.8\cdot r_0, \end{cases} \qquad (4)$$

D_T is the diameter of a separate receiving aperture of the synthesis telescope, and $B(f)$ is the SNR of the telescope. The latter function depends on the type of synthesized telescope.

From this standpoint two basic concepts are distinguished:[5] segmented telescopes. In which a quasi continuous "giant" mirror is assembled from a large number of almost adjoining aspherical fragments, and $D_T = D$; multimirror telescopes, consisting of a small number N_T of receiving telescopes with diameter D_T which are separated from one smother by am appreciable distance and which operate in the regime of coherent and in-phase addition of light fluxes In the common focal plane, and $D > D_T$. In the first case the OTF of the synthesis telescope is virtually identical to the OTF of a classical telescope and can be approximated as

$$B(f) = 1 - f/f_d, \qquad (5)$$

where $f_d = D/\lambda$ is the diffraction cutoff frequency. In the second case, with a more efficient, from the viewpoint of image formation, arrangement of N_T telescopes, it is equal to approximately

$$B(f) = 1/N_T. \qquad (6)$$

In what follows, so as not to limit the analysis, we shall assume that $B(f_r) = 0.1$, which corresponds to either $f_r = 0.9/$ or $N_T = 10$. In addition, we shall assume that $D \gg r_0$, but $D_T \approx r_0$, so that in both cases $r_e \approx r_0$.

For the further analysis the values of ρ_0 and ρ_b must be determined. It is shown in Ref. 6 that the equilibrium temperature t_e of the surface of a spherical object with radius r and having no internal energy sources and absorbing and emitting radiation as a black body with the coefficients α and ε is equal to

$$t_e = 645\cdot\left[0.0475\,\frac{\alpha}{\varepsilon} + 0.008\right]^{1/4}. \qquad (7)$$

The corresponding maximum λ_{max} and the peak W_{max} of the spectral density $W_t(\lambda)$ of the radiation are given by[6]

$$\lambda_{max} = A/t_e; \quad W_{max} = B\cdot t_e^5, \qquad (8)$$

where

$A = 2898$ μm · K,
$B = 1.29\cdot10^{-19}$ W · m^{-2} · μm^{-1} · K^{-5}.
Typically $\alpha/\varepsilon \approx 1.5$, which leads to the values $t_r \approx 340$ K, $\lambda_{max} = 8.5$ μm, and
$W_{max} = 59$ W · m^{-2} · μm^{-1}. At the same time in the transmission window 8 ... 12 μm at the average wavelength $\lambda_0 = 10$ μm and the width of the range $\Delta\lambda = 4$ μm the average radiation density
$W_0 = 55$ W · m^{-2} · μm^{-1}.

The density of the radiation from the object in unit solid angle is equal to $\varepsilon\cdot r^2\cdot W_0$ while the density of the received radiation in the plane of the aperture of the telescope is equal to $\dfrac{\varepsilon\cdot r^2}{R^2}\cdot W_0\cdot K_a^{\operatorname{cosec}\varphi_0}$, where R is the range to the object, φ_0 is the elevation angle of the object, and K_a is the transmittance of the atmosphere. As a result we find that the density ρ_0 of the radiation from unit solid angle per unit time per unit area of the receiving aperture in unit spectral range is given by

$$\rho_0 = \frac{\varepsilon}{\pi} W_0 K_a^{\operatorname{cosec}\varphi_0}. \qquad (9)$$

Substituting into Eq. (9) the characteristic values $\varepsilon = 0.6$, $K_a = 0.8$, and $\varphi_0 = \pi/6$ with $T_r = 340$ K and rewriting the density in terms of the number of photons according to the rule 1 $W = 5\cdot10^{18}\lambda$ photons · sec^{-1} (λ is given in μm), we have

$$\rho_0 = 3.4 \cdot 10^{20}\ \text{m}^{-2} \cdot \text{s}^{-1} \cdot \mu\text{m}^{-1} \cdot \text{env}^{-1}. \tag{10}$$

In the IR range the recording noise is caused by fluctuations of both the signal radiation from the object and the background thermal radiation of the atmosphere and the mirrors of the telescope. Assuming that both the atmosphere and the mirrors are characterized by the temperature $t_a = 300$ K, based on the data presented in Ref. 6, we conclude that the clear-sky radiation density with $\lambda = 10$ μm is equal to $5.3 \cdot 10^{19}\ \text{m}^{-2} \cdot \text{sec}^{-1} \cdot \mu\text{m}^{-1} \cdot \text{sr}^{-1}$. The effective radiation density of the telescope mirrors from a unit area into unit solid angle is equal to $\varepsilon_m \cdot K_m \cdot W_{t_a}(\lambda_0) / \pi$, where ε_m is the emissivity of the mirrors ($\varepsilon_m \approx 0.05$) and K_m is the number of "thermal" mirrors in the optical and mechanical channel ($K_m \sim 5$); this leads to the value $1.23 \cdot 10^{20}\ \text{m}^{-2} \cdot \text{sec}^{-1} \cdot \mu\text{m}^{-1} \cdot \text{sr}^{-1}$. As a result we find that the effective density of background photons is equal to

$$\rho_b = 1.8 \cdot 10^{20}\ \text{m}^{-2} \cdot \text{s}^{-1} \cdot \mu\text{m}^{-1} \cdot \text{env}^{-1}, \tag{11}$$

i.e., in the IR range (unlike the visible range) the signal power is equal to the background power. This leads to the fact that from the standpoint of the absolute magnitude of the achievable resolution it is preferable to form Images of large objects. Indeed, for $\theta_0 \sim \theta_r \ll \theta_a$ the ratio Q, as follows from Eq. (2), is proportional to θ_0 and increases monotonically as θ_0 increases, reaching saturation for $\theta_0 \gtrsim \theta_a$.

We shall make in these limiting cases quantitative estimates of the potential resolution, making the assumption that $T \gg T_c = 0.4$ s, $\Delta\lambda_c \ll \Delta\lambda = 4$ μm, $\xi = 0.6$, $\eta = 0.5$, $r_0 = 4$ m. For $\theta_0 \gg \theta_a$ we have

$$Q(f) = 6.8 \cdot 10^{-3} \sqrt{M}\, \xi \cdot \eta \cdot T_c \rho_0^2 / (\rho_0 + \rho_b) (r_0/f)^3 B(f) =$$
$$= 1.16 \cdot 10^{24} \sqrt{M} / f^3. \tag{12}$$

Solving the equation $Q(1/\theta_r) = 5$ we obtain the dependence of the resolution θ_r on the number of images M in the form

$$\theta_r = \frac{1.6 \cdot 10^{-8}}{M^{1/6}}\ \text{rad}, \tag{13}$$

which for $M = 100$ leads to the value $\theta_r = 0.8 \cdot 10^{-8}$ rad. We stress that the significant improvement of the resolution as compared with the visible range ($\theta r \sim 10^{-7}$ rad) is mainly connected with the significant decrease, examined above, in the effect of the atmospheric distortions, in particular, as the radius r_0 increases.

For a small object with $\theta_0 \sim \theta_r$ we have

$$Q(f) = 9.6 \cdot 10^{-3} \sqrt{M} \cdot \zeta\eta \frac{\rho_0^2}{\rho_b} \theta_0 \Delta\lambda^{1/2} T_c \cdot \left[\frac{r_0}{f} \right]^{5/2} \times$$
$$\times B(f) = 4.7 \cdot 10^{21} \theta_0 \cdot \frac{\sqrt{M}}{f^{5/2}}. \tag{14}$$

Solving the equation for $\theta_0 = \theta_r$ we obtain the dependence

$$\theta_r = \frac{10^{-6}}{M^{1/7}}, \tag{15}$$

which for $M = 100$ gives the value $\theta_r = 5 \cdot 10^{-7}$ rad.

In conclusion we note that to achieve a resolution of $\sim 10^{-8}$ rad at the wavelength $\lambda = 10$ μm it is necessary to construct a synthesis telescope with an aperture diameter $D = 10^3$ m, which is almost impossible. But, in building the currently more realistic synthesis telescopes with an equivalent diameter of about 25 m, which are intended for operation in both the visible ($\theta_d \sim 2 \cdot 10^{-8}$ rad) and infrared ($\theta_d \sim 4 \cdot 10^{-7}$ rad) ranges, because of the large margin with respect to the resolution (SNR) in the latter case optimization of the construction of the synthesis telescope Is best done for the visible range.

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