LASER BEAM INTENSITY RECONSTRUCTION FROM TEMPERATURE FIELD OF A THIN TARGET

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In the paper, an algorithm for reconstruction of the intensity distribution over the laser beam cross section from the temperature field of a thin target is described. The reasons for the solution instability and methods for their elimination are analyzed. The results of numerical reconstruction experiment are presented for various boundary conditions.

When solving a number of atmospheric problems, it is necessary to know the continuously varying parameters of a laser beam propagating in the atmosphere. When studying the regularities of variations of these parameters, a problem arises of measuring the intensity in the beam cross section. One of the possible methods of intensity measurement is its reconstruction from the temperature field on the target surface.

We published a number of papers[1–3] where the integral relationships and algorithms were obtained that allow one to reconstruct the laser radiation intensity from measurements of temperature on the target surface for various boundary conditions and arbitrary target thickness. There are some singularities in these integral relationships, and to eliminate them we had to apply regularization of a certain type. A case of a thin target was of interest in a number of experiments because it favored the simplification of these relationships.[1–3]

The relationships obtained in Ref. 1 by the imbedding method connect the target surface temperature $T(\rho, t)$ with the intensity distribution $I(\rho, t)$ of laser beam incident on a target in terms of the heat flux $q(\rho, t)$ reconstructed from this temperature (neglecting the heat loss, $q(\rho, t) = (1 - R)I(\rho, t)$, where R is the reflection coefficient). In the case of a thin target (generalized thermophysical Fourier parameter $F_g = a^2 t/L^2 > 1$) with various boundary conditions, the following relationships were obtained:

$$q(\rho, t) = \frac{k\,L}{a^2}\left[\frac{\partial}{\partial t}T(\rho, t) - a^2\,\Delta_\perp\,T(\rho, t)\right], \qquad (1)$$

$$q(\rho, t) = \frac{k}{L}\,T(\rho, t) + \frac{k\,L}{3\,a^2}\left[\frac{\partial}{\partial t}T(\rho, t) - a^2\,\Delta_\perp\,T(\rho, t)\right], \qquad (2)$$

for thermal–insulating and cooled targets, respectively, where a^2 and k are the thermal diffusivity and conductivity, respectively; L is the target thickness; $\rho = \{x, y\}$ are the transverse coordinates; t is time; and, $\Delta_\perp = \partial/\partial x^2 + \partial/\partial y^2$ is the transverse Laplacian operator.

The aim of the paper is numerical reconstruction of the intensity by Eqs. (1) and (2) that are simpler in comparison with integral relationships derived in Refs. 1–3 since they do not contain any singularities and are also subjected to the noise effects that are present in the initial data.

Let us use the following difference procedure[4] to realize numerically Eqs. (1) and (2). Let the target have the shape of a rectangular plate, and let us assume

$$h = x_0/N = y_0/N,\ \tau = t_{\max}/M,\ x_k = kh,\ y_m = hm,$$

$$t_\nu = \nu\tau,\ T^{\,\nu}_{k\,m} = T(x_k,\ y_m,\ t_\nu).$$

Let us introduce operators

$$\Lambda_1\,T^{\,\nu}_{k\,m} = -\,(T^{\,\nu}_{k-1\,m} - 2\,T^{\,\nu}_{k\,m} + T^{\,\mathrm{m}}_{k+1\,m})\ /\ h^2, \qquad (3)$$

$$\Lambda_2\,T^{\,\nu}_{k\,m} = -\,(T^{\,\nu}_{k\,m-1} - 2\,T^{\,\nu}_{k\,m} + T^{\,\nu}_{k\,m+1})\ /\ h^2, \qquad (4)$$

$$\Lambda\,T^{\,\nu}_{k\,m} = \Lambda_1\,T^{\,\nu}_{k\,m} + \Lambda_2\,T^{\,\nu}_{k\,m}\,, \qquad (5)$$

where Λ is the discrete analog of the Laplacian operator. Using these designations, we can write the following difference analog of Eqs. (1) and (2) for thermal–insulating and cooled targets, respectively:

$$q^{\nu-1}_{k\,m} = k\,L/a^2\,[(T^{\,\nu}_{k\,m} - T^{\,\nu-1}_{k+1\,m})\ /\ \tau - a^2\,\Lambda_1\,T^{\,\nu-1}_{k\,m}]\,, \qquad (6)$$

$$q^{\nu-1}_{k\,m} = (k/L)\,T^{\,\nu-1}_{k+1\,m} + [k\,L/3\,a^2]\,\times$$

$$\times\,[(T^{\,\nu}_{k\,m} - T^{\,\nu-1}_{k+1\,m})/\tau - a^2\,\Lambda_1\,T^{\,\nu-1}_{k\,m}]\,, \qquad (7)$$

where k, $m = 1, 2, \ldots, N - 1$; $\tau = 1, 2, \ldots, M$.

The intensity reconstruction for thermophysical situations (1) and (2) was simulated in the numerical experiment with the use of the above–described algorithm. Then it was supposed that the temperature distribution over the spatial coordinates $T(\rho, t)$ was measured at some fixed moment with a random error

$$T^{\,\nu}_{k\,m} = T(\rho_{k\,m},\ t_\nu) + \zeta^{\,\nu}_{k\,m}\,,$$

where $\zeta^{\,\nu}_{k\,m}$ obeyed the normal distribution law with zero mean and variance σ^2. The domain of the functions was given in the experiment as follows: $t \in [0, 1]$; $c,\ x,\ y \in [-1, 1]$; $N = 40$. The frequency of thermal image recording was 1/24, which corresponds to the temporal resolution of a thermal imager. The target was made of aluminium. The following function was chosen as a model of the initial intensity:

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$$q(\rho, t) = \exp \{- \alpha(x^2 + y^2)\} \, \Theta(\tau) \, f(\tau) \, , \qquad (8)$$

where $\Theta(\tau) = \begin{cases} 1, & \tau \ge 0, \\ 0, & t < 0, \ \ t \ge 1, \end{cases}$

and the form of $f(\tau)$ was

$$f_1(\tau) = 2.6\, I_0 \,[\, \exp((\tau - 0.5)^2 \alpha_2) - 0.8 \exp((\tau - 0.5)^2 \alpha_3)] \quad (9)$$

for thermal–insulated target and

$$f_2(\tau) = I_0 \exp\left\{ - \frac{(\tau - 0.5)^2}{(0.5)^2 - (\tau - 0.5)^2} \right\} \qquad (10)$$

for the cooled one, $I_0 = 1$ W/cm^2, $\tau = t/t_0$, $t_0 = 1$ s. It was chosen for both models $\alpha = 4 \ln(10)$, $\alpha_2 = 20$, and $\alpha_3 = 60$. The dependence (8) is shown in Figs. 1*a* and 2*a* at two different moments $t_8 = 0.3$ and $t_{16} = 0.6$, respectively (from all totality of temporal samples). The results of reconstruction for $\sigma = 0.02$ are shown in Figs. 1*b* and 2*b*.

Let us comment upon the results obtained. The plots are indicative of instability of the difference scheme in this statement. The error in calculating the *l*th temperature derivative ($l = 1, 2$) consists of two components

$$\delta = \delta_R + \delta_r,$$

where $\delta_R \sim h^p$ is the error in approximating the function in the vicinity of the expansion point (p is the order of accuracy), $\delta_r \sim 1/h^l$ is the inherent random error connected with the error in measuring (l is the order of differentiation).

The Tikhonov regularization methods[4] are applied to the problems of this class, as well as the smoothing spline methods[5,6]; however, these methods become more cumbersome when solving three–dimensional problem. The choice of one or another calculation procedure is determined by the required accuracy. So for the experimental data processing it is preferable to use simpler methods that ensure satisfactory accuracy for further processing.

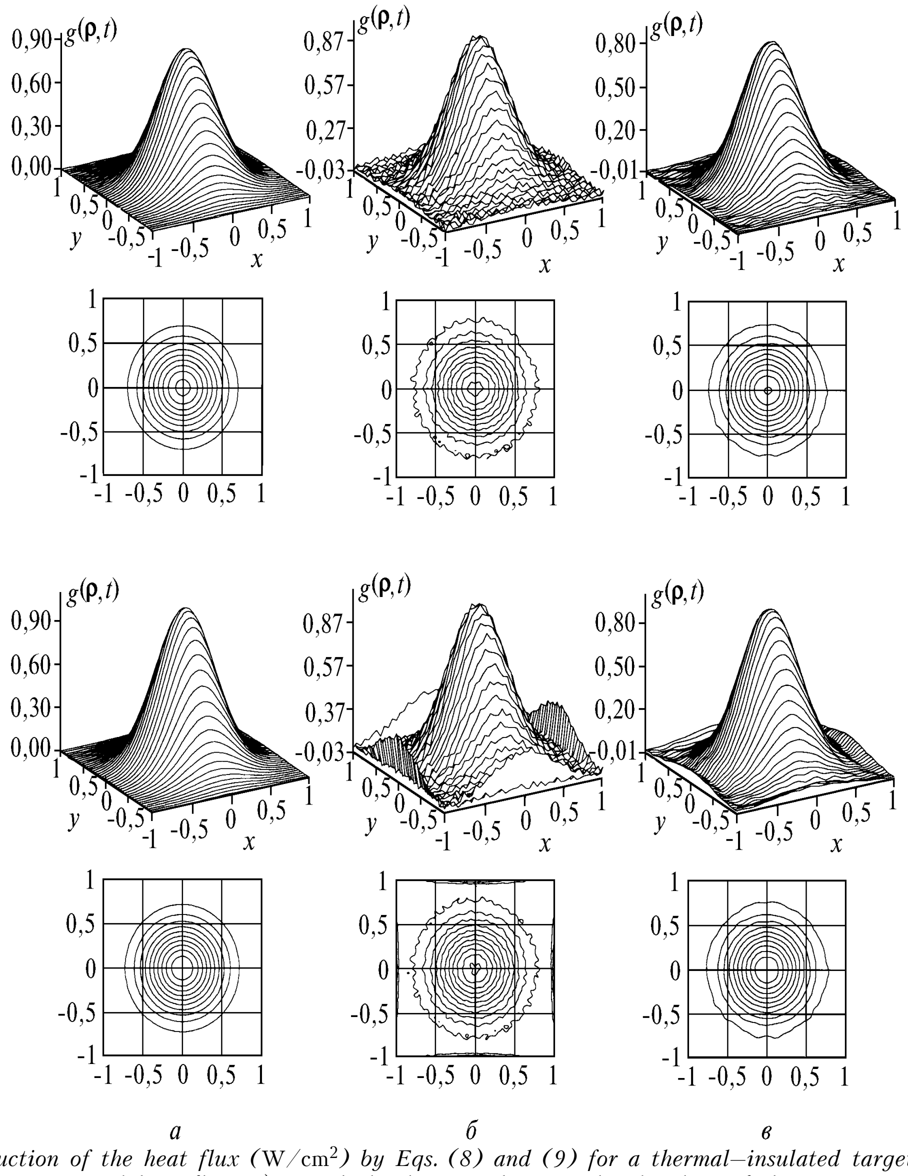

FIG. 1. Reconstruction of the heat flux (W/cm^2) by Eqs. (8) and (9) for a thermal–insulated target: a) initial model distribution, b) reconstructed heat flux, c) smoothed solution. The normalized values of the corresponding quantities are plotted on the axes.

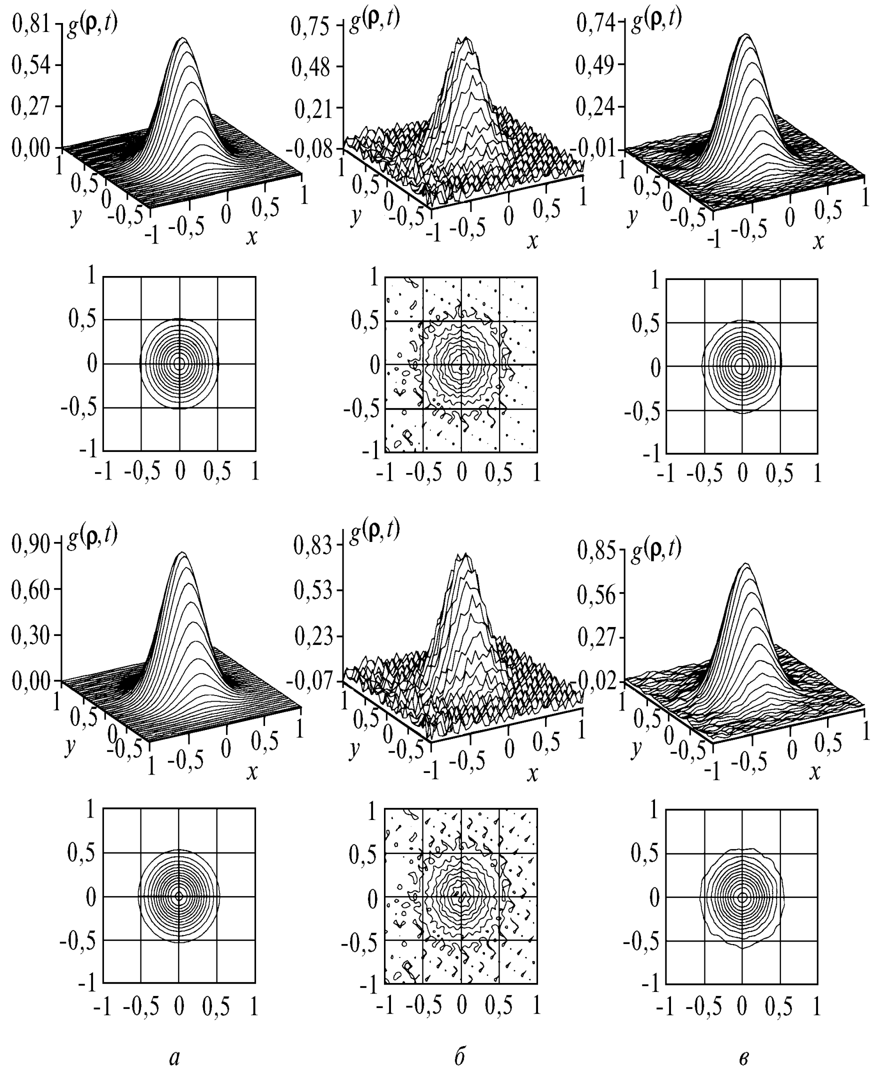

FIG. 2. Reconstruction of the heat flux (W/cm²) *by Eqs. (8) and (10) for a cooled target: a) initial model distribution, b) reconstructed heat flux, c) smoothed solution.*

The characteristic dependence of the error in the solution on the step size is shown in Fig. 3. The actual inherent error depends irregularly on the step size and randomly oscillates (see the dashed line in Fig. 3) within the limits determined by the majorant (the solid line). It is seen from the plot that regularization is efficient when the sampling step size increases, but in this case the resolution of algorithm decreases (the quality of reconstruction of fine structures deteriorates). So let us keep constant the sampling step size and decrease the effect of random errors δ_r. To do this, we must perform filtration of initial data. A simple and effective filter that ensures satisfactory accuracy is the convolution of initial data with a certain stabilizing function.[7,8]

We have chosen sinc(x) as a stabilizing multiplier. It is a function of the following form:

$$\mathrm{sinc}(x) = \sin(\kappa_{\max} x)/\pi x . \qquad (11)$$

This multiplier imposes limitation on the spectral width and cuts off all unnecessary high–frequency noise components. The parameter $\kappa_{\max}$ in Eq. (11) is the cutoff spatial frequency chosen according to the Kotel'nikov sampling theorem[7,9]

$$h = \pi/\kappa_{\max} .$$

Assuming that the spatial resolution of the algorithm is limited by instrumental resolution, we find the parameter $\kappa_{\max}$. Obviously, it is determined by the inequality

$$\pi/2\,h < \kappa_{\max} < \pi/h .$$

In our case, $h = 0.05$ and $\kappa_{\max} \approx 30$. The convolution and differentiation operators commute due to their linearity, so one can realize the smoothing procedure at the last stage of the algorithm. The result of convolution of the solution obtained with function (11) is shown in Figs. 1*c* and 2*c*. The maximum reconstruction error does not exceed 10%. In our opinion, these results are satisfactory. The presence of artefacts on the edges of the reconstructed flux is caused by the finite transverse target size and can be eliminated by way of increasing the target size.

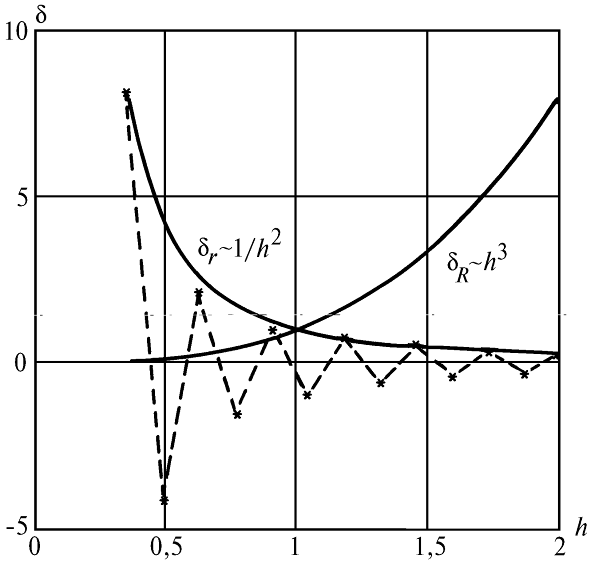

FIG. 3. Solution error as a function of the step size: δ_R is the error in approximation of the function; δ_r is the estimated majorant of the inherent random error connected with the error in measuring the function; dashed curve is for the inherent error randomly oscillating within the limits determined by the majorant.

In this paper, we have described the algorithm for reconstruction of the intensity from the temperature field of a thin target and have analyzed the reasons for the solution instability and the methods of their elimination. This algorithm is part of the software complex for experimental data processing, i.e., the laser beam intensity reconstruction from the temperature field of the heated surface of a target with arbitrary parameters.

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